

Generalized multi Galileons, covariantized new terms, and the no-go theorem for nonsingular cosmologies

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Introduction

Inflation

solve some problems of big-bang cosmology

- flatness, horizon

- consistent with observation

- CMB

cannot solve initial singularity

Galilean Genesis, Bounce

Creminelli et al. (2010)

Qiu et al. (2011)

~~initial singularity~~ $\dot{H} > 0$

NEC violation and stability conditions

perturbations around a Flat Friedmann sp.

$$S_s^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2} (\vec{\nabla} \zeta)^2 \right]$$

$\mathcal{G}_s > 0$: ~~ghost~~ instability

$\mathcal{F}_s > 0$: ~~gradient~~ instability

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + V(\phi) \quad (= G_2(\phi, X)) , \quad X = -\frac{1}{2}(\partial\phi)^2$$

$$\mathcal{F}_s = -M_{\text{pl}}^2 \frac{\dot{H}}{H^2} \quad \text{Violation of NEC } \dot{H} > 0 \rightarrow \mathcal{F}_s < 0$$

NEC violation and stability conditions

perturbations around a Flat Friedmann sp.

$$S_s^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[\mathcal{G}_s \dot{\zeta}^2 - \frac{\mathcal{F}_s}{a^2} (\vec{\nabla} \zeta)^2 \right]$$

$\mathcal{G}_s > 0$: ~~ghost~~ instability

$\mathcal{F}_s > 0$: ~~gradient~~ instability

$$\mathcal{L}_\phi = G_2(\phi, X) - G_3(\phi, X) \square \phi$$

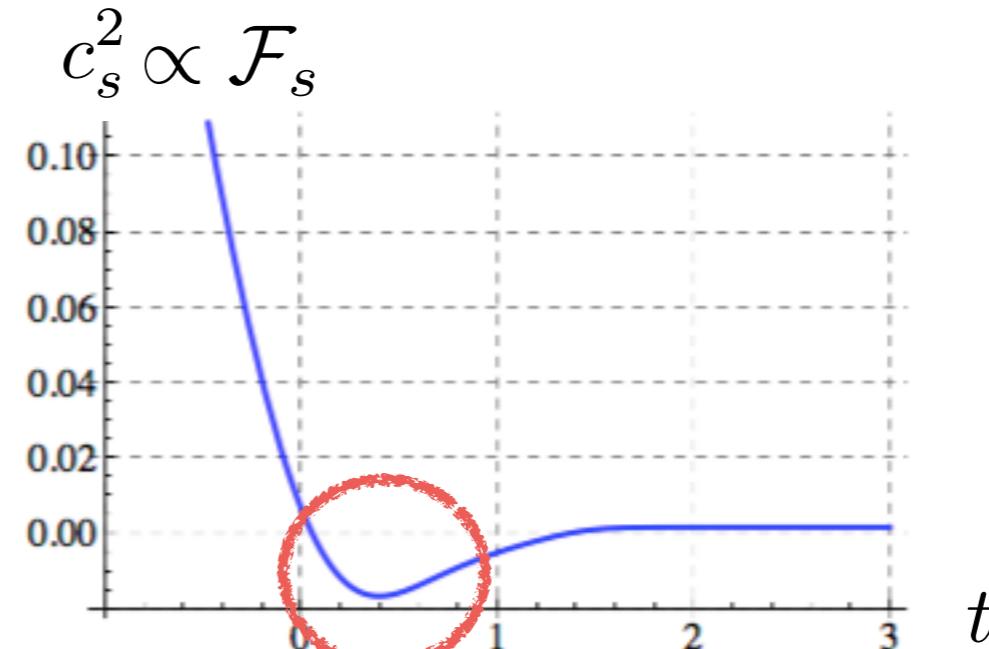
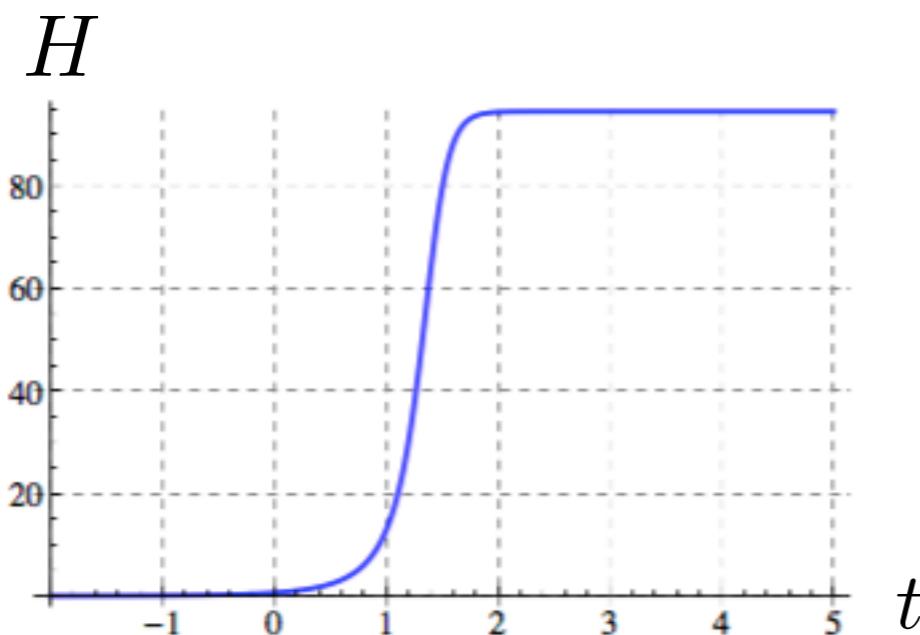
$$\mathcal{F}_s = \frac{1}{a} \frac{d}{dt} \left(\frac{a \mathcal{G}_T^2}{\Theta} \right) - \mathcal{F}_T \quad \begin{aligned} \Theta &= -\dot{\phi} X G_{3X} + H M_{\text{pl}}^2 \\ \mathcal{G}_T &= \mathcal{F}_T = M_{\text{pl}}^2 \end{aligned}$$

Violation of NEC $\dot{H} > 0 \cancel{\rightarrow} \mathcal{F}_s < 0$

Inflation from Minkowski sp.

D. Pirtskhalava et al. (2014)

$$\mathcal{L} = \frac{1}{2} M_{\text{pl}}^2 R + f^2 \frac{e^{2\pi}}{1 + \beta e^{2\pi}} (\partial\pi)^2 + \frac{f^2}{\Lambda^3} (\partial\pi)^2 \square\pi + \frac{f^3}{2\Lambda^3} (\partial\pi)^4$$



Stable NEC violation does not indicate stable at all time.

→ model dependent or independent?

No-go theorem in single field theories

$$\mathcal{L}_\phi = G_2(\phi, X) - G_3(\phi, X)\square\phi$$

Instabilities of **nonsingular solutions**

$$a > 0 \text{ and } H, \dot{H}, \dots < \infty$$

are model independent.

M. Libanov et al. (2016)

→ Extension to Horndeski theory T. Kobayashi (2016)
G.W. Horndeski (1974)

– most general scalar tensor theory
having second-order field equations

$$\mathcal{L}_\phi = G_2(\phi, X) - G_3(\phi, X)\square\phi + \dots$$

model space

No-go theorem

- 2nd-order eom
- flat Friedmann sp.
- single scalar field



There is a bounce model which gradient instabilities occurs at normal phase which NEC is not violated.

A. Easson et al.(2011)

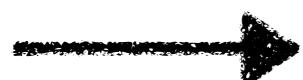
There are other fields such as perfect fluid.

model space

No-go theorem

- 2nd-order eom
- flat Friedmann sp.
- single scalar field

**Dominant field transfers its energy
to other fields.**



gradient ~~instability~~ ...?

multi scalar tensor theories

Generalized multi Galileons Padilla et al. (2013)

- general multi-scalar tensor theory having second-order field equations

$$\begin{aligned}\mathcal{L} = & G_2(X^{IJ}, \phi^K) - G_{3L}(X^{IJ}, \phi^K) \square \phi^L + G_4(X^{IJ}, \phi^K) R \\ & + G_{4,\langle IJ \rangle} (\square \phi^I \square \phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla_\mu \nabla_\nu \phi^J) \\ & + G_{5L}(X^{IJ}, \phi^K) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi^L \\ & - \frac{1}{6} G_{5I,\langle JK \rangle} (\square \phi^I \square \phi^J \square \phi^K - 3 \square \phi^{(I} \nabla_\mu \nabla_\nu \phi^{J)} \nabla^\mu \nabla^\nu \phi^K) \\ & + 2 \nabla_\mu \nabla_\nu \phi^I \nabla^\nu \nabla^\lambda \phi^J \nabla_\lambda \nabla^\mu \phi^K\end{aligned}$$
$$X^{IJ} = -\frac{1}{2} \nabla_\mu \phi^I \nabla^\mu \phi^J$$

This is not the most general multi-scalar tensor theory having second-order field equations unlike the Horndeski theory.

multi scalar tensor theories

Generalized multi Galileons Padilla et al. (2013)

- general multi-scalar tensor theory having second-order field equations

$$C_{\mu\nu} - C_{\mu\nu}(x^{IJ}\phi^K) - C_{\mu\nu}(x^{IJ}\phi^K)\square_\nu L + C_{\mu\nu}(x^{IJ}\phi^K)D$$

All terms of Generalized multi galileons and new terms

→ **most general multi-scalar tensor theory
having second-order field equations**

New terms

at flat sp. → Covariantization

Allys (2016)

Akama,Kobayashi (2017)

multi scalar tensor theories

Generalized multi Galileons Padilla et al. (2013)

- general multi-scalar tensor theory having second-order field equations

$$\begin{aligned}\mathcal{L} = & G_2(X^{IJ}, \phi^K) - G_{3L}(X^{IJ}, \phi^K) \square \phi^L + G_4(X^{IJ}, \phi^K) R \\ & + G_{4,\langle IJ\rangle} (\square \phi^I \square \phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla_\mu \nabla_\nu \phi^J) \\ & + G_{5L}(X^{IJ}, \phi^K) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi^L \\ & - \frac{1}{6} G_{5I,\langle JK\rangle} (\square \phi^I \square \phi^J \square \phi^K - 3 \square \phi^{(I} \nabla_\mu \nabla_\nu \phi^{J)} \nabla^\mu \nabla^\nu \phi^K) \\ & + 2 \nabla_\mu \nabla_\nu \phi^I \nabla^\nu \nabla^\lambda \phi^J \nabla_\lambda \nabla^\mu \phi^K\end{aligned}$$
$$X^{IJ} = -\frac{1}{2} \nabla_\mu \phi^I \nabla^\mu \phi^J$$

New terms

at flat sp. \longrightarrow Covariantization

Allys (2016)

Akama,Kobayashi (2017)

Perturbations around a Flat Friedmann sp.

$$S_h^{(2)} = \frac{1}{8} \int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2} (\vec{\nabla} h_{ij})^2 \right]$$

$$\mathcal{G}_T = 2 \left[G_4 - 2X^{IJ}G_{4IJ} + \dots \right] > 0$$

$$\mathcal{F}_T = 2 \left[G_4 - X^{IJ}(\ddot{\phi}^K G_{5IJK} + G_{5I,J}) \right] > 0$$

$$S_Q^{(2)} = \frac{1}{2} \int dt d^3x a^3 \left[\mathcal{K}_{IJ} \dot{Q}^I \dot{Q}^J - \frac{1}{a^2} \mathcal{D}_{IJ} \vec{\nabla} Q^I \cdot \vec{\nabla} Q^J + \dots \right]$$
$$\phi^I(t, \vec{x}) = \phi^I(t) + Q^I(t, \vec{x})$$

$\mathcal{K}_{IJ}, \mathcal{D}_{IJ}$ are positive definite.

**focus on only this condition
to discuss gradient instability**

Positive definite of \mathcal{D}_{IJ}

$$\mathbf{v}^T \mathcal{D} \mathbf{v} > 0 , \quad \mathbf{v} : \text{arbitrary and } \mathbf{v} \neq 0$$

$$\mathbf{v} = \begin{pmatrix} \dot{\phi}^1 \\ \dot{\phi}^2 \\ \vdots \\ \vdots \\ \dot{\phi}^N \end{pmatrix} , \quad \mathbf{v}^T \mathcal{D} \mathbf{v} > 0 \leftrightarrow 2D_{IJ}X^{IJ} > 0$$

$$\mathcal{D}_{IJ} = \mathcal{C}_{IJ} - \frac{\mathcal{J}_{(I}\mathcal{B}_{J)}}{\Theta} + \frac{1}{a} \frac{d}{dt} \left(\frac{a\mathcal{B}_I\mathcal{B}_J}{2\Theta} \right)$$

$$\mathcal{C}_{IJ}X^{IJ} = 2H(\dot{\mathcal{G}}_T + H\mathcal{G}_T) - \dot{\Theta} - H\Theta - H^2\mathcal{F}_T$$

$$\dot{\phi}^I \mathcal{J}_I + \ddot{\phi}^I \mathcal{B}_I + 2\dot{H}\mathcal{G}_T = 0$$

⋮
⋮

Positive definite of \mathcal{D}_{IJ}

$$\mathbf{v}^T \mathcal{D} \mathbf{v} > 0 , \quad \mathbf{v} : \text{arbitrary and } \mathbf{v} \neq 0$$

$$\mathbf{v} = \begin{pmatrix} \dot{\phi}^1 \\ \dot{\phi}^2 \\ \vdots \\ \vdots \\ \dot{\phi}^N \end{pmatrix} , \quad \mathbf{v}^T \mathcal{D} \mathbf{v} > 0 \Leftrightarrow 2D_{IJ}X^{IJ} > 0$$

→ $2D_{IJ}X^{IJ} = 2H^2 \left(\frac{1}{a} \frac{d\xi}{dt} - \mathcal{F}_T \right) > 0$

$$\xi = \frac{a\mathcal{G}_T^2}{\Theta}, \Theta = -\dot{\phi}X^{JK}G_{3IJK} + 2HG_4 + \dots$$

→ $\frac{d\xi}{dt} > a\mathcal{F}_T \underline{> 0}$

from nonsingular and stability condition

$$a > 0$$

$$\mathcal{F}_T > 0$$

Positive definite of \mathcal{D}_{IJ}

$$\rightarrow \frac{d\xi}{dt} > a\mathcal{F}_T > 0$$

\rightarrow same discussion with the case of the Horndeski theory

Non-singular and stable solutions

$$\rightarrow \int_{-\infty}^{\infty} a\mathcal{F}_T dt < \infty \quad \text{or} \quad \int_{-\infty}^{\infty} a\mathcal{F}_T dt < \infty$$

“geodesically incomplete for graviton”

Y. Cai et al. (2016) P. Creminelli et al. (2016)

multi scalar tensor theories

Generalized multi Galileons Padilla et al. (2013)

- general multi-scalar tensor theory having second-order field equations

$$\begin{aligned}\mathcal{L} = & G_2(X^{IJ}, \phi^K) - G_{3L}(X^{IJ}, \phi^K) \square \phi^L + G_4(X^{IJ}, \phi^K) R \\ & + G_{4,\langle IJ \rangle} (\square \phi^I \square \phi^J - \nabla_\mu \nabla_\nu \phi^I \nabla_\mu \nabla_\nu \phi^J) \\ & + G_{5L}(X^{IJ}, \phi^K) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi^L \\ & - \frac{1}{6} G_{5I,\langle JK \rangle} (\square \phi^I \square \phi^J \square \phi^K - 3 \square \phi^{(I} \nabla_\mu \nabla_\nu \phi^{J)} \nabla^\mu \nabla^\nu \phi^K) \\ & + 2 \nabla_\mu \nabla_\nu \phi^I \nabla^\nu \nabla^\lambda \phi^J \nabla_\lambda \nabla^\mu \phi^K\end{aligned}$$
$$X^{IJ} = -\frac{1}{2} \nabla_\mu \phi^I \nabla^\mu \phi^J$$

New terms

at flat sp. \longrightarrow Covariantization

Allys (2016)

Akama,Kobayashi (2017)

New terms

Allys (2016)

$$\mathcal{L}_{\text{ext}} = \mathcal{L}_{\text{ext1}} + \mathcal{L}_{\text{ext2}} + \mathcal{L}_{\text{ext3}}$$

$$\mathcal{L}_{\text{ext1}} = A_{[IJ][KL]M} \delta_{\nu_1 \nu_2 \nu_3}^{\mu_1 \mu_2 \mu_3} \partial_{\mu_1} \phi^I \partial_{\mu_2} \phi^J \partial^{\nu_1} \phi^K \partial^{\nu_2} \phi^L \partial_{\mu_3} \partial^{\nu_3} \phi^M$$

$$\begin{aligned} \mathcal{L}_{\text{ext2}} = & A_{[IJ][KL](MN)} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} \phi^I \partial_{\mu_2} \phi^J \\ & \times \partial^{\nu_1} \phi^K \partial^{\nu_2} \phi^L \partial_{\mu_3} \partial^{\nu_3} \phi^M \partial_{\mu_4} \partial^{\nu_4} \phi^N \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{ext3}} = & A_{[IJK][LMN]O} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \partial_{\mu_1} \phi^I \partial_{\mu_2} \phi^J \partial_{\mu_3} \phi^K \\ & \times \partial^{\nu_1} \phi^L \partial^{\nu_2} \phi^M \partial^{\nu_3} \phi^N \partial_{\mu_4} \partial^{\nu_4} \phi^O \end{aligned}$$

These terms **do not exist**
in the single field theory(galileon theory).

Covariantization of new terms

Akama,Kobayashi (2017)

1. $\partial_\mu \rightarrow \nabla_\mu$

2. 2nd-order eom

$$\mathcal{L}_{\text{ext}} = \mathcal{L}_{\text{ext1}} + \mathcal{L}_{\text{ext2}} + \mathcal{L}_{\text{ext3}}$$

$$\begin{aligned}\mathcal{L}_{\text{ext1}} = & A_{[IJ][KL]M} \delta_{\nu_1 \nu_2 \nu_3}^{\mu_1 \mu_2 \mu_3} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \nabla^{\nu_1} \phi^K \nabla^{\nu_2} \phi^L \\ & \times \nabla_{\mu_3} \nabla^{\nu_3} \phi^M\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\text{ext2}} = & A_{[IJ][KL](MN)} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \\ & \times \nabla^{\nu_1} \phi^K \nabla^{\nu_2} \phi^L \nabla_{\mu_3} \nabla^{\nu_3} \phi^M \nabla_{\mu_4} \nabla^{\nu_4} \phi^N\end{aligned}$$

→ higher derivative

$$\begin{aligned}\mathcal{L}_{\text{ext3}} = & A_{[IJK][LMN]O} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \nabla_{\mu_3} \phi^K \\ & \times \nabla^{\nu_1} \phi^L \nabla^{\nu_2} \phi^M \nabla^{\nu_3} \phi^N \nabla_{\mu_4} \nabla^{\nu_4} \phi^M\end{aligned}$$

Covariantization of new terms

Akama,Kobayashi (2017)

$$\mathcal{L}_{\text{ext}} = \mathcal{L}_{\text{ext1}} + \mathcal{L}_{\text{ext2}} + \mathcal{L}_{\text{ext3}}$$

$$\begin{aligned} \mathcal{L}_{\text{ext1}} = & A_{[IJ][KL]M} \delta_{\nu_1 \nu_2 \nu_3}^{\mu_1 \mu_2 \mu_3} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \nabla^{\nu_1} \phi^K \nabla^{\nu_2} \phi^L \\ & \times \nabla_{\mu_3} \nabla^{\nu_3} \phi^M \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{ext2}} = & A_{[IJ][KL](MN)} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \\ & \times \nabla^{\nu_1} \phi^K \nabla^{\nu_2} \phi^L \nabla_{\mu_3} \nabla^{\nu_3} \phi^M \nabla_{\mu_4} \nabla^{\nu_4} \phi^N \end{aligned}$$

$$+ B_{[IJ][KL]} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} R^{\nu_3 \nu_4}_{\mu_3 \mu_4} \quad \text{where} \\ \times \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \nabla^{\nu_1} \phi^K \nabla^{\nu_2} \phi^L \quad B_{[IJ][KL],(MN)} = \frac{1}{2} A_{[IJ][KL](MN)}$$

$$\begin{aligned} \mathcal{L}_{\text{ext3}} = & A_{[IJK][LMN]O} \delta_{\nu_1 \nu_2 \nu_3 \nu_4}^{\mu_1 \mu_2 \mu_3 \mu_4} \nabla_{\mu_1} \phi^I \nabla_{\mu_2} \phi^J \nabla_{\mu_3} \phi^K \\ & \times \nabla^{\nu_1} \phi^L \nabla^{\nu_2} \phi^M \nabla^{\nu_3} \phi^N \nabla_{\mu_4} \nabla^{\nu_4} \phi^M \end{aligned}$$

Effect on scalar perturbations

$$S^{(2)} = \frac{1}{2} \int dt d^3x \left(\dots - \frac{1}{a^2} \mathcal{C}_{IJ} \partial_i Q^I \partial^i Q^J + \dots \right)$$

$$\mathcal{C}_{IJ} \rightarrow \mathcal{C}_{IJ} + \mathcal{C}_{IJ}^{\text{ext}}$$

$$\begin{aligned} \mathcal{C}_{IJ}^{\text{ext}} = & 32H(-A_{[IK][JL]M}X^{KL}\dot{\phi}^M + 2HB_{[IK][JL]}X^{KL} \\ & + 4HB_{[IK][JL],}X^{KL}X^{MN}) \end{aligned}$$

No-go argument in generalized multi-galileons

$$2D_{IJ}X^{IJ} = 2H^2 \left(\frac{1}{a} \frac{d\xi}{dt} - \mathcal{F}_T \right) > 0$$

$$\longrightarrow \frac{d\xi}{dt} > a\mathcal{F}_T > 0$$

Effect on scalar perturbations

$$S^{(2)} = \frac{1}{2} \int dt d^3x \left(\dots - \frac{1}{a^2} \mathcal{C}_{IJ} \partial_i Q^I \partial^i Q^J + \dots \right)$$

$$\mathcal{C}_{IJ} \rightarrow \mathcal{C}_{IJ} + \mathcal{C}_{IJ}^{\text{ext}}$$

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$$\mathcal{D}_{IJ} = \mathcal{C}_{IJ} + \dots, \quad \mathcal{C}_{IJ}^{\text{ext}} X^{IJ} = 0$$

→ **do not change the no-go argument**

$$(\mathcal{D}_{IJ} X^{IJ} \rightarrow \mathcal{D}_{IJ} X^{IJ})$$

Summary

No-go theorem in multi scalar theories

■ Generalized multi Galileons

Non-singular cosmological solutions

→ **unstable** unless $\int_{-\infty}^{\infty} a\mathcal{F}_T dt < \infty$ is satisfied
geodesically incomplete for graviton

■ New terms

do not affect the proof.

The number of scalar fields is not related with gradient instability.

Summary

model space

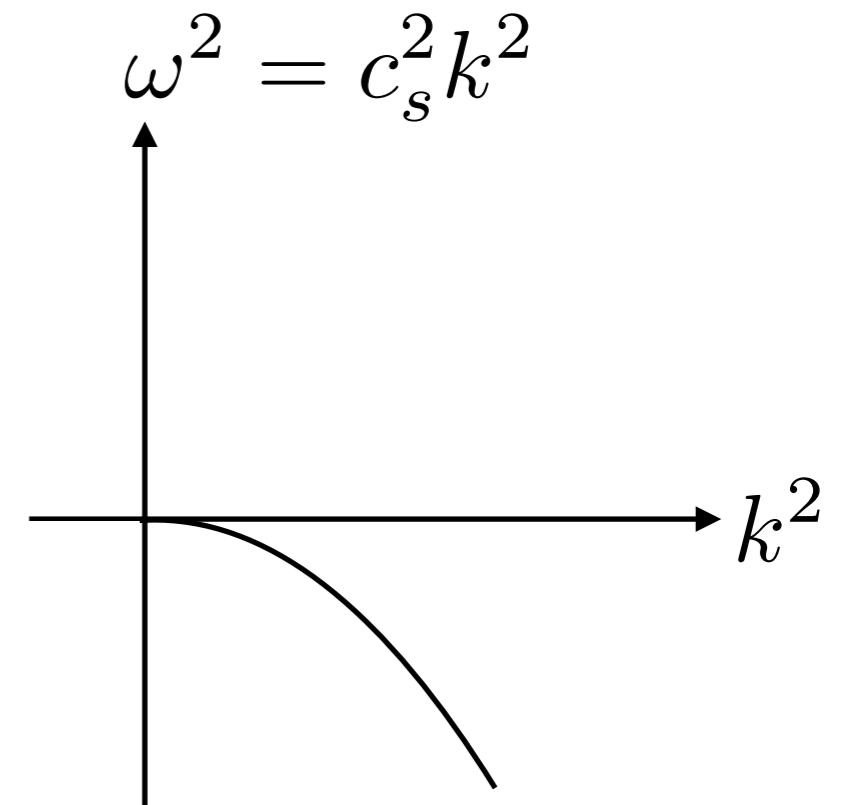
No-go theorem

- 2nd-order eom
- flat Friedmann sp.
- spatial higher derivative
(Misonoh et al.(2017))

Horndeski theory

$$\omega^2 = c_s^2 k^2, \quad c_s^2 < 0$$

High frequency mode diverges.
(gradient instability)

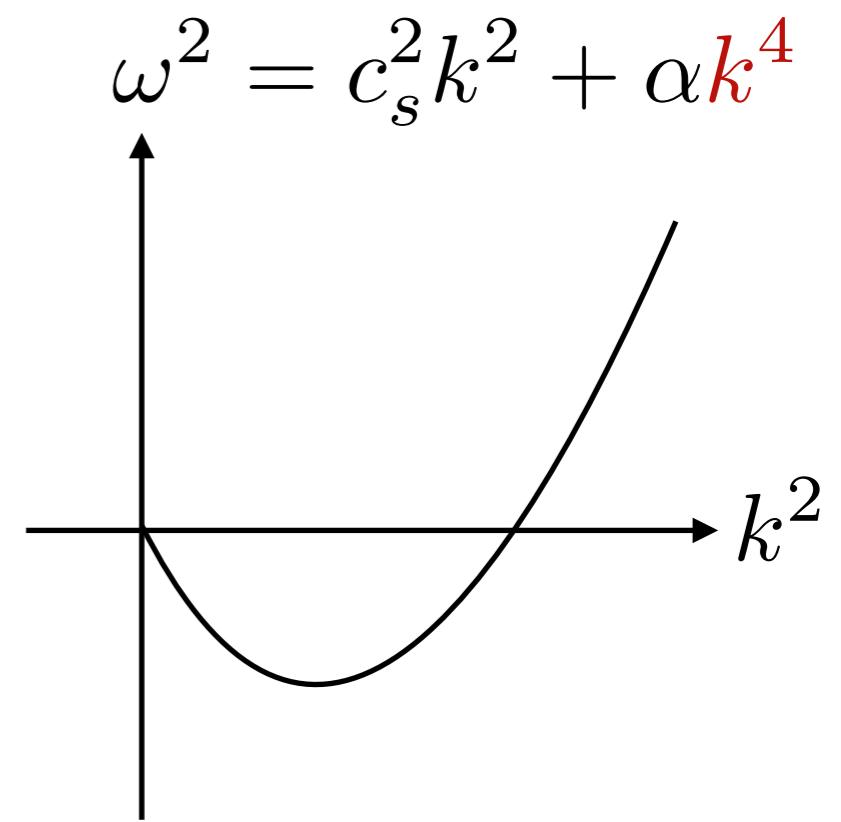


Beyond Horndeski theory

$$\omega^2 = c_s^2 k^2 + \alpha k^4 + \dots,$$

$$c_s^2 < 0, \quad \alpha > 0$$

**High frequency mode
is stabilized.**



Summary

model space

No-go theorem

- 2nd-order eom
- flat Friedmann sp.

- spatial higher derivative
(Misonoh et al.(2017))
 - non-flat Friedmann sp.
(in preparation)
- :

Summary

model space

No-go theorem

- 2nd-order eom
- flat Friedmann sp.
- open Friedmann sp.

- spatial higher derivative
(Misonoh et al.(2017))

- closed Friedmann sp.

⋮