Gravitational waves from a new parametrization of inflation

- 1605.04871
- 1705.02712
- + in prep

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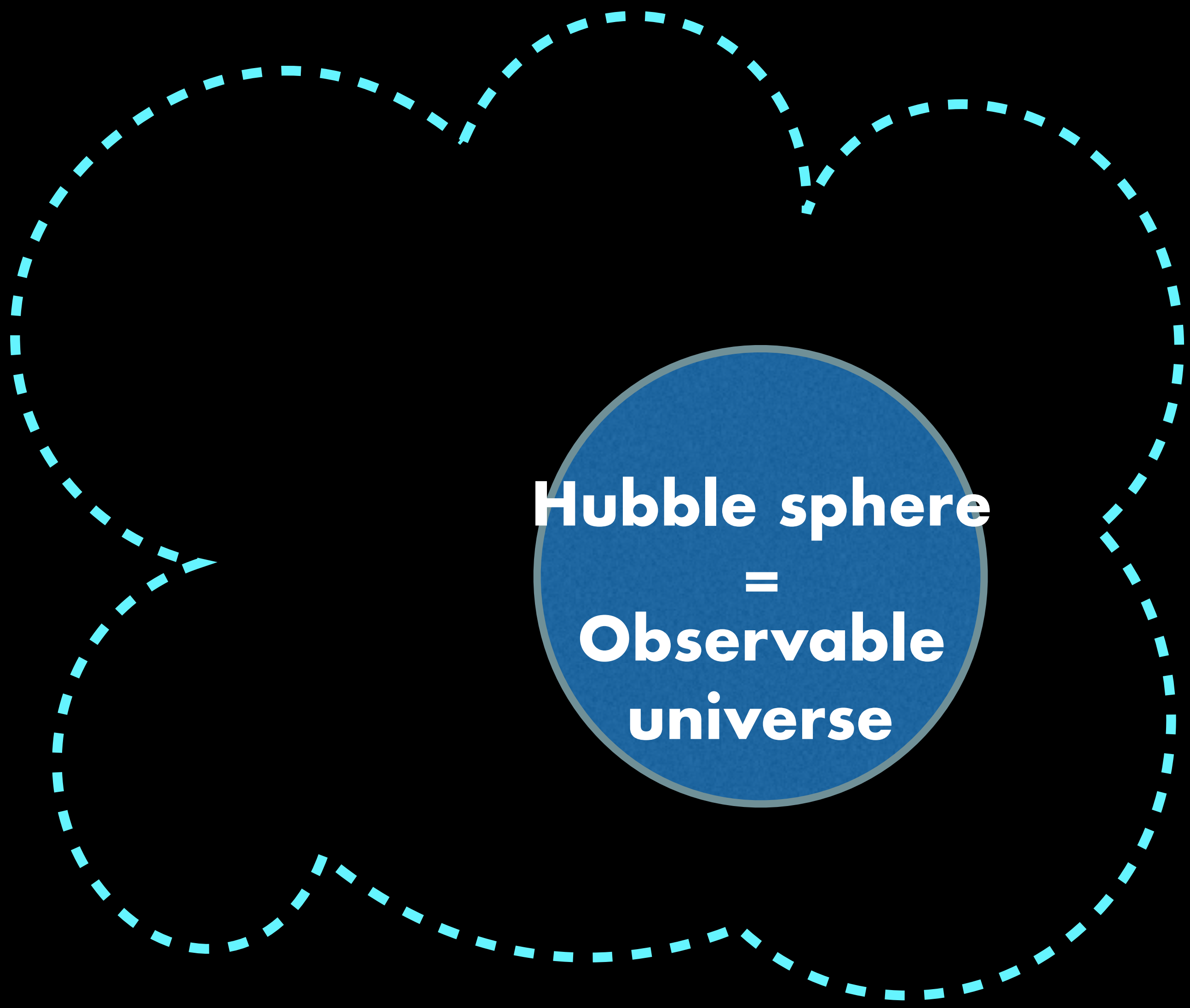
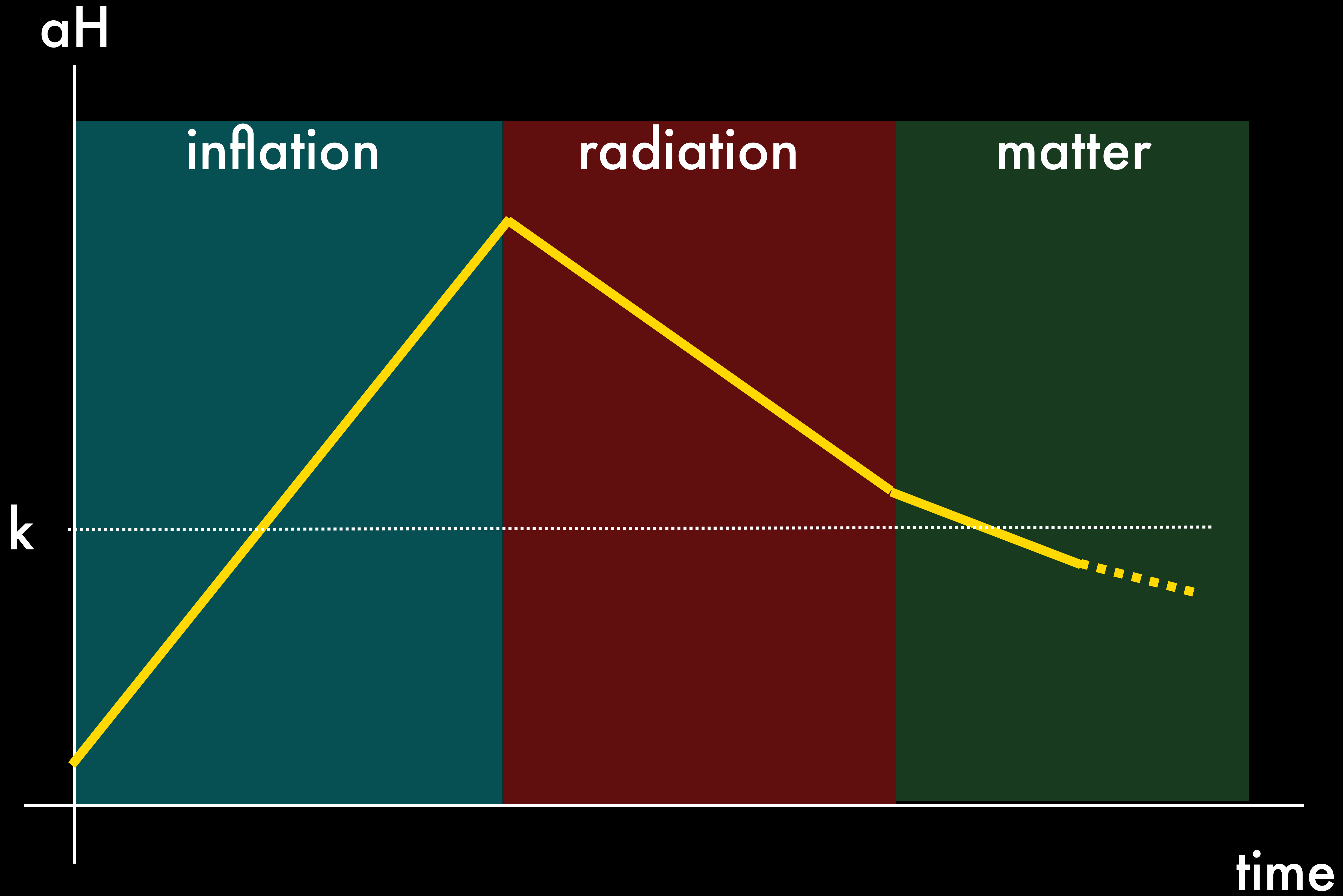
$$a(t) = \text{scale factor}$$

$$H(t) = \text{Hubble parameter} = \frac{\dot{a}}{a}$$

$$c(aH)^{-1} = \text{Hubble radius}$$




aH
inflation
radiation
matter
k
time

Slow-roll approximation

Friedmann equation

$$H^2 = \frac{1}{3}\left(V(\phi) + \frac{\dot{\phi}^2}{2}\right)$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2$$

"Hubble slow roll"

$$\epsilon \equiv 2\left(\frac{H'}{H}\right)^2$$

$$\epsilon \ll 1$$

Slow-roll approximation: $H \simeq$ constant

How much inflation? "e-fold"

The common definition

$$N \equiv \ln \frac{a_{\rm end}}{a_{\rm initial}}$$

The physical definition

$$\tilde{N} \equiv \ln \frac{(aH)^{-1}_{\rm initial}}{(aH)^{-1}_{\rm end}}$$

- Need this number to be around **60** to solve horizon problem
- Ñ<N, but roughly equal in slow-roll models

Phenomenological parametrization

$$\mathscr{H}(\phi) = aH$$

Inflation is parametrized by the (inverse) Hubble radius.

$$\mathscr{H}(\phi) \quad \Longrightarrow \quad E_1 = \mathscr{H}'/\mathscr{H} \quad \Longrightarrow \quad \sqrt{\epsilon} = \frac{\sqrt{E_1^2+2} - E_1}{\sqrt{2}}$$

$$\Downarrow$$

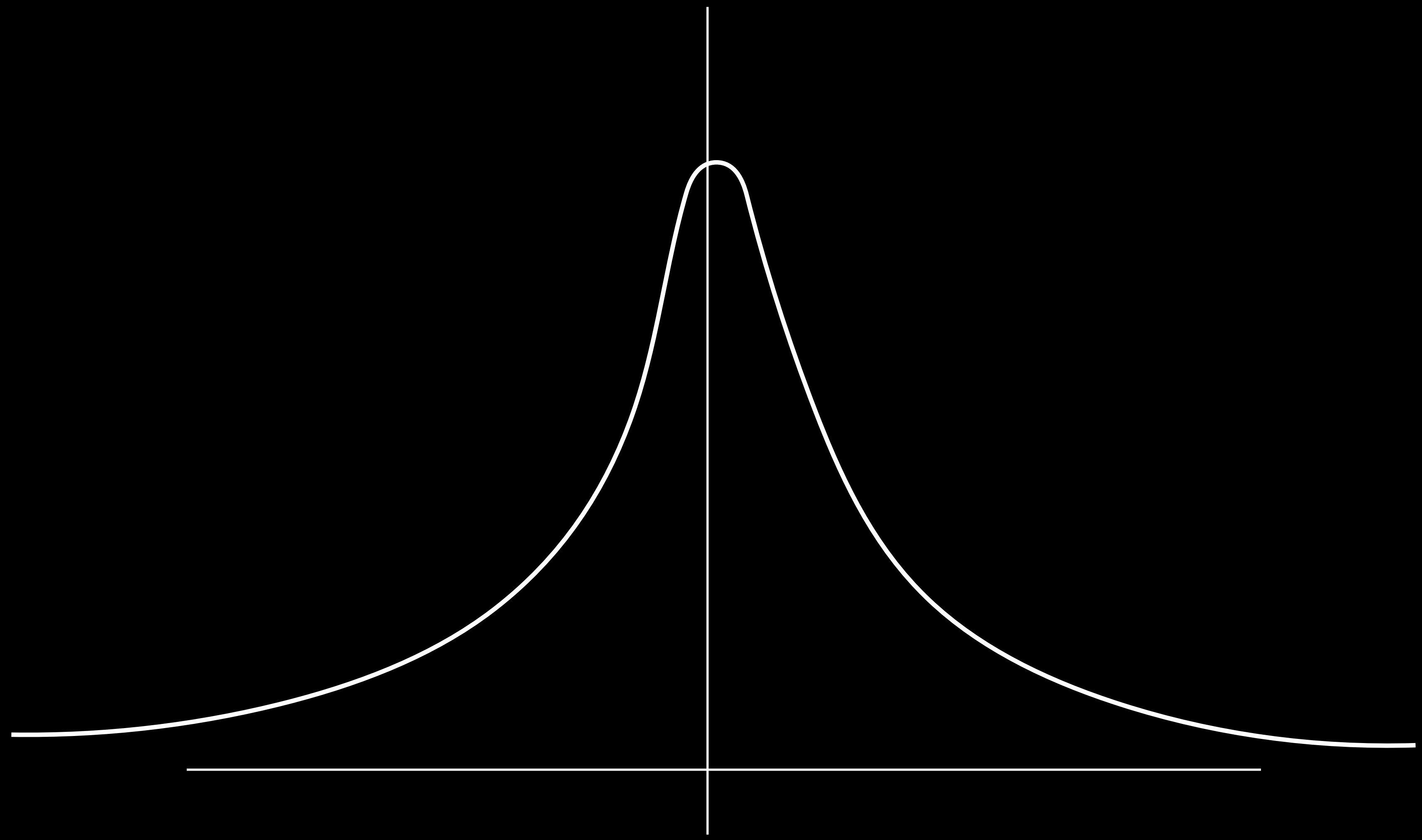
$$V(\phi) = H^2(3-\epsilon) \quad \Longleftarrow \quad H(\phi) = H_{\rm end} \exp\left(-\int_{\phi_{\rm end}}^{\phi} \sqrt{\epsilon/2}\, \mathrm{d}\phi\right)$$

Choose any increasing function $\mathscr{H}(\phi)$

An interesting model

$$\mathscr{H}(\phi) \propto e^{-(\alpha\phi)^n}$$

"Generalised Gaussian"



An interesting model

$$\mathscr{H}(\phi) \propto e^{-(\alpha\phi)^n}$$

"Generalised Gaussian"

When n=2

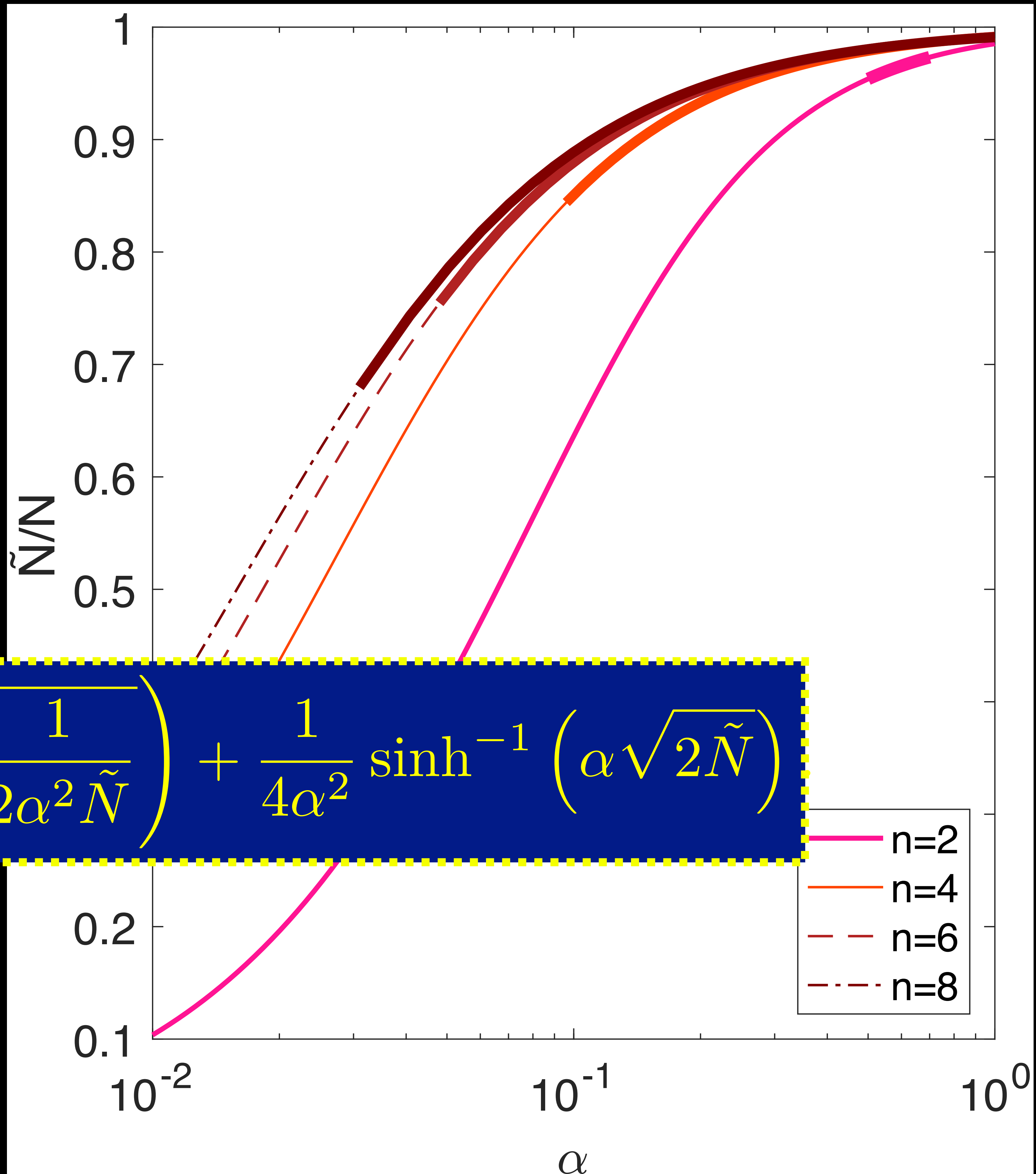
$$\mathscr{H}(\phi) \propto e^{-(\alpha\phi)^2} \quad \Leftrightarrow \quad V(\phi) \propto \phi^k, \quad k = \frac{1}{2\alpha^2}$$

For all power-law potentials,
the (Hubble radius)$^{-1}$ evolves like the Gaussian

[SC'17]

N vs Ñ

$$\mathscr{H}(\phi) \propto e^{-(\alpha\phi)^n}$$

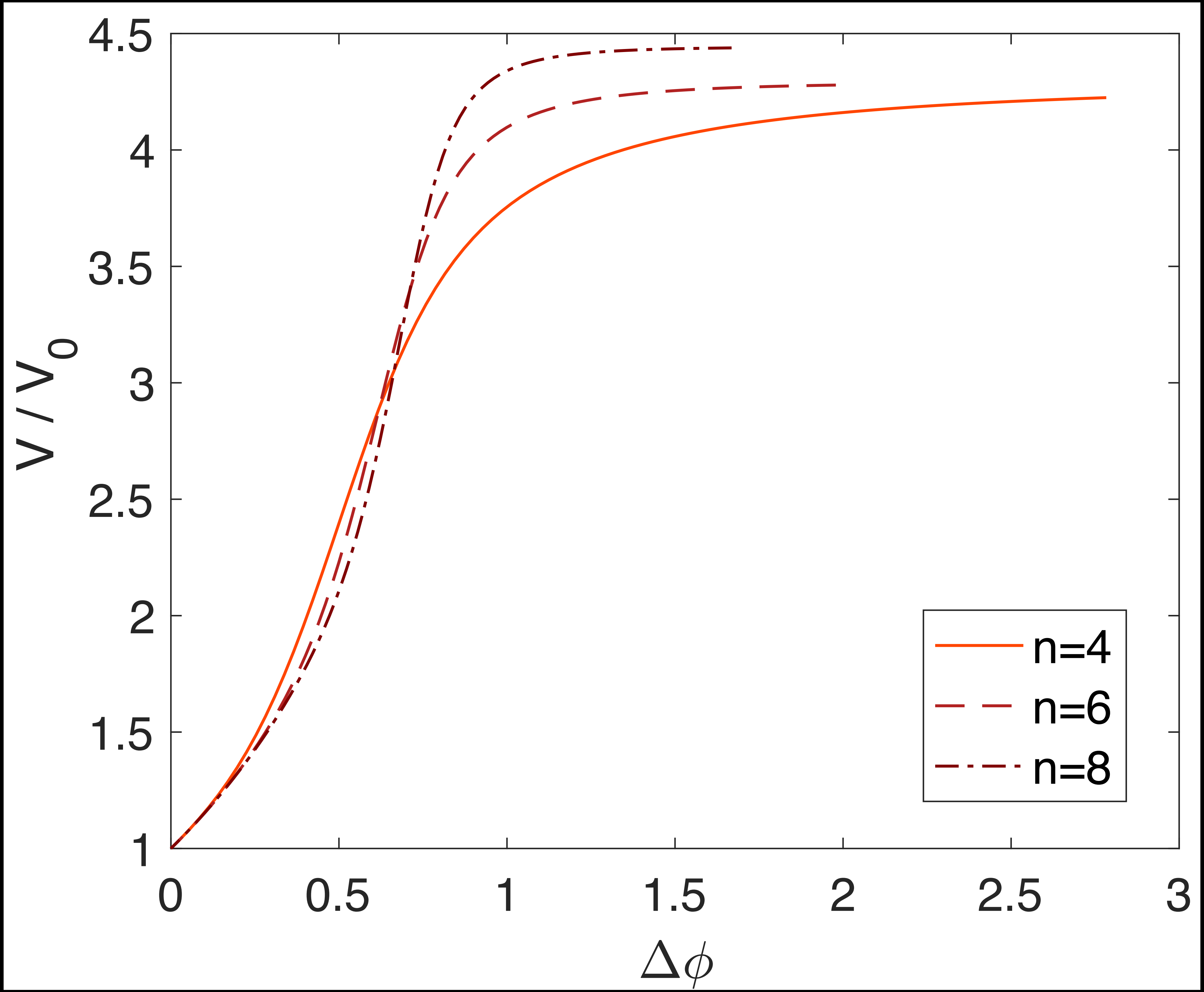


For n=2

$$N = \frac{\tilde{N}}{2}\left(1 + \sqrt{1 + \frac{1}{2\alpha^2\tilde{N}}}\right) + \frac{1}{4\alpha^2}\sinh^{-1}\left(\alpha\sqrt{2\tilde{N}}\right)$$

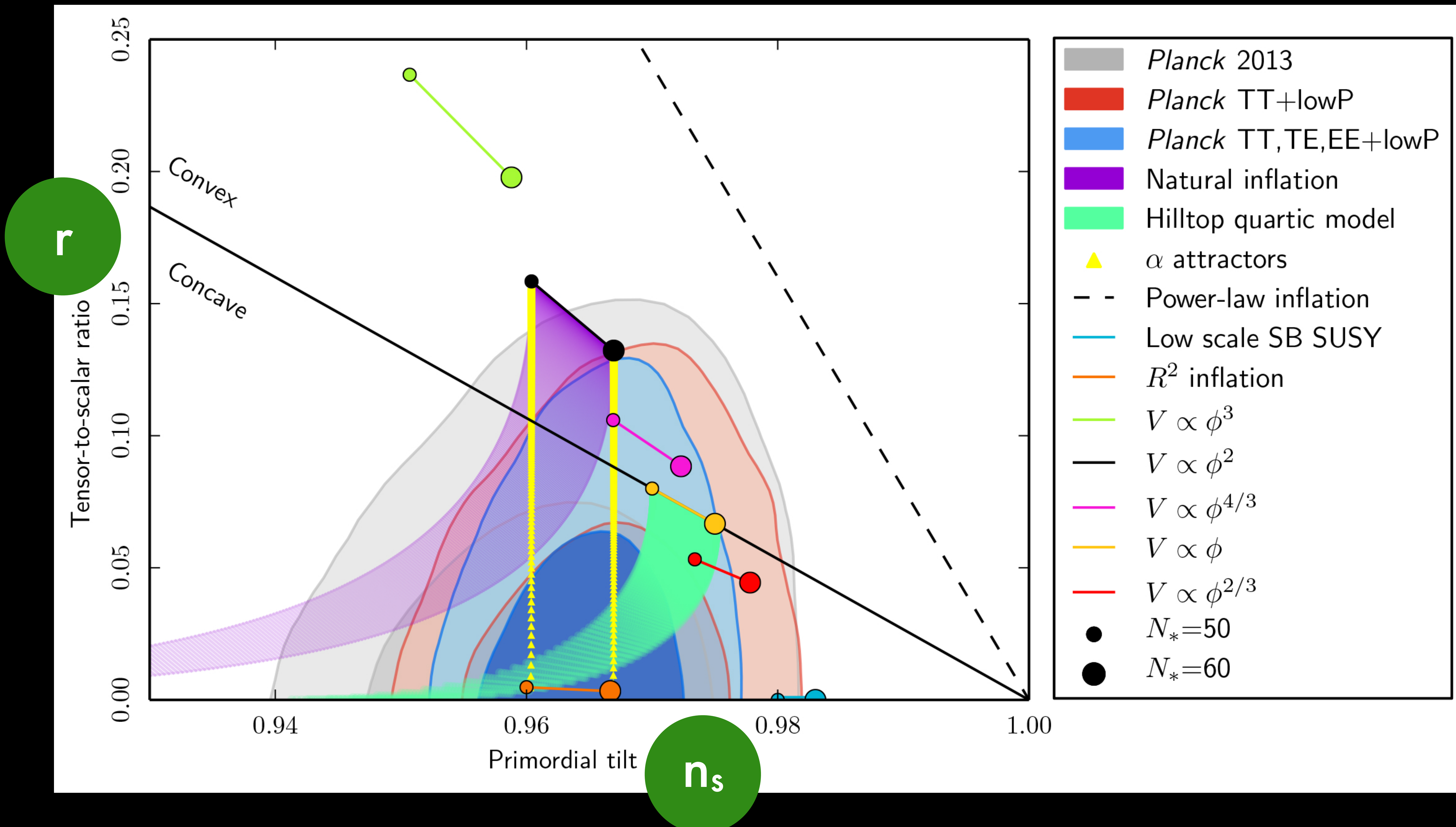
[SC'17]

The potential

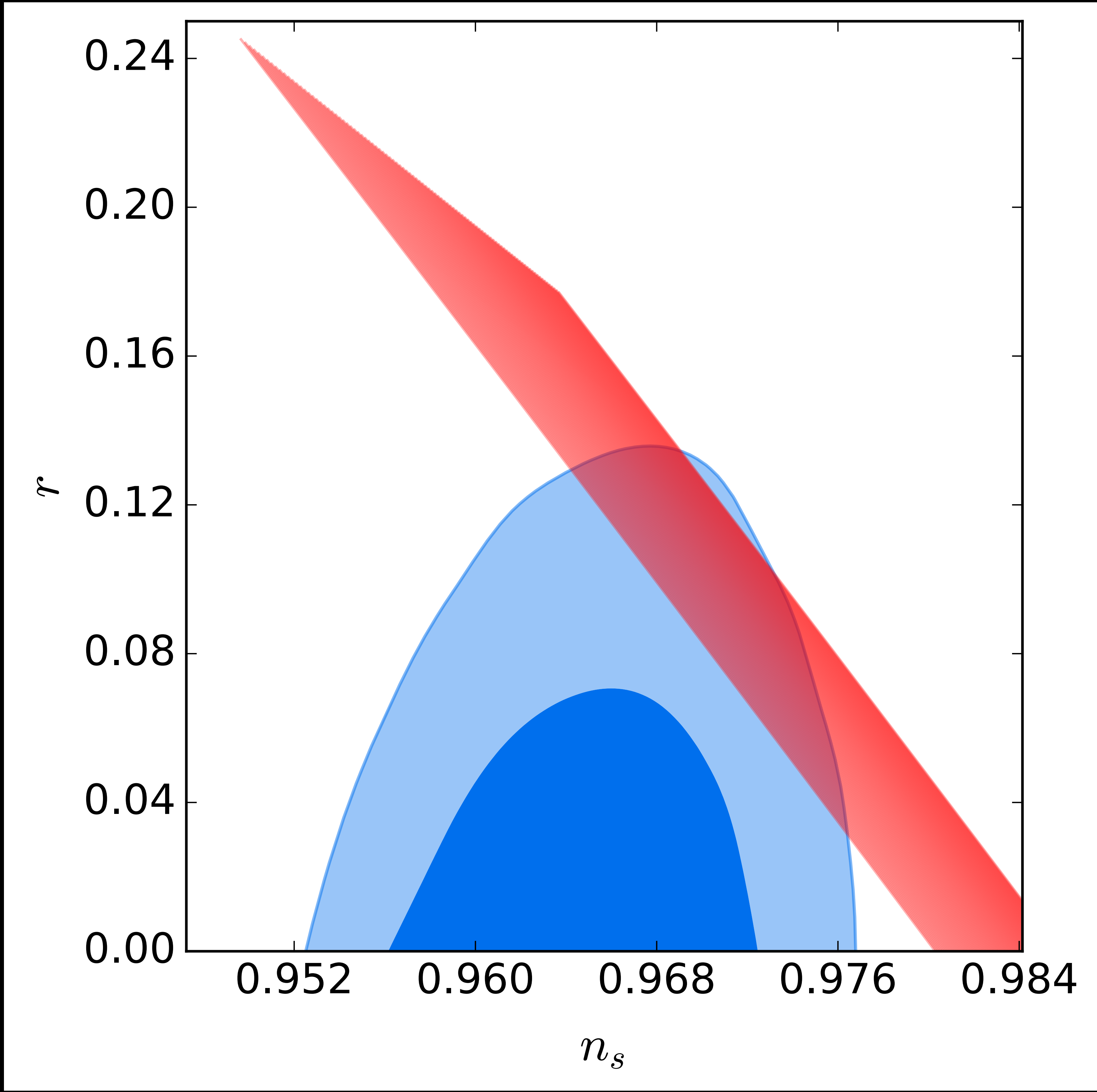


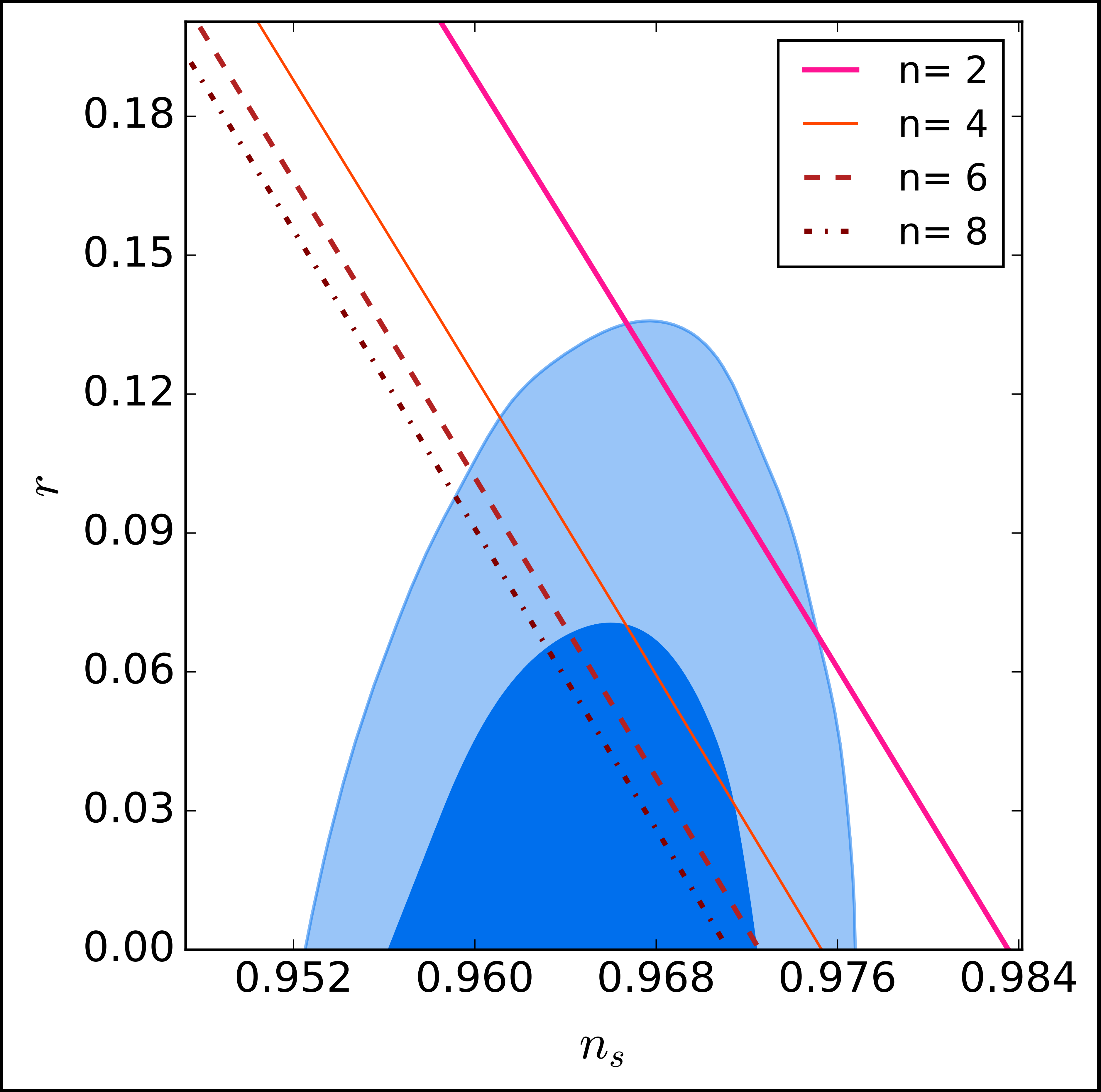
[SC'17]

Planck's constraints

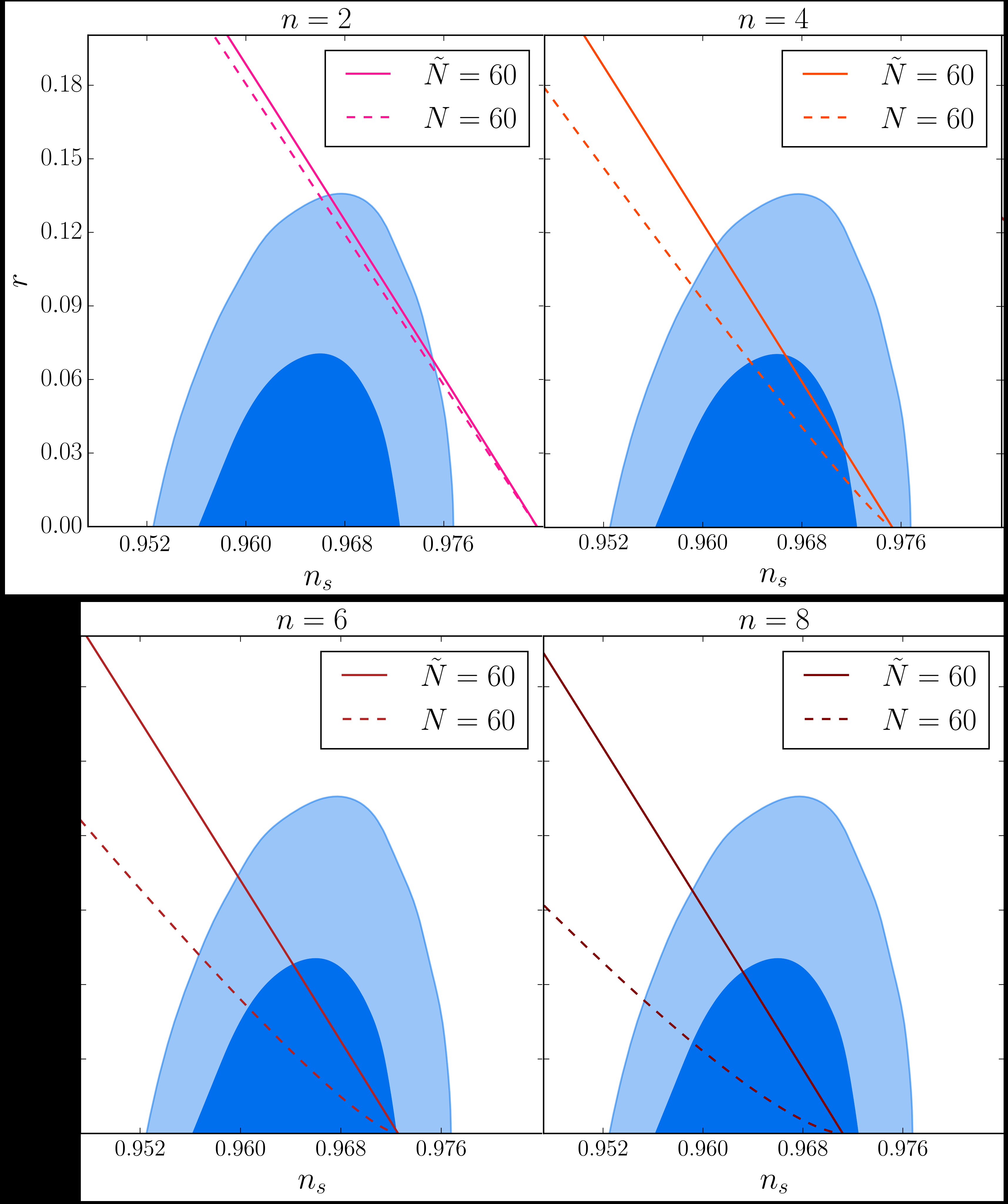


GG model with n=2



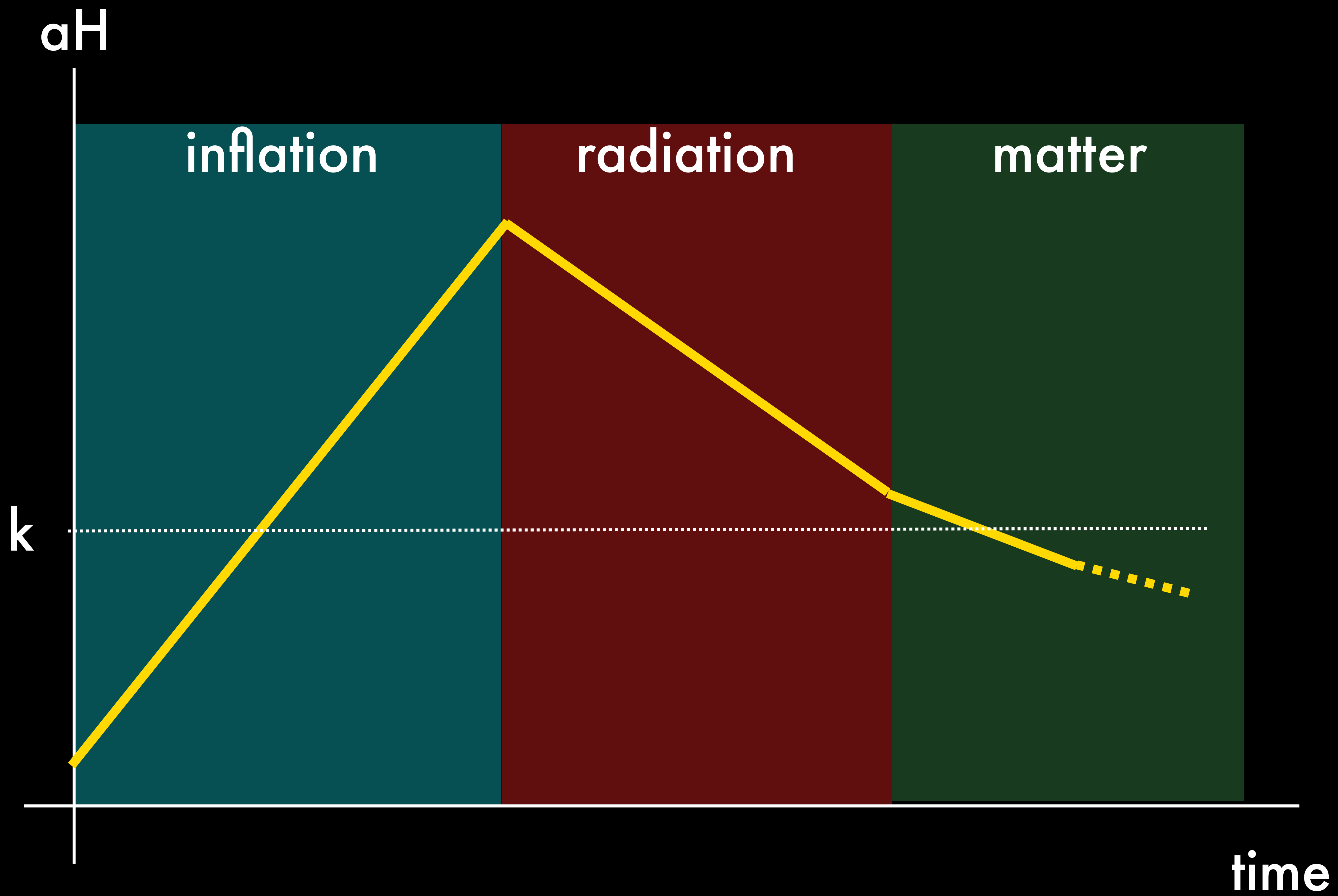

n= 2
n= 4
n= 6
n= 8
0.18
0.15
0.12
0.09
0.06
0.03
0.00
r
0.952
0.960
0.968
0.976
0.984
n_s

[SC'17]



$n = 2$
$n = 4$
$n = 6$
$n = 8$
$\tilde{N} = 60$
$N = 60$
r
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.952
0.960
0.968
0.976
n_s

More about efolding (properly this time)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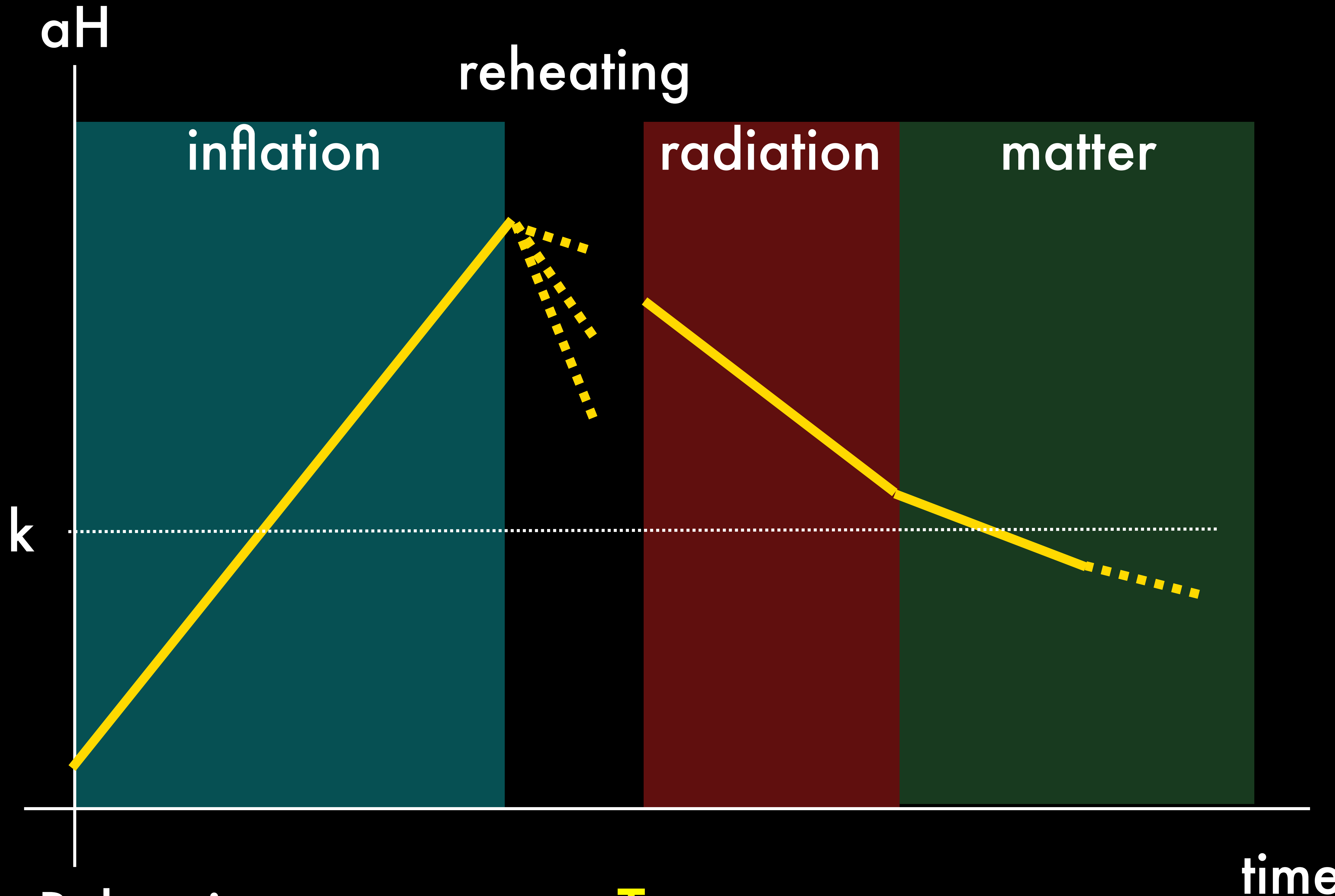
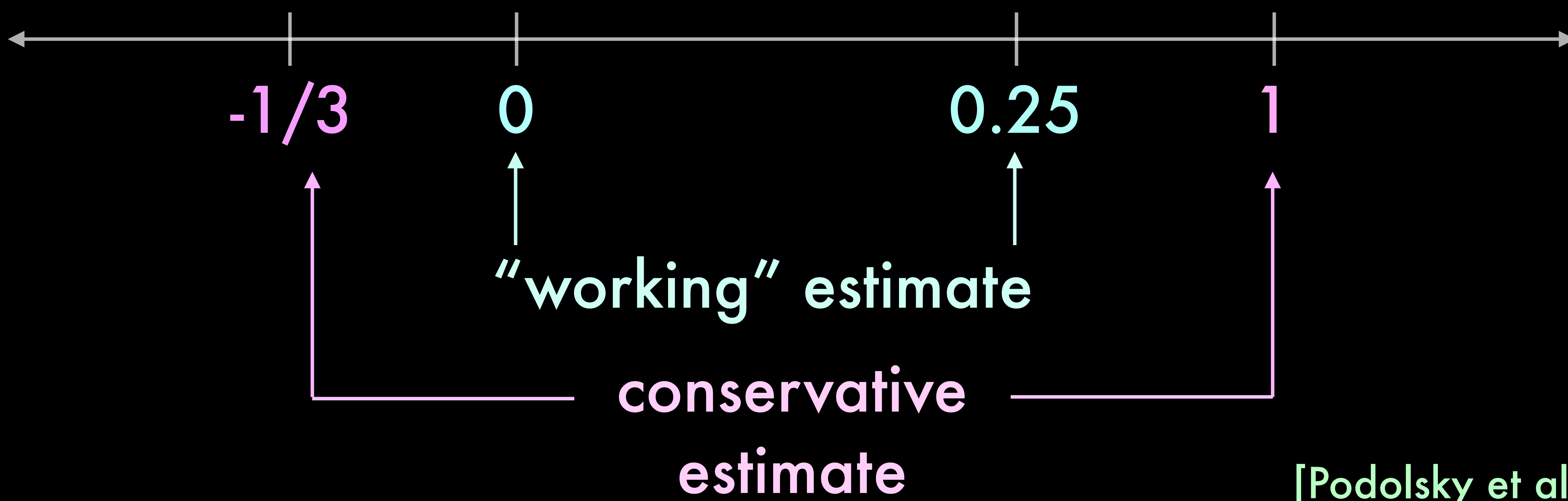


- Reheating temperature T_{reh}
- Mean equation of state $\bar{w}$

Martin & Ringeval'10
Muñoz & Kamionskowski'15

$$10^2\ \mathrm{MeV} \lesssim T_{\rm reh} \lesssim 10^{15}\ \mathrm{GeV}$$

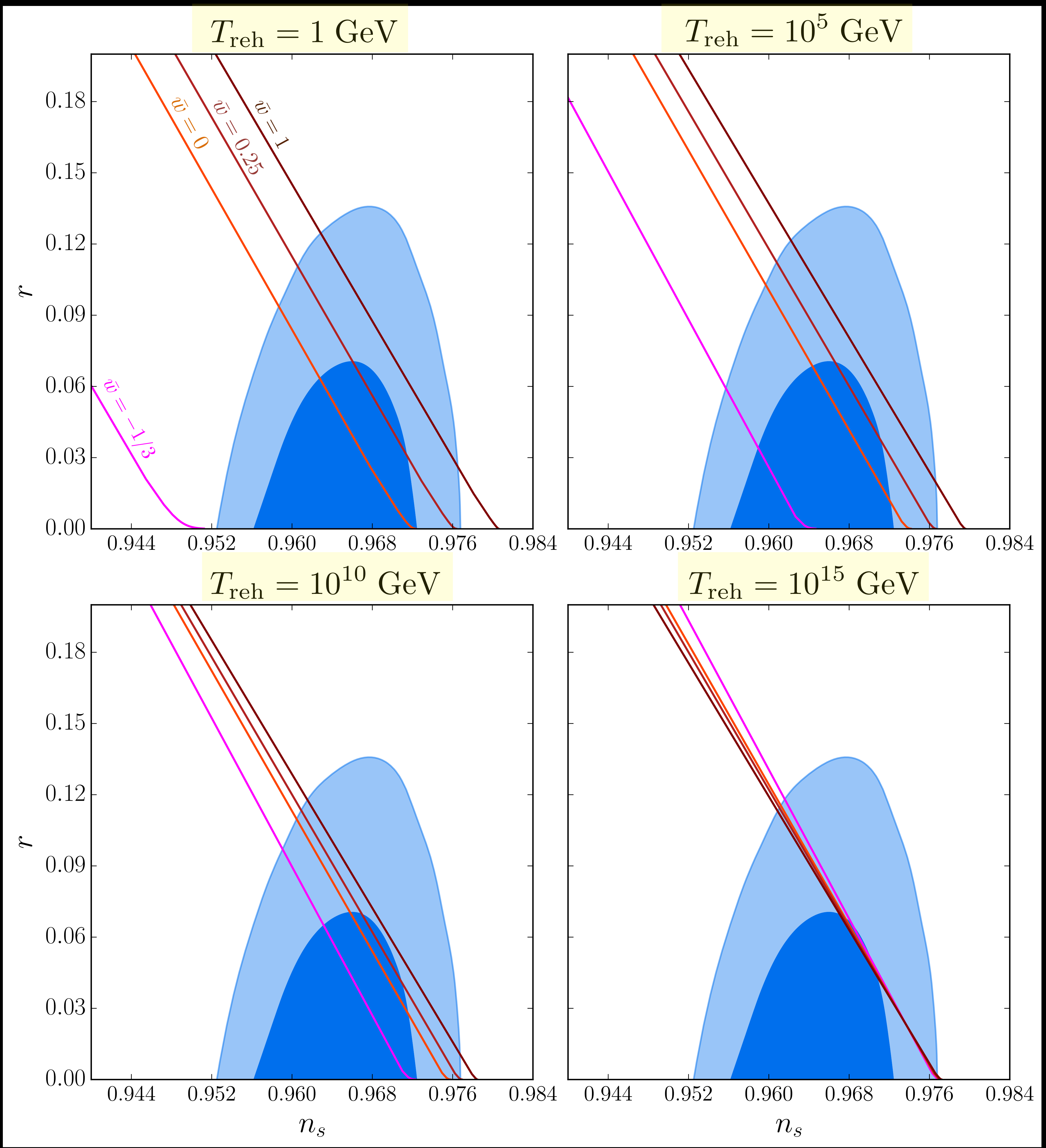
$$\bar{w}$$



[Podolsky et al'06]

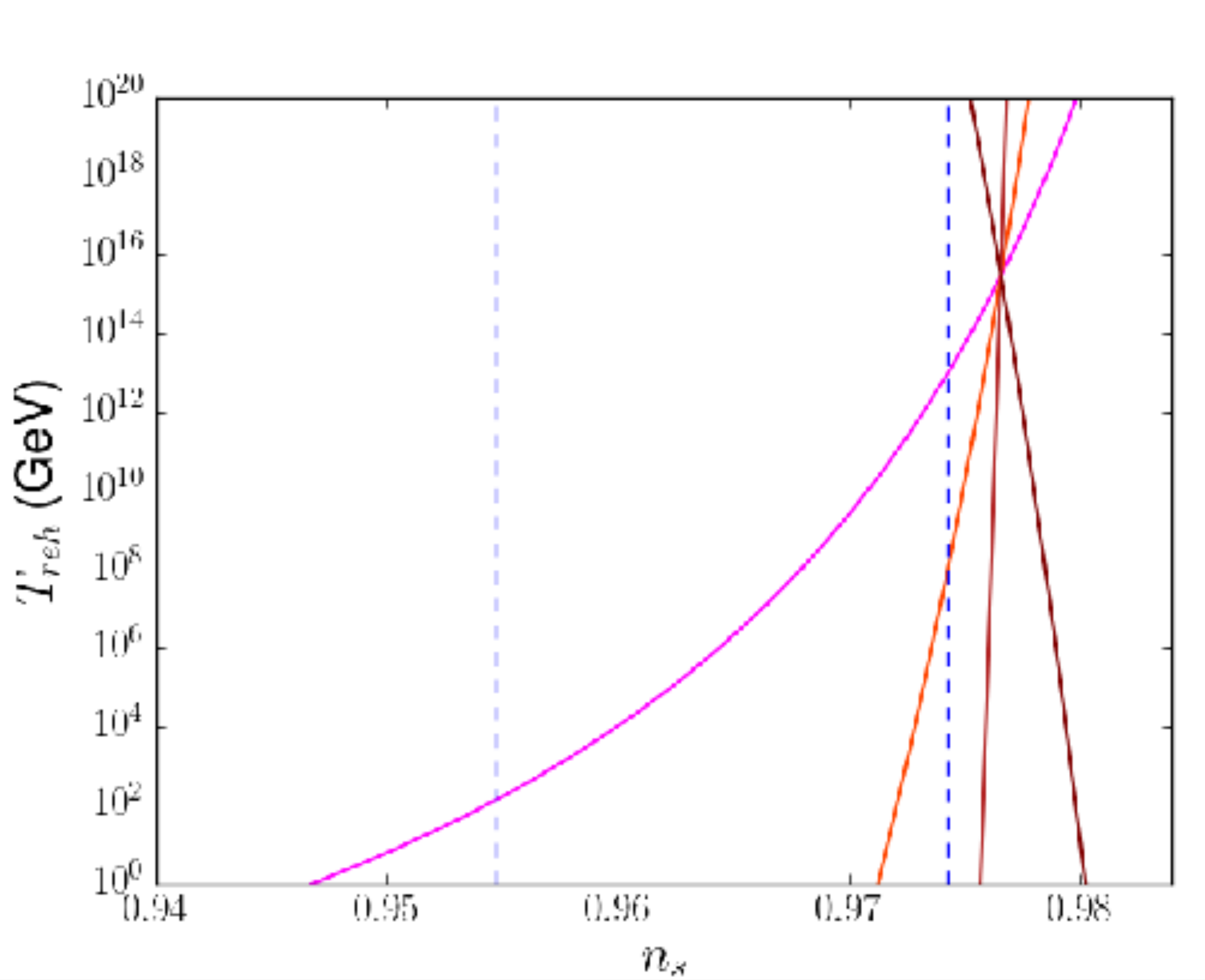
These 2 parameters determine N.

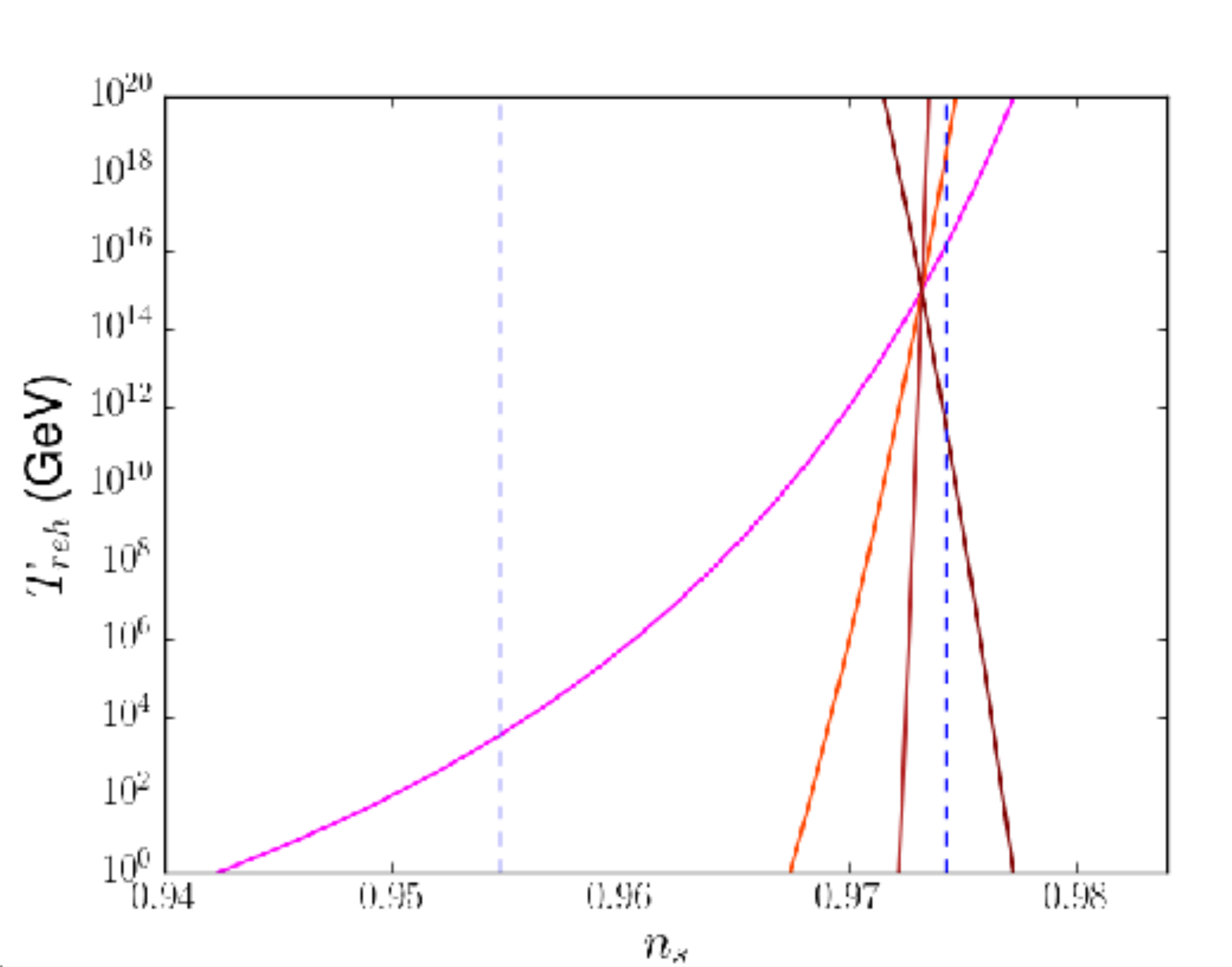
[Martin+Ringeval'10]



$T_{\rm reh} = 1$ GeV
$T_{\rm reh} = 10^5$ GeV
$T_{\rm reh} = 10^{10}$ GeV
$T_{\rm reh} = 10^{15}$ GeV
$\bar{w} = 0$
$\bar{w} = 0.25$
$\bar{w} = 1$
$\bar{w} = -1/3$
r
n_s
0.00
0.03
0.06
0.09
0.12
0.15
0.18
0.944
0.952
0.960
0.968
0.976
0.984

n=4





n=2, α= $1/\sqrt{2}$

$(V(\phi) \sim \phi)$

[see also Rehagen&Gelmini'15]

n=8, α=1

Primordial GW amplitude

Energy density in stochastic GW background

$$\Omega_{\rm gw}(k) \equiv \frac{1}{\rho_{\rm crit}}\frac{{\rm d}\rho_{\rm gw}}{{\rm d}\ln k} = \frac{1}{12H_0^2}k^2 P_T(k)$$

$$\Omega_{\rm gw}h^2 = 4.36\times 10^{-15}\, r\exp\left(\int_{\phi_{\rm gw}}^{\phi_*}\frac{2}{E_1+\sqrt{E_1^2+2}}{\rm d}\phi\right)$$

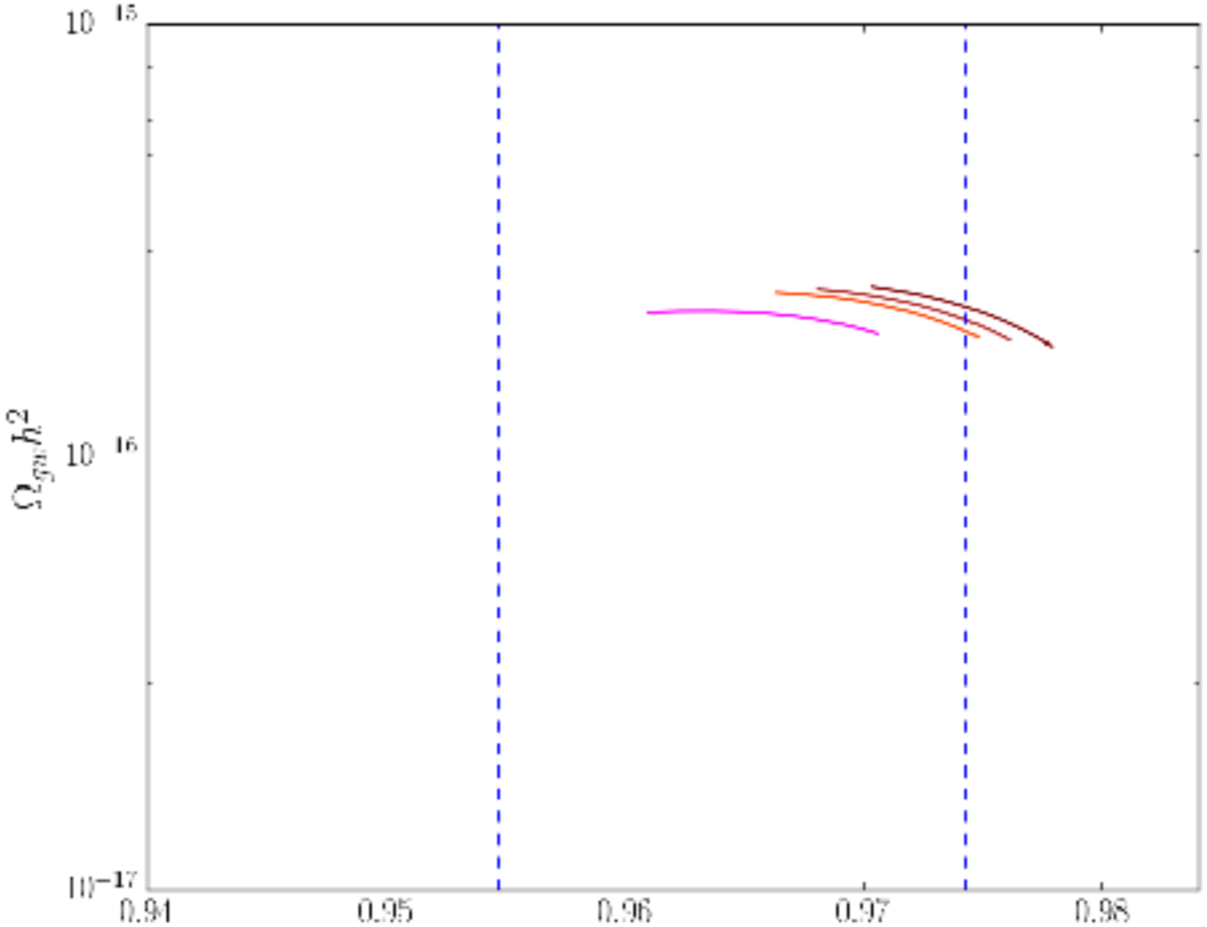
[SC+Efstathiou'06]

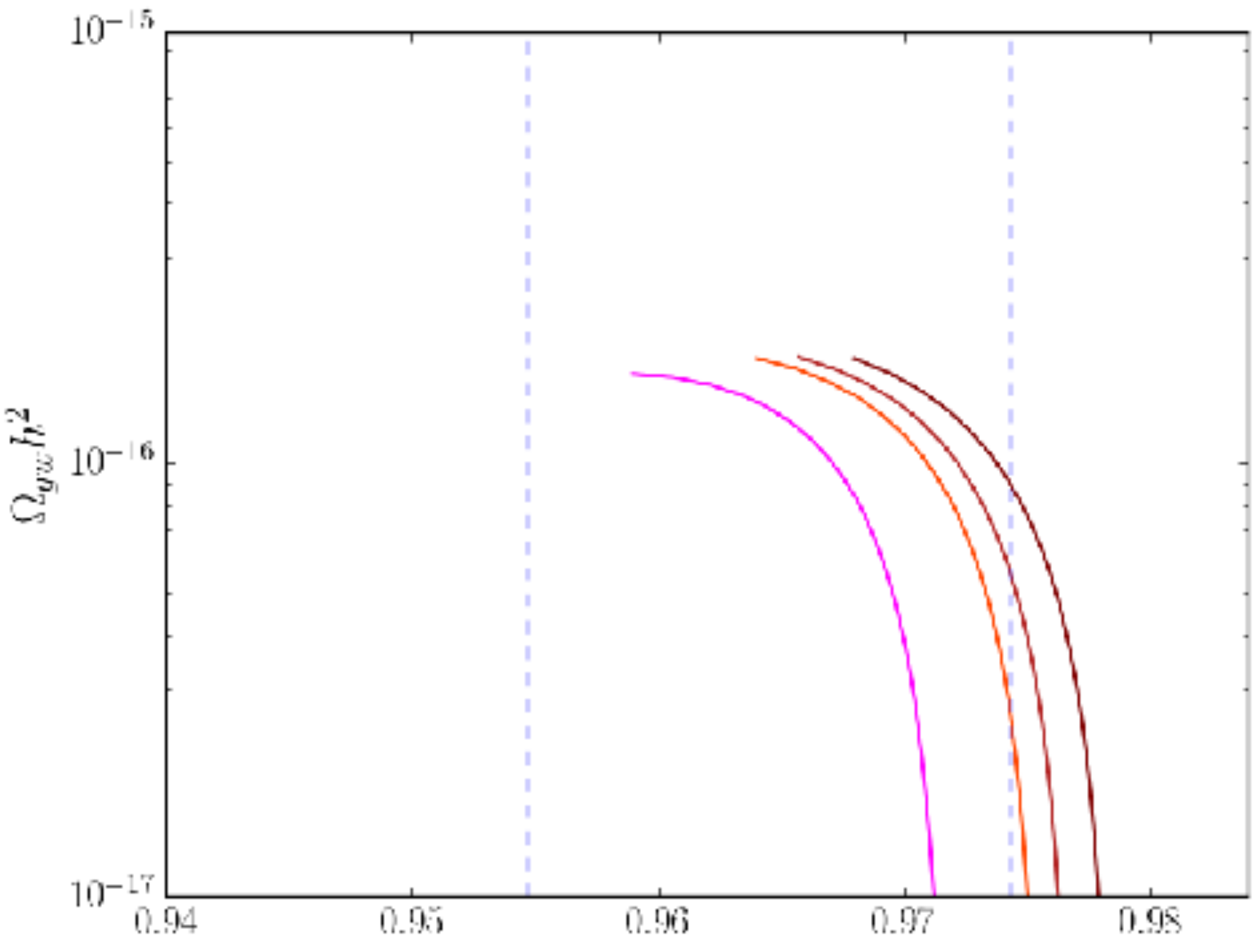
Big Bang Observer / DECIGO will probe

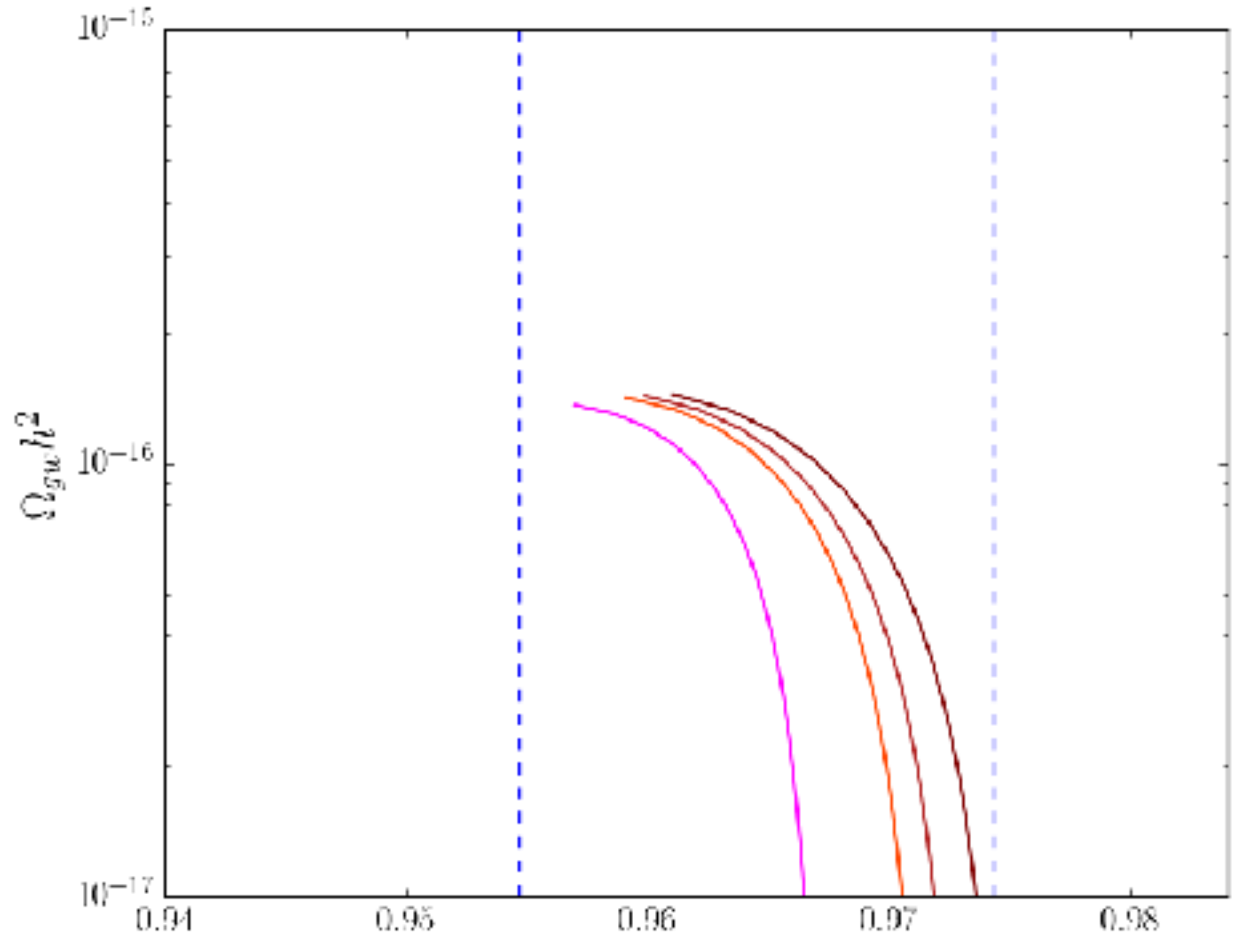
$$\Omega_{\rm gw}h^2 \sim 10^{-15} - 10^{-16} \quad \text{at} \quad 0.1 - 1\ \text{Hz}$$

$\Omega_{\rm gw} h^2$ @ 1Hz

n=2, 4, 8







Conclusions

- Phenomenological model building using the (inverse) Hubble radius $\mathscr{H} = aH$.
- Generalised Gaussian model = plateaus of increasing order, laterally pushing power laws into Planck's constraints.
- Gaussian (n=2) $\leftrightharpoons$ power-law correspondence.

  More to be explored.
- Generic statement like "N=60" doesn't work for GG models.
- Using a simple modelling of reheating, GW amplitudes shown to be able to reach BBO/ DECIGO sensitivities.