

Degenerate Higher Order (Scalar-Tensor) Theories

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Based on [[arXiv:1601.04658](#)], [[arXiv:1602.03119](#)], [[arXiv:1608.08135](#)] and [[arXiv:1703.01623](#)]

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Historical Overview

- 1971 – Lovelock
 - the most general metric theory of gravity yielding conserved second order equations of motion in arbitrary number of dimensions
- 1974 – Horndeski
 - the most general scalar-tensor theory of gravity yielding conserved second order equations of motion in 4 dimensions
- 2008 – Galileons [Nicolis, Rattazzi, Trincherini]
 - a set of terms within 4-dimensional EFT obeying the symmetry $\phi \rightarrow \phi + c + b_\mu x^\mu$ (in a non-trivial way)
- 2009 – Covariant / Generalized Galileons [Deffayet et al.]
 - rediscovery of Horndeski

Second order field equations

What's wrong with higher order field equations?

Ostrogradsky theorem – 1850

Assumptions: 1) single variable 2) non-degenerate (nd)

- Newton, i.e. $L = L(q, \dot{q})$

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} = 0 \quad \xrightarrow{\text{nd}} \quad \ddot{q} = F(q, \dot{q})$$

$$p \equiv \frac{\partial L}{\partial \dot{q}} \quad \text{nd} \implies \dot{q} = f(q, p) \quad H(q, p) = p f - L(q, f)$$

- Higher derivative $L = L(q, \dot{q}, \ddot{q})$

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}} + \frac{d^2}{dt^2} \frac{\partial L}{\partial \ddot{q}} = 0 \quad \xrightarrow{\text{nd}} \quad \dddot{q} = F(q, \dot{q}, \ddot{q}, \dddot{q})$$

$$Q \equiv \dot{q}, \quad p \equiv \frac{\partial L}{\partial \dot{q}} - \frac{d}{dt} \frac{\partial L}{\partial \ddot{q}}, \quad P \equiv \frac{\partial L}{\partial \ddot{q}}, \quad \text{nd} \implies \ddot{q} = f(q, Q, P)$$

$$H(q, Q, p, P) = p Q + Pf - L(q, Q, f)$$

H linear in $p \Rightarrow$ unbounded energy

WAY OUT: break the assumptions

Evading the Ostrogradsky instability

Assumptions:

1) two variables

2) degenerate

Langlois & Noui [arXiv:1510.06930]

- $$\bullet \quad L = L(q_1, \dot{q}_1, \ddot{q}_1, q_2, \dot{q}_2) \quad \rightarrow \quad L(q_1, Q, \dot{Q}, q_2, \dot{q}_2) + \lambda(\dot{q}_1 - Q)$$

$$p_1 \equiv \frac{\partial L}{\partial \dot{q}_1} = \lambda \quad P \equiv \frac{\partial L}{\partial \dot{Q}} \quad p_2 \equiv \frac{\partial L}{\partial \dot{q}_2}$$

Primary constraint $\psi(P, p_2) \approx 0$

$$\det \mathbb{H} = 0$$

$$\mathbb{H} = \begin{pmatrix} \frac{\partial^2 L}{\partial \dot{Q}^2} & \frac{\partial^2 L}{\partial \dot{Q} \partial \dot{q}_2} \\ \frac{\partial^2 L}{\partial \dot{q}_2 \partial \dot{Q}} & \frac{\partial^2 L}{\partial \dot{q}_2^2} \end{pmatrix}$$

Generalization:

Motohashi et al. [arXiv:1603.09355]; Klein & Roest [arXiv:1604.01719]

- $$\bullet \quad L = L(\ddot{\phi}_m, \dot{\phi}_m, \phi_m, \dot{q}_\alpha, q_\alpha) \quad v_m^A = (\delta_m^n, V_m^\alpha) \quad V_m^\alpha \equiv -L_{\ddot{\phi}_m \dot{q}_\beta} L_{\dot{q}_\beta \dot{q}_\alpha}^{-1}$$

$$\text{Primary constraints} \iff 0 = P_{(mn)} \equiv v_m^A L_{\dot{\psi}_A \dot{\psi}_B} v_n^B$$

$$\text{Secondary constraints} \iff 0 = S_{[mn]} \equiv 2 v_m^A L_{\dot{\psi}_A \psi_B} v_n^B$$

Evading the Ostrogradsky instability

Field theories:

[arXiv:1703.01623]

- $L = L(\partial_\mu \partial_\nu \phi_m, \partial_\mu \phi_m, \phi_m, \partial_\mu q_\alpha, q_\alpha)$ $v_m^A = (\delta_m^n, V_m^\alpha)$ $V_m^\alpha \equiv -L_{\ddot{\phi}_m \dot{q}_\beta} L_{\dot{q}_\beta \dot{q}_\alpha}^{-1}$
 $\psi_A \equiv (\dot{\phi}_m, q_\alpha)$

Primary constraints \iff $0 = P_{(mn)} \equiv v_m^A L_{\dot{\psi}_A \dot{\psi}_B} v_n^B$

Secondary constraints \iff $0 = (S_i)_{(mn)} \equiv 2 v_m^A L_{\dot{\psi}_{(A} \partial_i \psi_{B)}} v_n^B$

$$0 = S_{[mn]} \equiv 2 v_m^A L_{\dot{\psi}_{[A} \psi_{B]}} v_n^B + 2 v_{[m}^A L_{\dot{\psi}_A \partial_i \psi_{B}}} \partial_i v_{n]}^B - \partial_i \left(v_m^A L_{\dot{\psi}_{[A} \partial_i \psi_{B]}}} v_n^B \right)$$

Lorentz invariance \implies $(S_i)_{(mn)} = 0$ if $P_{(mn)} = 0$

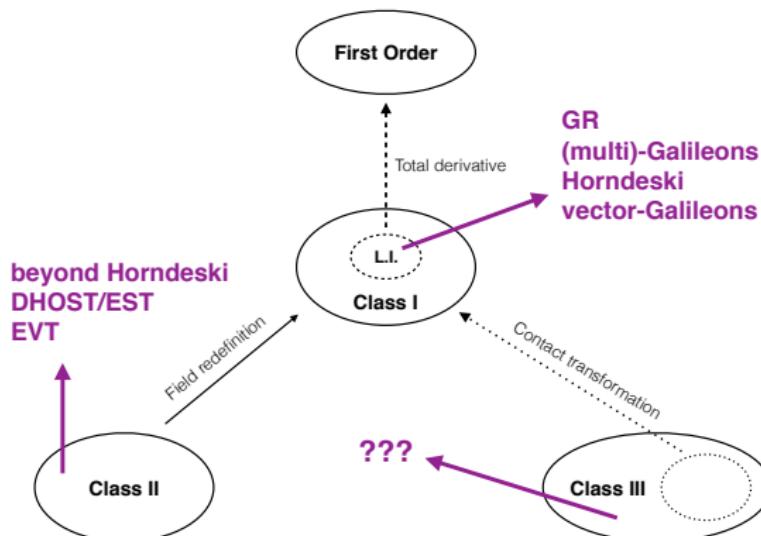
1 primary constraint \implies \exists 1 secondary constraint

Kind of theories

$$\bullet \quad L = L(\partial_\mu \partial_\nu \phi_m, \partial_\mu \phi_m, \phi_m, \partial_\mu q_\alpha, q_\alpha)$$

$$v_m^A = (\delta_m^n, V_m^\alpha) \quad V_m^\alpha \equiv -L_{\ddot{\phi}_m \dot{q}_\beta} L_{\dot{q}_\beta \dot{q}_\alpha}^{-1}$$

- **Class I:** $V_m^\alpha = 0 \rightarrow$ trivial primary constraints
- **Class II:** $V_m^\alpha \neq V_m^\alpha(\partial_\mu \partial_\nu \phi_n, \partial_\mu q_\beta) \rightarrow$ linear primary constraints
- **Class III:** $V_m^\alpha = V_m^\alpha(\partial_\mu \partial_\nu \phi_n, \partial_\mu q_\beta) \rightarrow$ nonlinear primary constraints



Scalar-Tensor Theories

two fields $(g_{\mu\nu}, \phi)$

Second derivative of ϕ : $L = L(\nabla_\mu \partial_\nu \phi) \longrightarrow L(\nabla_\mu A_\nu) + \lambda^\mu (\partial_\mu \phi - A_\mu)$

Covariant 3+1 decomposition: $t^\mu = \partial/\partial t = N n^\mu + N^\mu$

$$\begin{cases} g_{\mu\nu} = h_{\mu\nu} - n_\mu n_\nu \\ A_\mu = \hat{A}_\nu h_\mu^\nu - A_* n_\mu \end{cases}$$

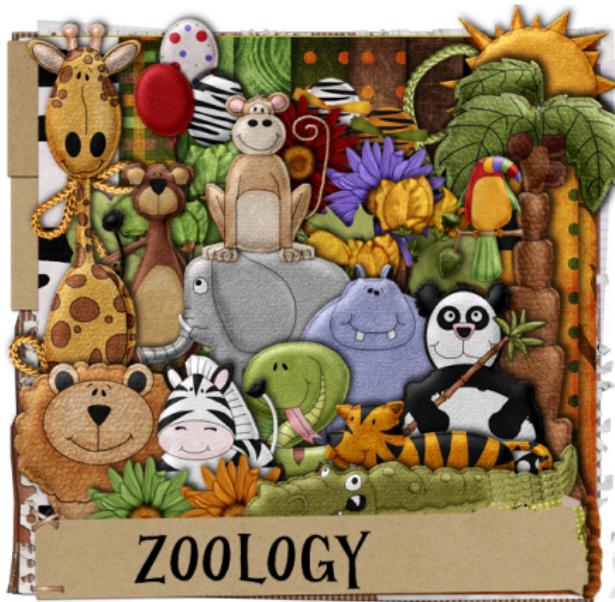
$$\nabla_\mu A_\nu = \lambda_{\mu\nu} V + \Lambda_{\mu\nu}^{\rho\sigma} K_{\rho\sigma} + \dots \quad V \equiv n^\mu \nabla_\mu A_* \sim \ddot{\phi}$$

$$\mathbb{H} = \begin{pmatrix} \mathcal{A} & \mathcal{B}^{ij} \\ \mathcal{B}^{kl} & \mathcal{K}^{ij,kl} \end{pmatrix}, \quad \mathcal{A} \equiv \frac{\partial^2 L}{\partial V^2}, \quad \mathcal{B}^{ij} \equiv \frac{\partial^2 L}{\partial V \partial K_{ij}}, \quad \mathcal{K}^{ij,kl} \equiv \frac{\partial^2 L}{\partial K_{ij} \partial K_{kl}}$$

$$\det \mathbb{H} = 0 \implies \text{ghost free}$$

Degenerate Scalar-Tensor Theories

$$S = \int d^4x \sqrt{-g} \left(f_2 R + C_{(2)}^{\mu\nu\rho\sigma} \phi_{\mu\nu} \phi_{\rho\sigma} + f_3 G_{\mu\nu} \phi^{\mu\nu} + C_{(3)}^{\mu\nu\rho\sigma\alpha\beta} \phi_{\mu\nu} \phi_{\rho\sigma} \phi_{\alpha\beta} \right)$$



- 24 classes of theories with 17 free functions [[arXiv:1608.08135](https://arxiv.org/abs/1608.08135)]
- linear effective approach ETofDE ([Vernizzi's talk](#))
- non-linear scales... ???

Summary

- It is possible to avoid the Ostrogradsky instability in higher order (scalar-tensor) theories in a non-trivial way
- Brand new theories
- New phenomenology

Thank you