Moduli Vacuum Misalignment and Precise Predictions in String Inflation

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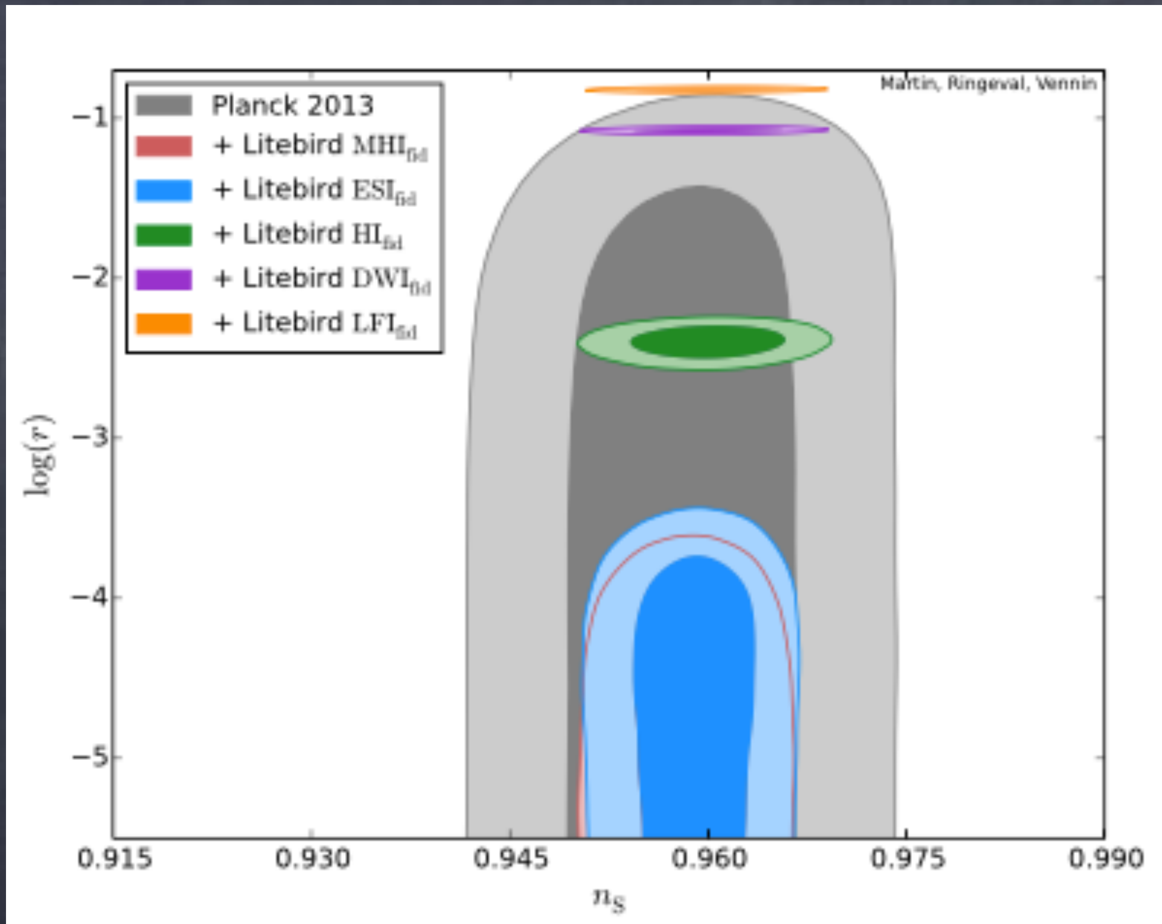
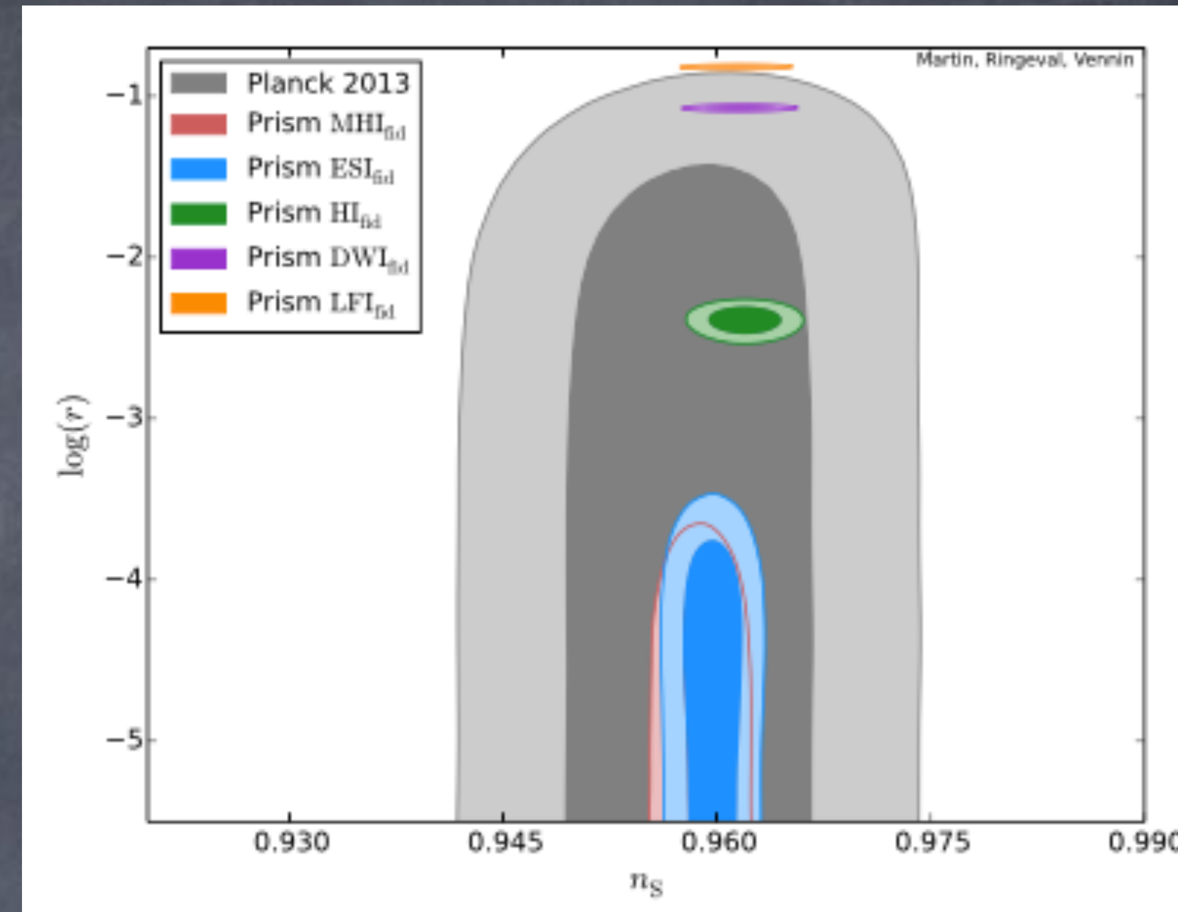
Take Home

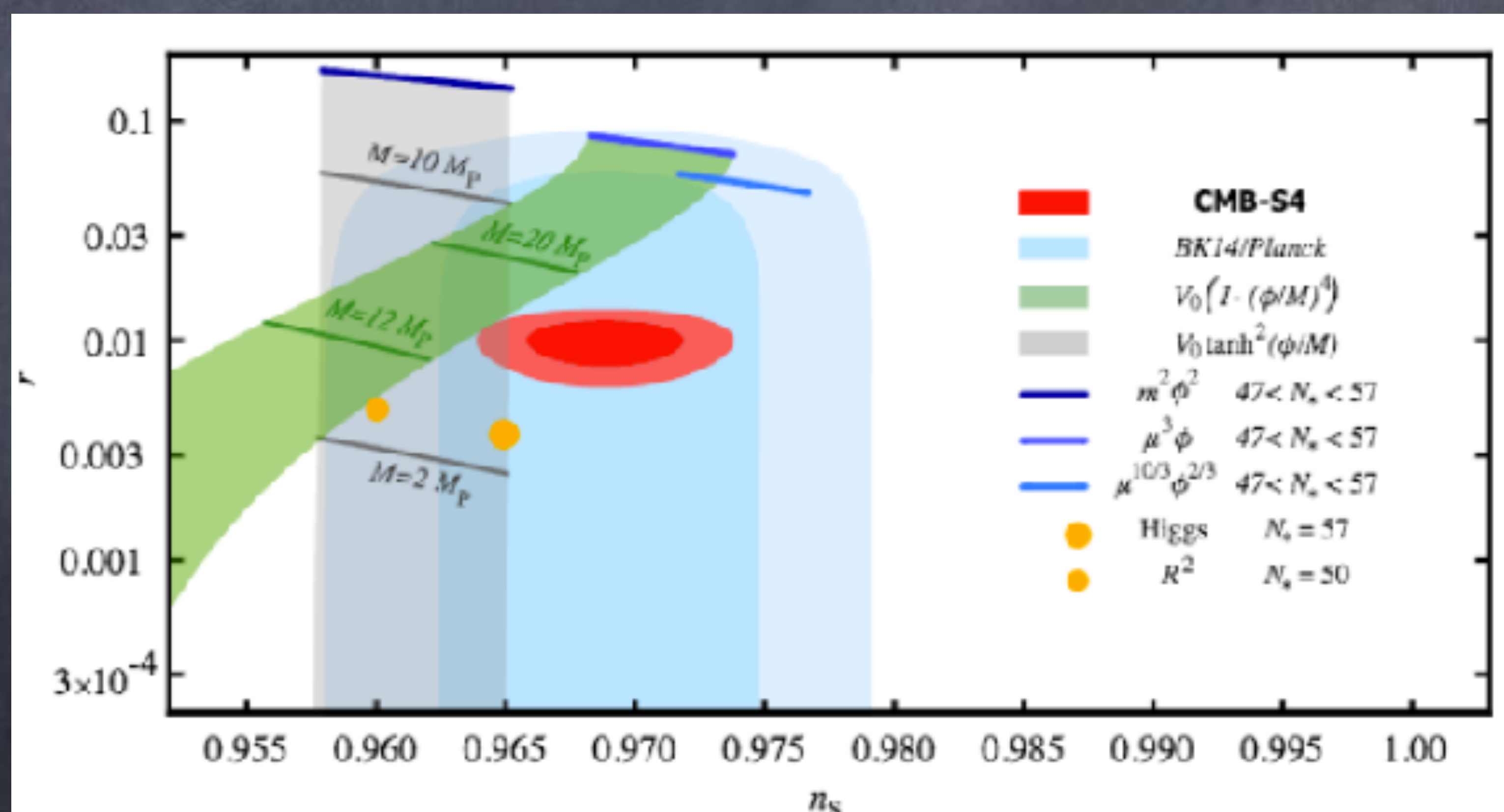
The number of e-foldings is sensitive to the post-inflationary history of the universe.

The generic presence of light scalar fields (in SUSY/String Theory) leads to a late-time period of matter domination which lowers the required number of e-foldings and, in turn, modifies the exact predictions of any inflationary model.



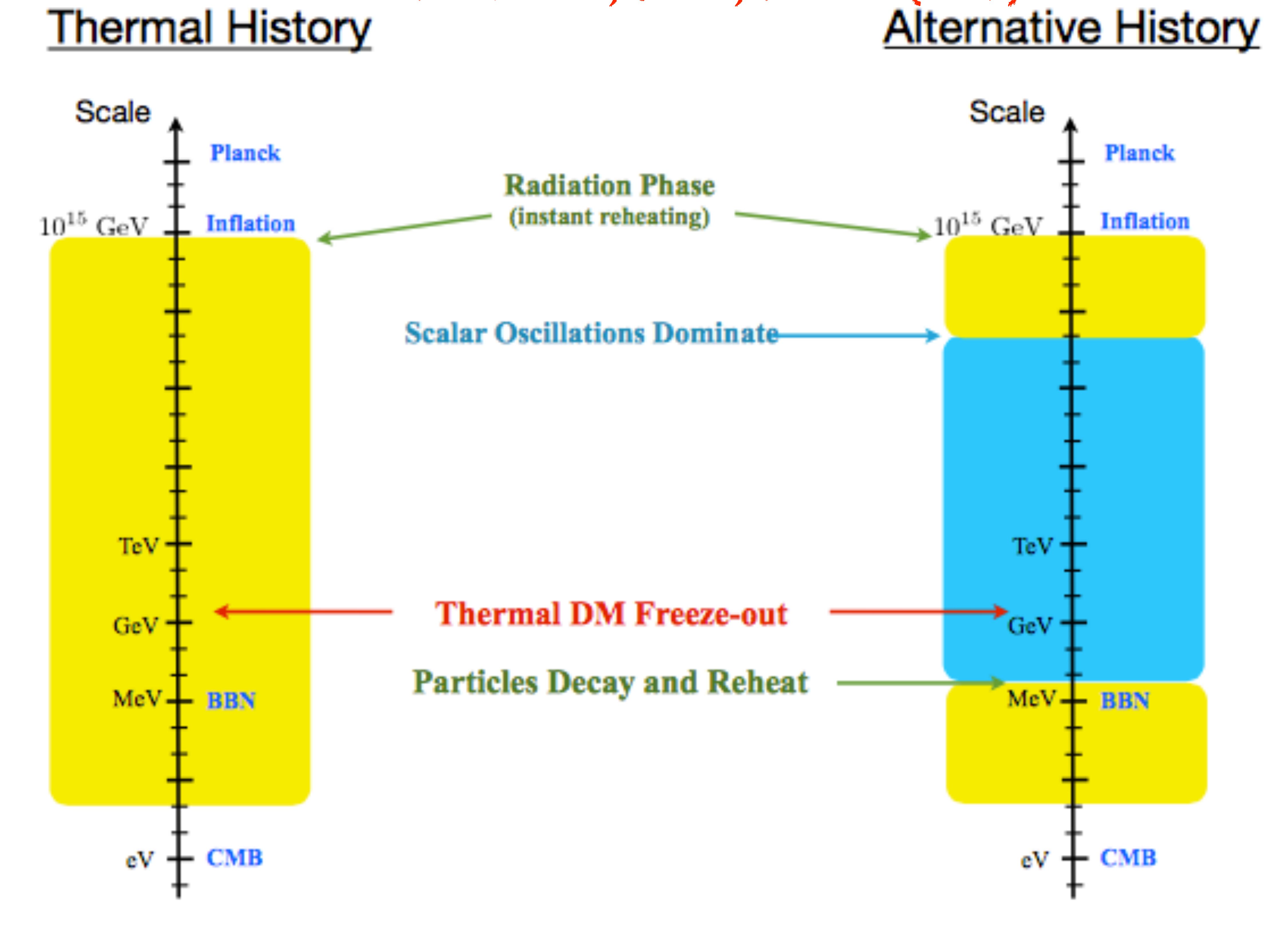
Martin,
Ringeval,
Vennin



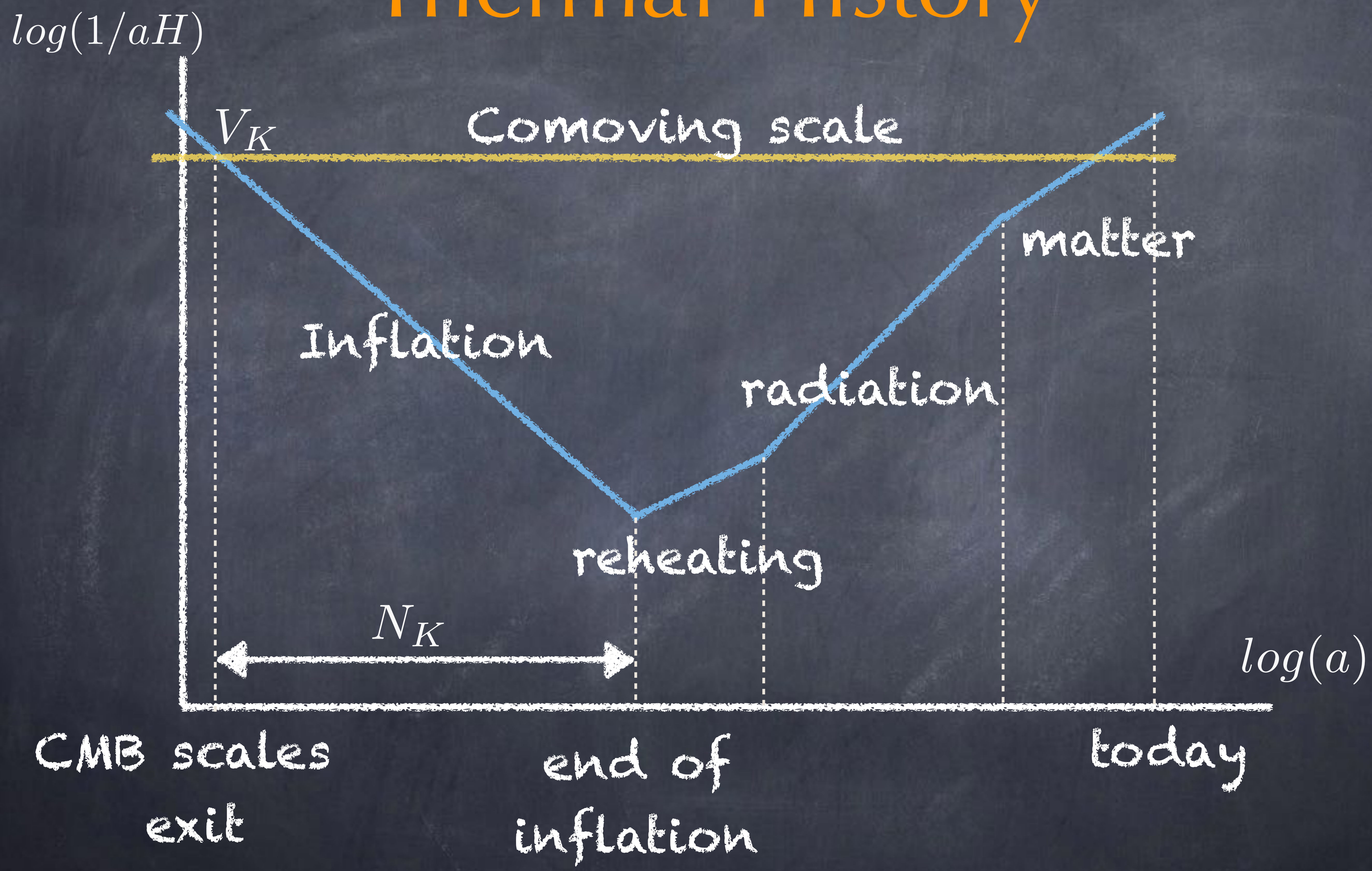


precise measurements of spectral index is crucial! sensitivity $n_s \sim 10^{-3}$

From: Kane, Sinha, Watson (2015)



Thermal History



Making predictions ..

$$N_{inf} + \frac{1}{4}(1 - 3w_{rh})N_{rh} = 55 + \frac{1}{4}lnr + \frac{1}{4}ln(\rho_k/\rho_{end})$$

$$N_{inf} = 55 \pm 5$$

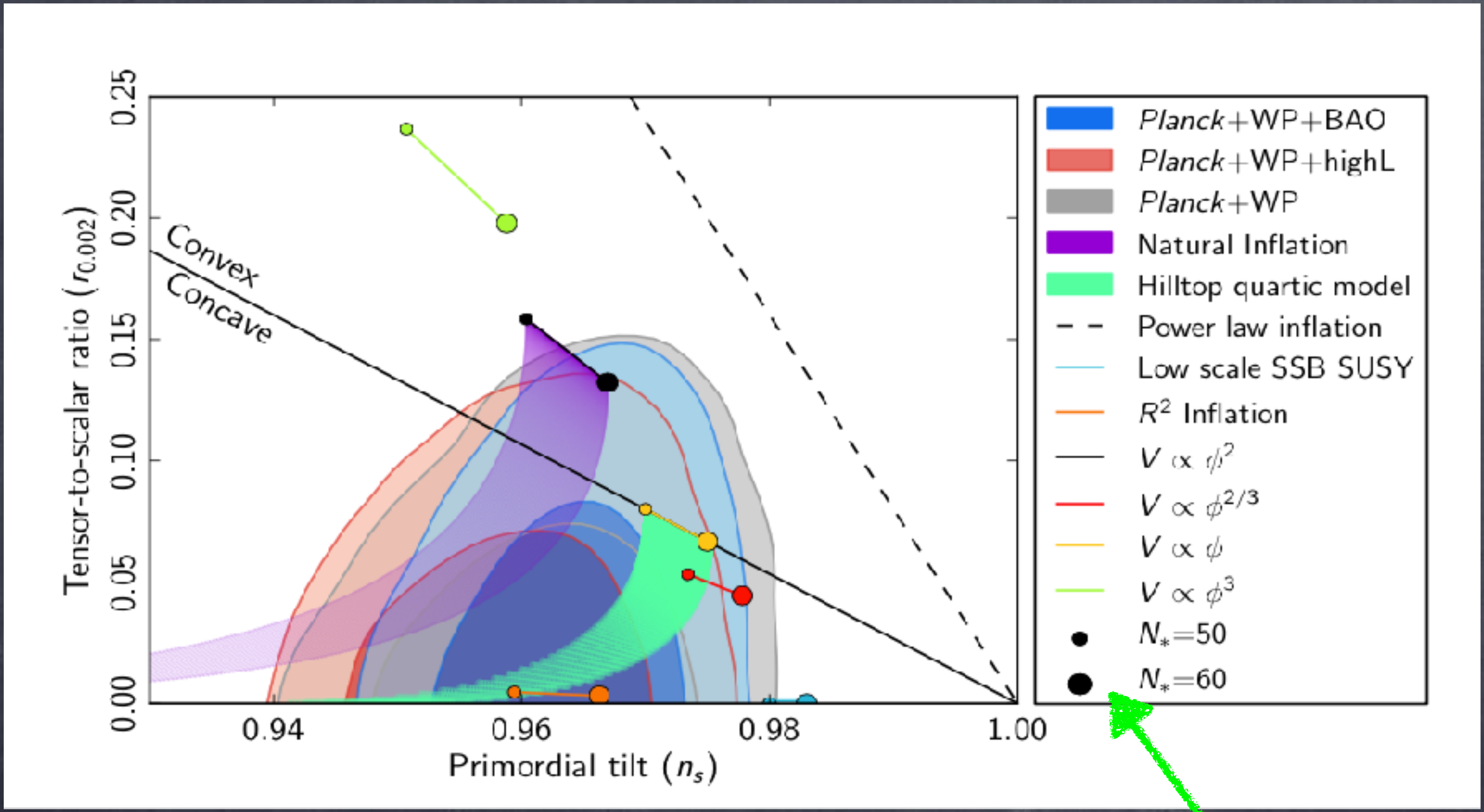
Liddle, Leach (2003)

PLANCK (2016)

'Theoretical prior'

compute observables in terms of N_{inf} and see whether it fits data for N = 50-60!

$$V(\chi) = \frac{1}{2}m^2\chi^2 \qquad n_s - 1 = -\frac{2}{N_k} \qquad r = 8/N_K$$



'Theoretical prior'

How does making predictions change for modular cosmology?

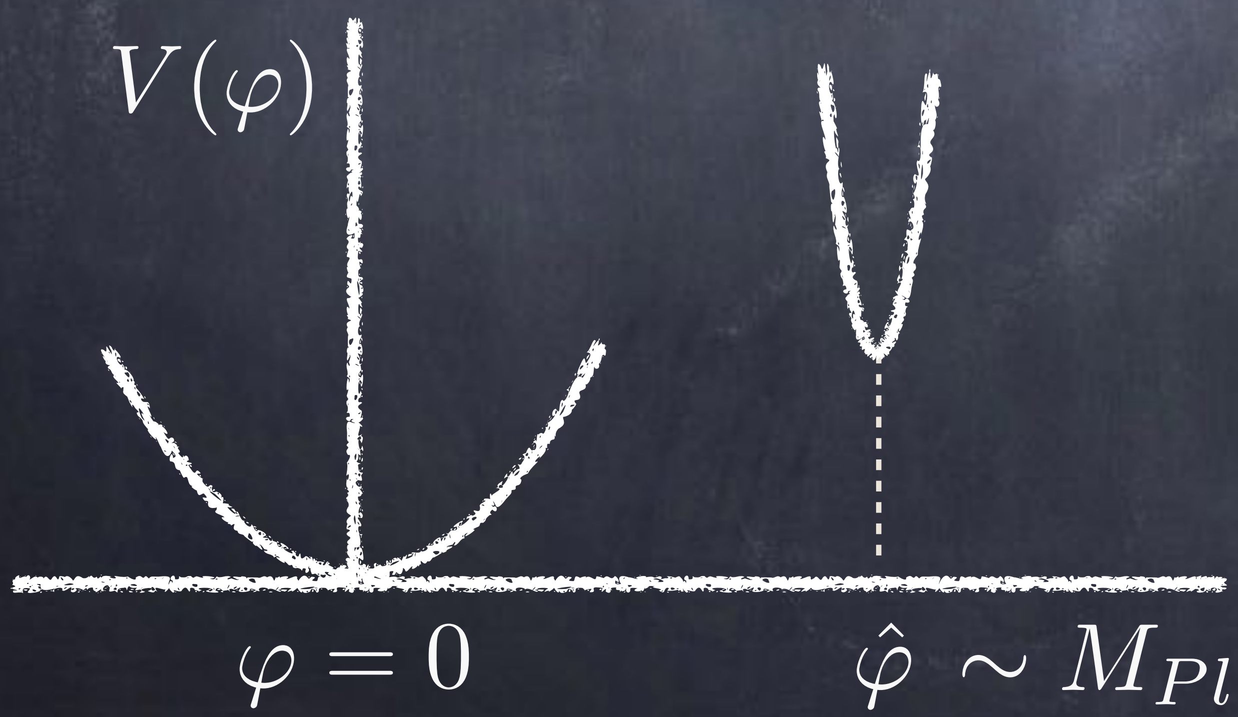
A typical case

$$\mathcal{L} \supset -\frac{1}{2}m^2\varphi^2 - \frac{1}{2}H^2(\varphi - \hat{\varphi})^2 - V_{inf}(\chi)$$

$m < H_{inf}$

post-inflationary moduli mass

minima during inflation



$$Y = \hat{\varphi}/M_{Pl} \sim 1$$

Dine, Randall, Thomas

Supergravity

$$V = e^{K[\varphi,\bar{\varphi}]} V_0[\varphi, \chi] \sim H^2 M_{Pl}^2 f\left(\frac{\varphi}{M_{Pl}}\right)$$

$V'' \sim H^2$ $\eta -$ problem

Scale of variations M_{Pl} $Y = \hat{\varphi}/M_{Pl} \sim 1$

Dine, Randall, Thomas

Toy example ..

Dvali

$$V = (m_{3/2}^2 - a^2 H^2)|\varphi|^2 + \frac{1}{2M_{Pl}^2}(m_{3/2}^2 + b^2 H^2)|\varphi|^4$$

$$\hat{\varphi} \sim (a/b) M_{Pl}$$

(talk by Fuminori Hasegawa)

Different source of moduli stabilisation during and after inflation

Inflation ⟶ reheating ⟶ radiation

N_{inf} N_{rh1} N_{rad}

reheating ⟵ moduli (matter)

N_{rh2} 'non-standard' N_{mod}

BBN

today

K.D, Maharana

arXiv:1409.7037[hep-ph]

Liddle, Leach (2003)

$$N_{inf} + \frac{1}{4}(1-3w_{rh1})N_{rh1} + \frac{1}{4}N_{mod} + \frac{1}{4}(1-3w_{rh2})N_{rh2}$$

$$= 55.43 + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

non-thermal history

Constraint ..

$$\Gamma_{mod} \sim \frac{m_\varphi^3}{16\pi M_{Pl}^2} \qquad N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

initial displacement $Y = \hat{\varphi}/M_{Pl}$

$$\frac{1}{6}\overbrace{ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1-3w_{rh1})N_{re1} + \frac{1}{4}(1-3w_{rh2})}^{\text{post-inflationary details}}N_{re2}$$

$$= 55.43 - \underbrace{N_{inf} + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})}_{\text{inflationary details}}$$

inflationary potentials

Implications I

Central value of e-folding shifts

$$N_{inf} = 55 \pm 5$$

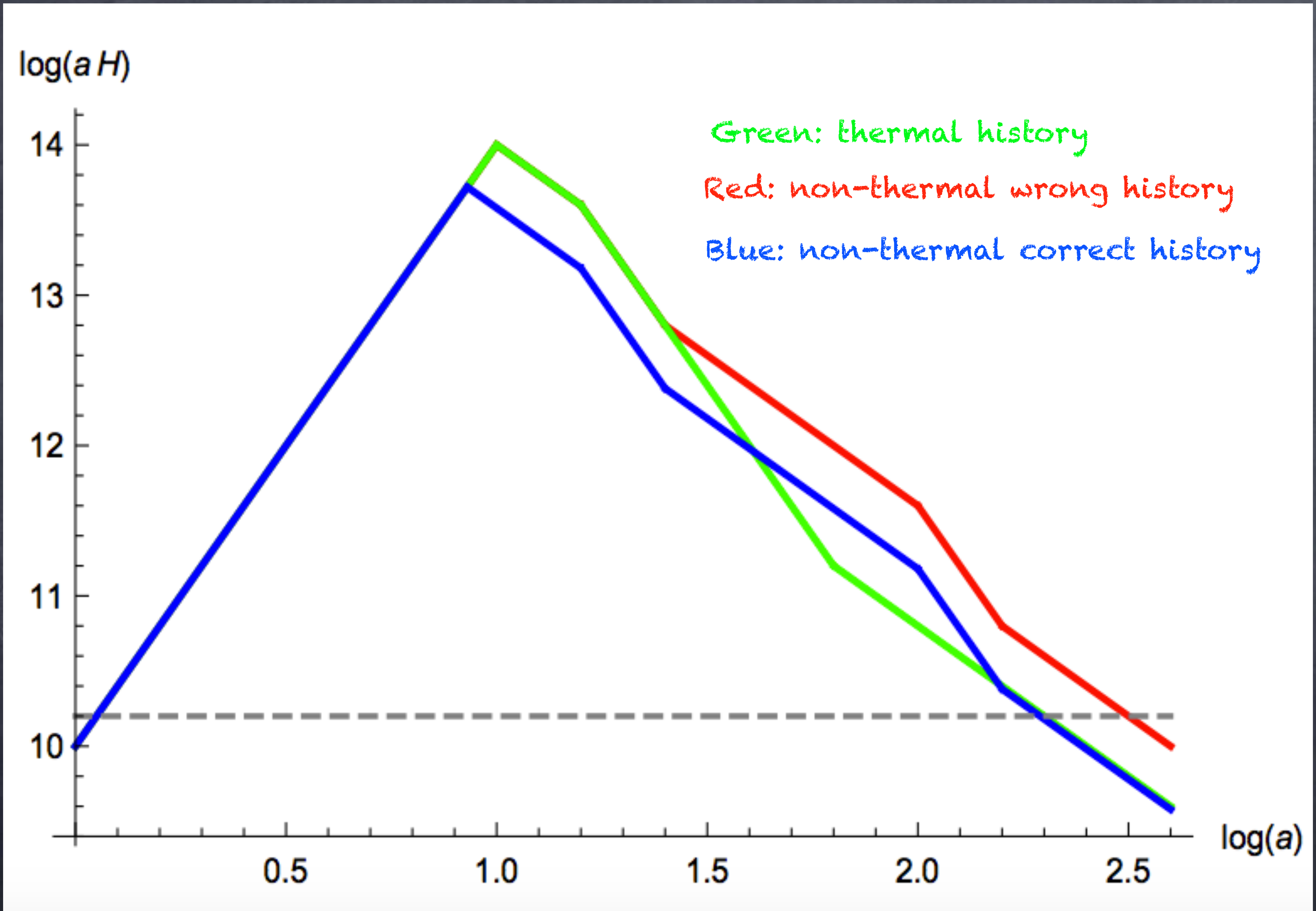
$$\boxed{N_{inf} = \left(55 - \frac{1}{3} ln\left(\frac{\sqrt{16\pi} M_{pl} Y^2}{m_\varphi}\right)\right) \pm 5}$$

For $m_\varphi \sim 10^3$ TeV : $N_{inf} = 41 \quad - \quad 51$

For $m_\varphi \sim 10^6$ TeV : $N_{inf} = 43 \quad - \quad 53$

(used to be 50 - 60)

(Y ~ 0.1 assumed)


log(a H)
14
13
12
11
10
Green: thermal history
Red: non-thermal wrong history
Blue: non-thermal correct history
0.5
1.0
1.5
2.0
2.5
log(a)

Implications II

$$\frac{1}{6}ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \underbrace{\frac{1}{4}(1-3w_{rh1})N_{re1} + \frac{1}{4}(1-3w_{rh2})N_{re2}}_{\text{usually positive definite}}$$

$$= 55.43 - N_{inf} + \frac{1}{4}ln\ r + \frac{1}{4}ln(\frac{\rho_k}{\rho_{end}})$$

analytical/numerical understanding of reheating: $w_{re} < 1/3$

Ellis, Garcia, Nanopoulos, Olive (2015)

Small field models

$$m_\varphi \gtrsim \sqrt{16\pi} M_{\rm pl} Y^2 \; e^{-3\left(55.43 - N_k + \frac{1}{4}\ln(\frac{\rho_{\rm k}}{\rho_{\rm end}}) + \frac{1}{4}\ln r\right)}$$

- conservative estimate r ~ 0.01 stronger the bound
- potential plateau like .. ratio of energy densities negligible
- take Y = 0.1, then for N = 50

$$m_\varphi \gtrsim 4.5 \times 10^6 TeV$$

much stronger than BBN bound

What to calculate now?

Cicoli, K.D, Maharana, Quevedo

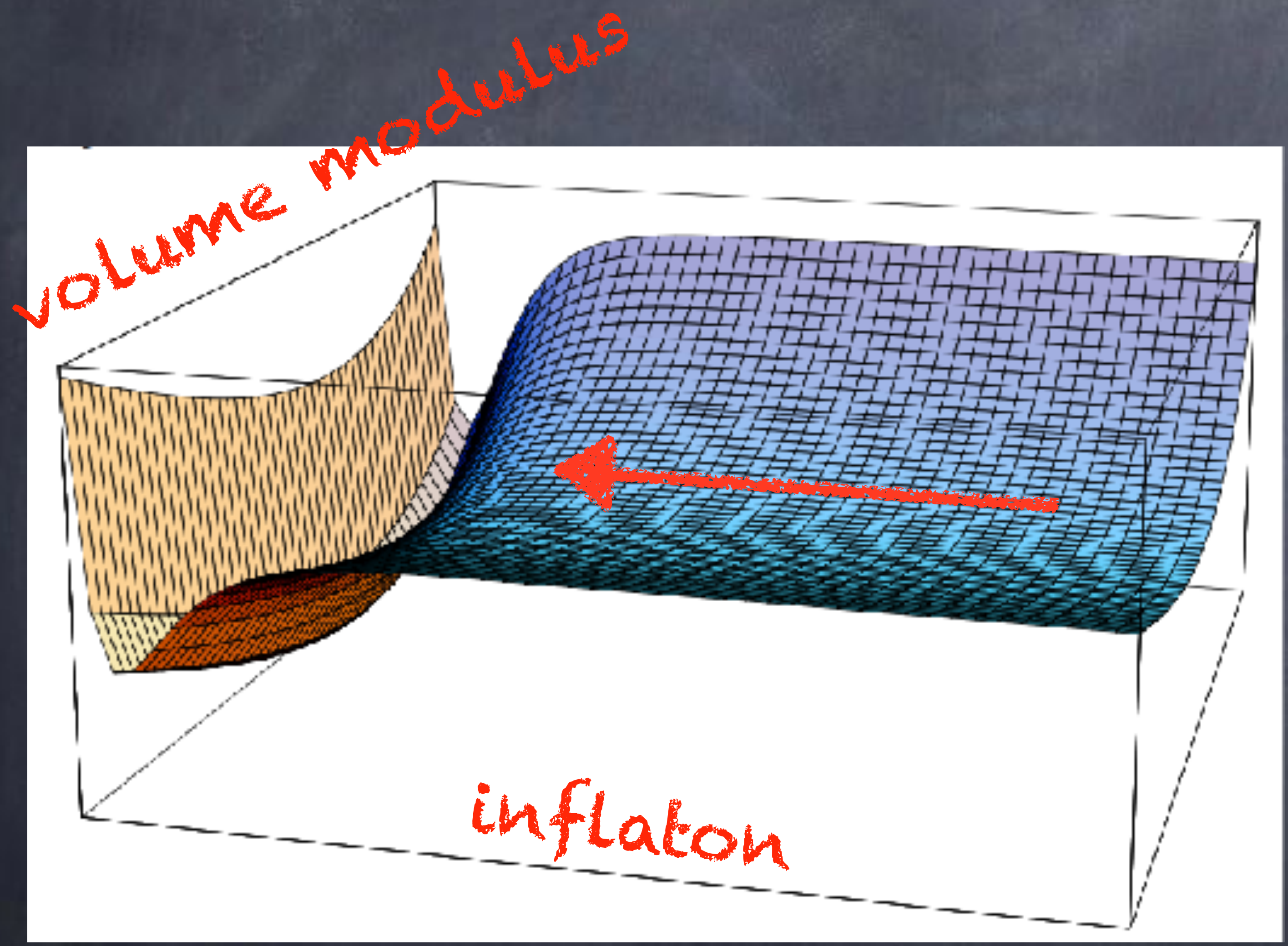
$$N_{mod} \sim \frac{2}{3} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\phi^2})$$

$$\frac{1}{6} ln(\frac{16\pi M_{Pl}^2 Y^4}{m_\varphi^2}) + \frac{1}{4}(1 - 3w_{rh1})N_{re1} + \frac{1}{4}(1 - 3w_{rh2})N_{re2}$$

$$= 55.43 - N_{inf} + \frac{1}{4} ln\ r + \frac{1}{4} ln(\frac{\rho_k}{\rho_{end}})$$

Kahler moduli inflation

Conlon, Quevedo (2005)



- In LVS scenario of string theory.
- A concrete set-up where inflationary potential is known
- Post-inflationary modulus domination happens

$$\Gamma_{\tau_n} \simeq 0.1 \frac{m_{\tau_n}^3}{M_s^2} \gg \quad \Gamma_\nu \simeq \frac{m_\nu^3}{16\pi M_{pl}^2}$$

Cicoli, Mazumdar (2010)

Details ..

Volume $\mathcal{V} = \alpha\left(\tau_1^{3/2} - \sum_{i=2}^{n} \lambda_i \tau_i^{3/2}\right)$

$$V = \sum_{i=2}^{n} \frac{8(a_i A_i)^2 \sqrt{\tau_i}}{3\mathcal{V}\lambda_i} e^{-2a_i\tau_i} - \sum_{i=2}^{n} \frac{4a_i A_i W_0}{\mathcal{V}^2}\tau_i e^{-a_i\tau_i} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3} + \frac{D}{\mathcal{V}^\gamma}$$

$$m_{\tau_i}^2 \simeq \frac{W_0^2(\ln\mathcal{V})^2 M_{\rm pl}^2}{\mathcal{V}^2} \gg m_{\mathcal{V}}^2 \simeq \frac{W_0^2 M_{\rm pl}^2}{\mathcal{V}^3 \ln\mathcal{V}}$$

taking all small moduli at minima: two field potential

$$V_{\rm inf} = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n-1}\left[\frac{\lambda_i\alpha}{a_i^{3/2}}\right](\ln\mathcal{V})^{3/2} - \frac{\xi}{2}\right) + \frac{D}{\mathcal{V}^\gamma} - \frac{4a_n A_n W_0}{\mathcal{V}^2}\tau_n e^{-a_n\tau_n}$$

modulus

inflaton

Shift in Volume Modulus

Potential experienced by the volume modulus depends on the inflaton: Vacuum misalignment

Other modulus are not shifted and having masses much larger than the Hubble scale.

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n}\left[\frac{\lambda_i \alpha}{a_i^{3/2}}\right](\ln \mathcal{V})^{3/2} - \frac{\hat{\xi}}{2}\right) + \frac{D}{\mathcal{V}^\gamma}$$

$$Y = \frac{\delta\varphi}{M_{pl}} = \sqrt{\frac{2}{3}}\delta\phi \simeq 2\sqrt{\frac{2}{3}}R\phi_* \simeq 0.1 - 1$$

We validate the SUGRA expectations with explicit calculations

e-foldings

t1: end of inflation t2: inflation decay time

short matter dominated epoch until inflaton decays

$$N_{\mathrm{mod1}} = \ln\left(\frac{a(t_2)}{a(t_1)}\right) = \frac{1}{3}\ln\left(\frac{\rho_{\tau_n}(t_1)}{\rho_{\tau_n}(t_2)}\right) \simeq \frac{2}{3}\ln\left(\frac{H(t_1)}{\Gamma_{\tau_n}}\right) \simeq \frac{2}{3}\ln\left(\frac{10\beta^{1/2}\mathcal{V}^{1/2}}{W_0^2(\ln\mathcal{V})^3}\right)$$

At t_eq, radiation and volume modulus oscillations density become equal

$$N_{\mathrm{mod2}} \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}(\ln\mathcal{V})^{5/2}Y^4}{10\beta^2}\right) \approx \frac{2}{3}\ln\left(\frac{16\pi\mathcal{V}^{5/2}Y^4}{10P^2R^2(\ln\mathcal{V})^{1/2}}\right)$$

A benchmark example

$$W_0 = \alpha = \lambda_i = 1, a_i = 2\pi, g_s = 0.06$$

$$\mathcal{V}_{\rm in} \simeq 1.38 \cdot 10^5, \beta \simeq 3.88$$

$$\tau_n \simeq 1.12 \cdot 10^8 \, N_e^{-4} \qquad \text{and} \qquad r = 16\epsilon \simeq 1.04 \cdot 10^{-10}$$

plays important role

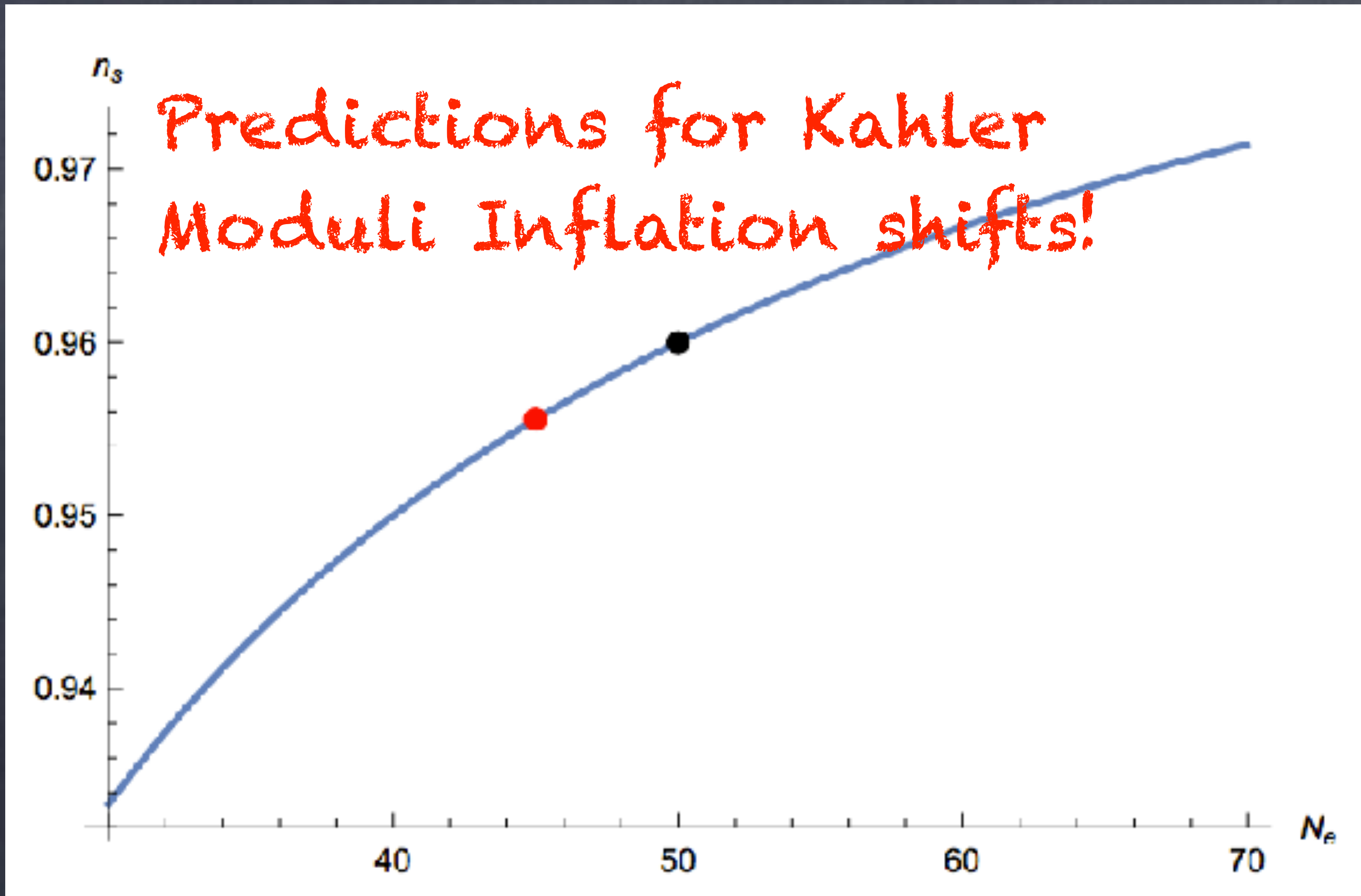
$$m_{\mathcal{V}} \sim 10^8 - 10^9 \quad GeV$$

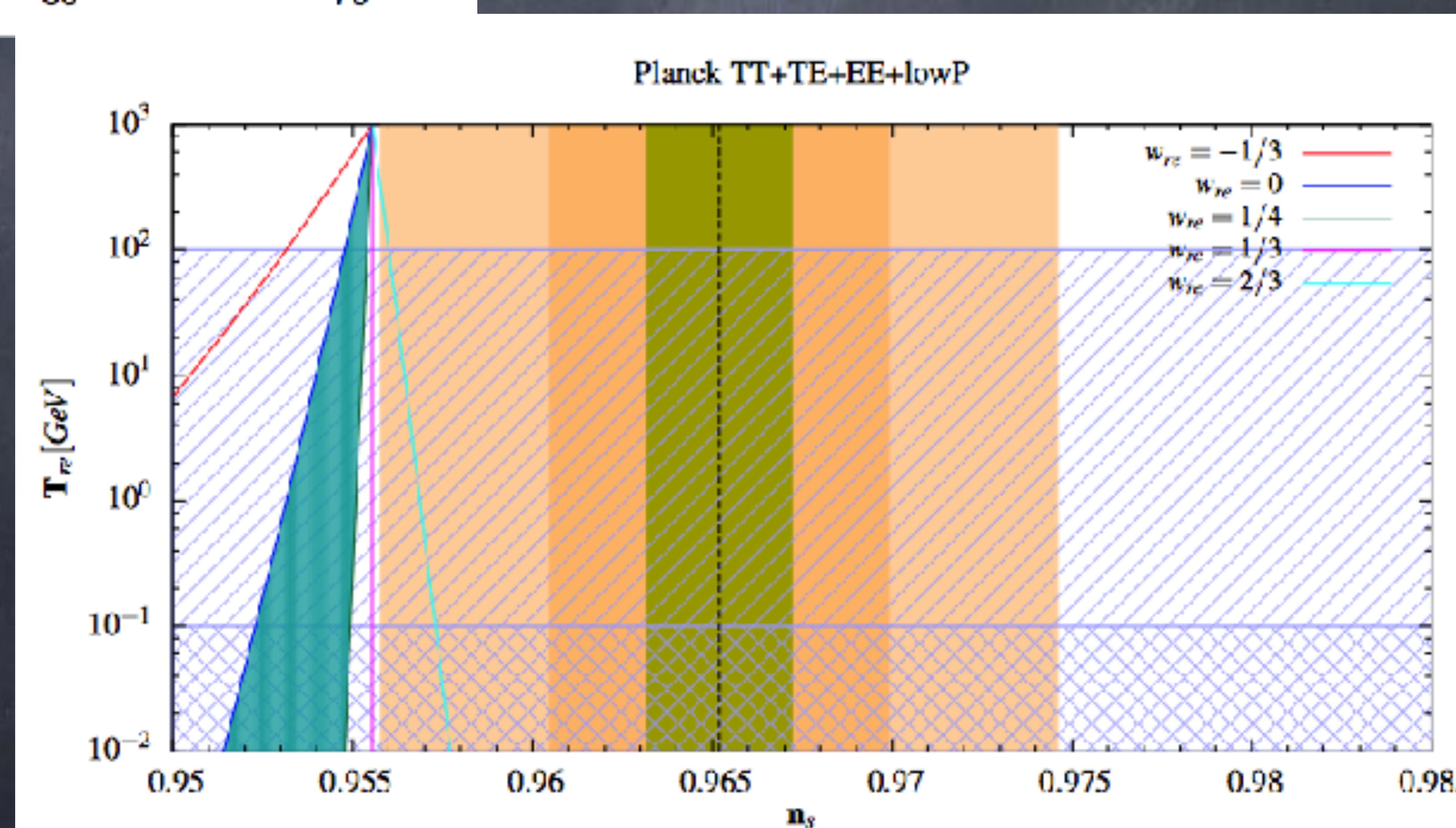
$$N_{\rm mod1} \simeq 0.99 \qquad \text{and} \qquad N_{\rm mod2} \simeq 25.4$$

$$N_e \simeq 44.65 + \frac{1}{4} \ln\left(\frac{\rho_*}{\rho_{\rm end}}\right) \simeq 45 \qquad \Rightarrow \qquad \tau_n \simeq 27.3 \qquad \text{and} \qquad n_s \simeq 0.955$$

$$T_{\rm rh} \gtrsim 10^3 \quad GeV$$

OK with BBN bound


Predictions for Kahler Moduli Inflation shifts!
n_s
0.97
0.96
0.95
0.94
40
50
60
70
N_e


Planck TT+TE+EE+lowP
$T_{re}\,[GeV]$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
$w_{re} = -1/3$
$w_{re} = 0$
$w_{re} = 1/4$
$w_{re} = 1/3$
$w_{re} = 2/3$
0.95
0.955
0.96
0.965
0.97
0.975
0.98
0.985
n_s

Conclusions

- modulus dominated cosmology is a generic feature of string/sugra motivated scenario

$$\hat{N}_{inf} = 55 - \frac{1}{3}\left(\frac{\sqrt{16\pi}M_{Pl}Y^2}{m_\varphi}\right) \simeq 45$$

- Explicit calculations for Kahler Moduli inflation

Take Home

The number of e-foldings is sensitive to the post-inflationary history of the universe.

The generic presence of light scalar fields (in SUSY/String Theory) leads to a late-time period of matter domination which lowers the required number of e-foldings and, in turn, modifies the exact predictions of any inflationary model.

Thank You

Shift in Volume Modulus

Potential experienced by the volume modulus depends on the inflaton: Vacuum misalignment

Other modulus are not shifted and having masses much larger than the Hubble scale.

$$V = -\frac{3W_0^2}{2\mathcal{V}^3}\left(\sum_{i=2}^{n}\left[\frac{\lambda_i \alpha}{a_i^{3/2}}\right](\ln \mathcal{V})^{3/2} - \frac{\hat{\xi}}{2}\right) + \frac{D}{\mathcal{V}^\gamma}$$

$$V(\mathcal{V}_*) = \frac{\partial V(\mathcal{V}_*)}{\partial \mathcal{V}} = 0 \qquad -\frac{3W_0^2}{2}\, e^{-3\phi_*}\left(P\phi_*^{3/2} - \frac{\hat{\xi}}{2}\right) + D\, e^{-2\phi_*} = 0$$

$$\frac{3W_0^2}{2}\, e^{-3\phi_*}\left(3P\phi_*^{3/2} - \frac{3}{2}P\phi_*^{1/2} - \frac{3\hat{\xi}}{2}\right) - 2D\, e^{-2\phi_*} = 0$$

$$\phi \equiv \ln \mathcal{V} \quad \text{and} \quad P \equiv \alpha \sum_{i=2}^{n} \lambda_i\, a_i^{-3/2} = \frac{\alpha}{R}\, \lambda_n\, a_n^{-3/2} \qquad R \equiv \frac{\lambda_n\, a_n^{-3/2}}{\sum_{i=2}^{n} \lambda_i\, a_i^{-3/2}} \ll 1$$

Shift in Volume Modulus

$$\phi_*^{3/2} - \frac{3}{2}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

$$D = \frac{9W_0^2}{4} P e^{-\phi_*} \phi_*^{1/2}$$

$$V_{\rm in}(\phi) = -\frac{3W_0^2}{4} e^{-3\phi} \left[2P\,(1-R)\,\phi^{3/2} - \hat{\xi} - 3P\phi_*^{1/2} e^{(\phi-\phi_*)} \right]$$

$$(1-R)\phi_{\rm in}^{3/2} - \frac{1}{2}(1-R)\phi_{\rm in}^{1/2} - e^{(\phi_{\rm in}-\phi_*)}\phi_*^{1/2} - \frac{\hat{\xi}}{2P} = 0$$

$$V_{\rm in}(\phi) = V(\phi) + \delta V(\phi)$$

$$\delta V(\phi) = \frac{3W_0^2}{2} e^{-3\phi} P R\, \phi^{3/2}$$

$$\delta\phi = -\frac{\delta V'(\phi_*)}{V''(\phi_*)} = 4R\,\frac{\phi_* + \frac{\hat{\xi}}{2P}\,\phi_*^{1/2}}{2\phi_* - 1} \simeq 2R\phi_*$$

$$\delta\phi = \phi_{\rm in} - \phi_*$$

$$Y = \frac{\delta\varphi}{M_{pl}} = \sqrt{\frac{2}{3}}\delta\phi \simeq 2\sqrt{\frac{2}{3}} R\phi_* \simeq 0.1 - 1$$

We validate the generic arguments with explicit calculations

Inflationary Phenomenology

$$V = V_0 - \frac{4W_0 a_n A_n}{\mathcal{V}_{\rm in}^2}\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\exp\left[-a_n\left(\frac{3\mathcal{V}_{\rm in}}{4\lambda}\right)^{2/3}\sigma^{4/3}\right]$$

Canonical inflaton field $\quad V(\sigma) = C_0(1 - e^{-b\sigma})$

Effective single field dynamics

Volume modulus is stabilised during inflation and heavy

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{32\mathcal{V}_{\rm in}^3}{3\beta^2 W_0^2\lambda_n} a_n^2 A_n^2 \sqrt{\tau_n}\,(1 - a_n\tau_n)^2\, e^{-2a_n\tau_n}\,,$$

$$\eta = M_{\rm pl}^2\frac{V''}{V} = -\frac{4\mathcal{V}_{\rm in}^2}{3\beta W_0\lambda_n\sqrt{\tau_n}} a_n A_n\left[\left(1 - 9a_n\tau_n + 4a_n^2\tau_n^2\right)e^{-a_n\tau_n}\right].$$

Inflationary Phenomenology

$$N_e(\sigma) = \int_{\sigma_{\rm end}}^{\sigma} \frac{1}{\sqrt{2\epsilon(\sigma)}}\, d\sigma \;\simeq\; \frac{3\beta W_0 \lambda_n}{16\mathcal{V}_{\rm in}^2 a_n^{3/2} A_n} \frac{e^{a_n\tau_n}}{(a_n\tau_n)^{3/2}}$$

$$\epsilon \simeq \left(\frac{3\lambda_n}{8a_n^{3/2}\mathcal{V}_{\rm in}}\right)\frac{1}{N_e^2\sqrt{a_n\tau_n}} \quad \ll \quad \eta \simeq -\frac{1}{N_e}$$

COBE normalisation: $\dfrac{V^{3/2}}{M_{\rm pl}^3\, V'} = 5.2\times 10^{-4}$

$$\tau_n \simeq 7.31\cdot 10^{-14}\left(\frac{6\pi\lambda_n}{g_s\,\beta\, e^{K_{\rm cs}}}\right)^2\left(\frac{\mathcal{V}_{\rm in}^4}{W_0^4\, a_n^4}\right)\frac{1}{N_e^4}$$

$$r = 16\epsilon \simeq 16\times 3.7\cdot 10^6\left(\frac{g_s\,\beta\, e^{K_{\rm cs}}}{16\pi}\right)\left(\frac{W_0^2}{\mathcal{V}_{\rm in}^3}\right) \ll 10^{-4} N_e^{-3}$$

$$n_s = 1 + 2\eta - 6\epsilon \simeq 1 - \frac{2}{N_e}$$

spectral index depends only on N_e