Weak lensing with finite beams

Pierre Fleury

Presentation based on [1706.09383]
with J. Larena (UCT) and J.-P. Uzan (IAP)

Weak lensing

Light bending generates image distorsions:

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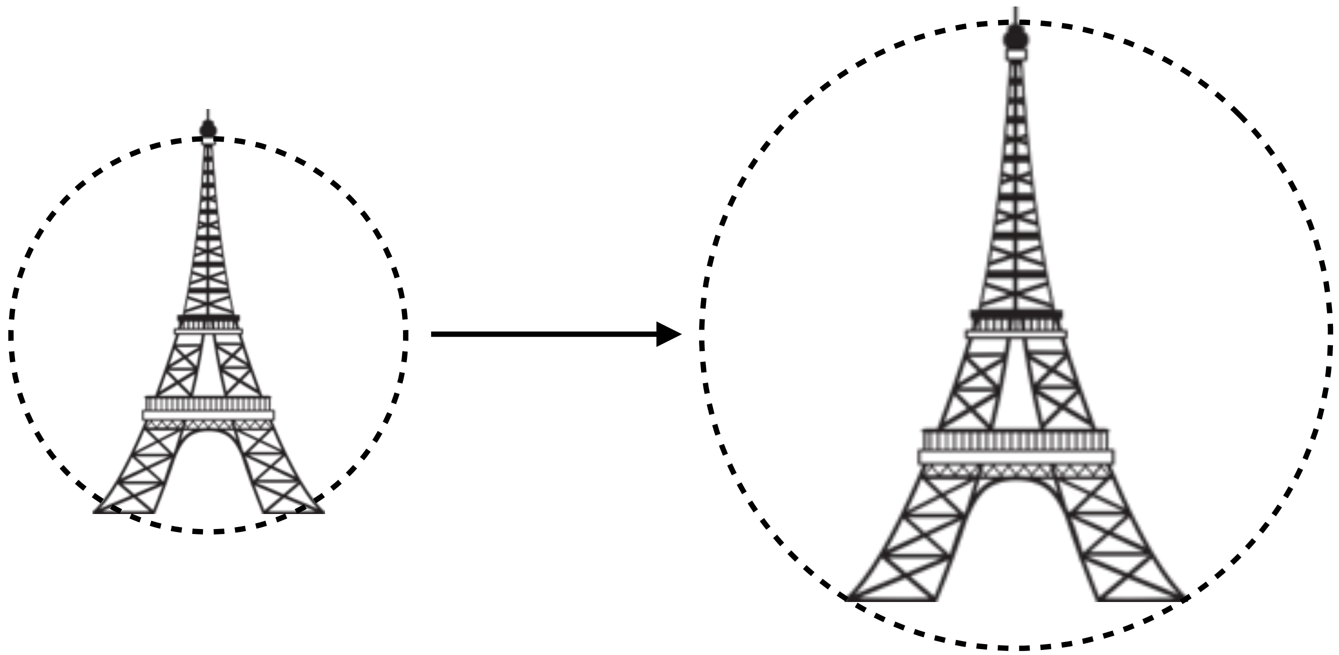
1. convergence



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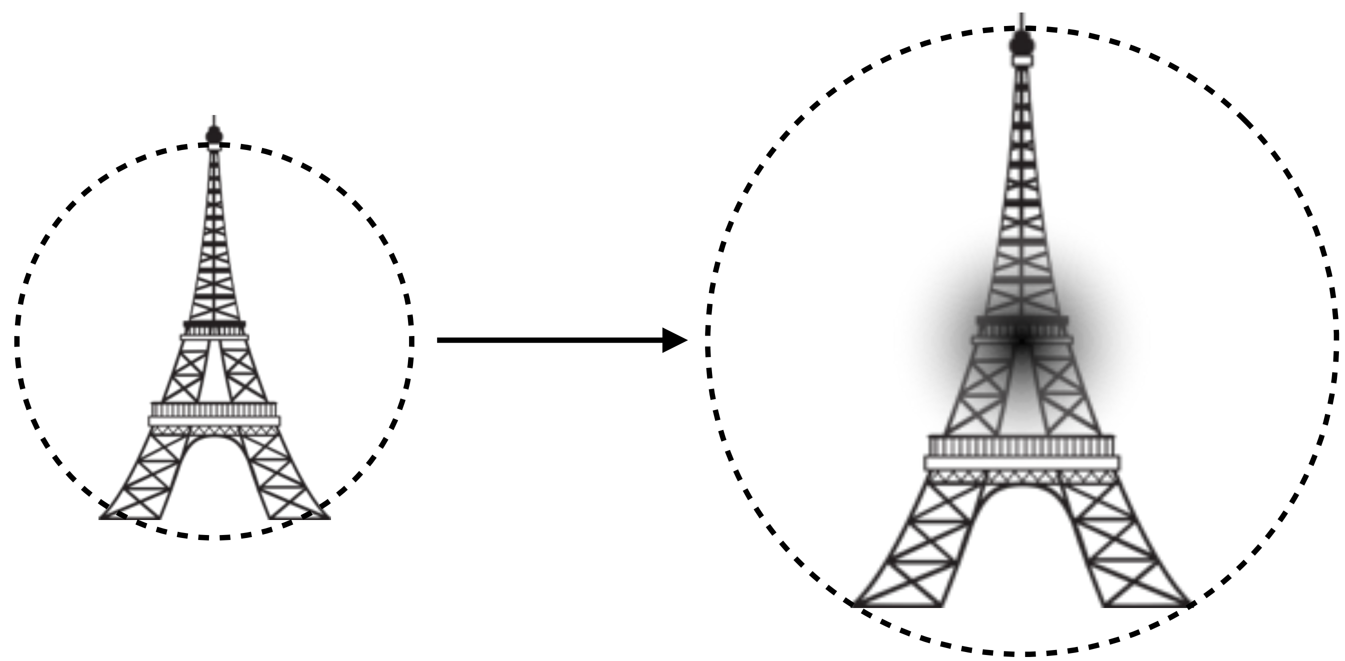

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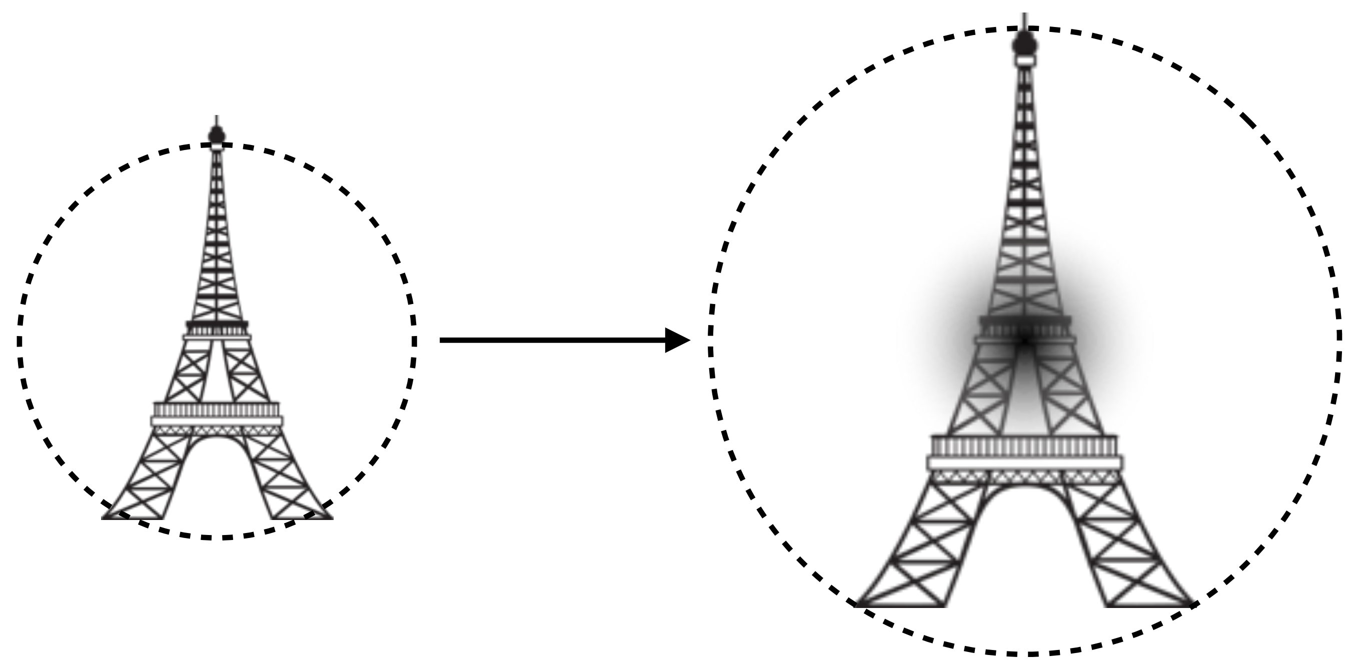
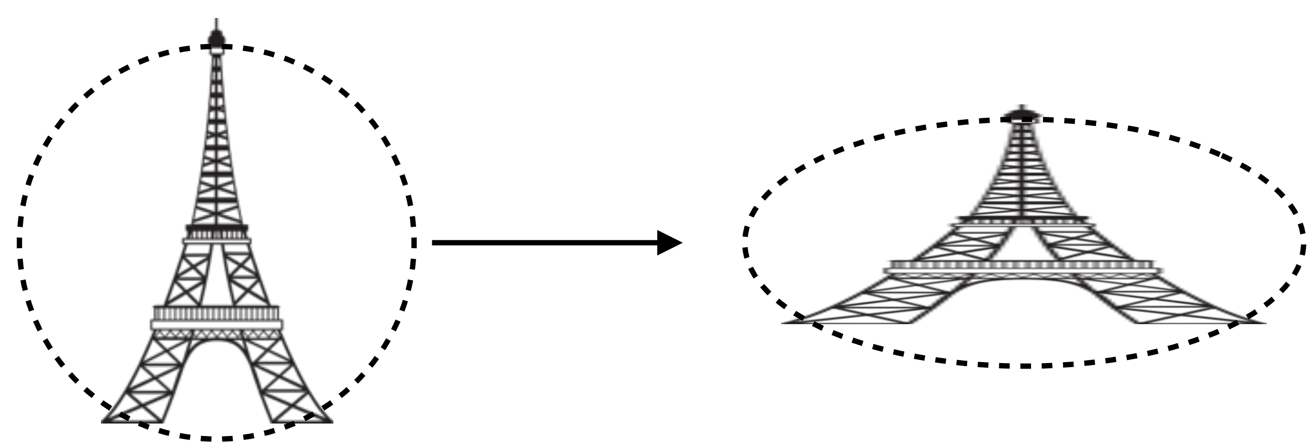


2. shear



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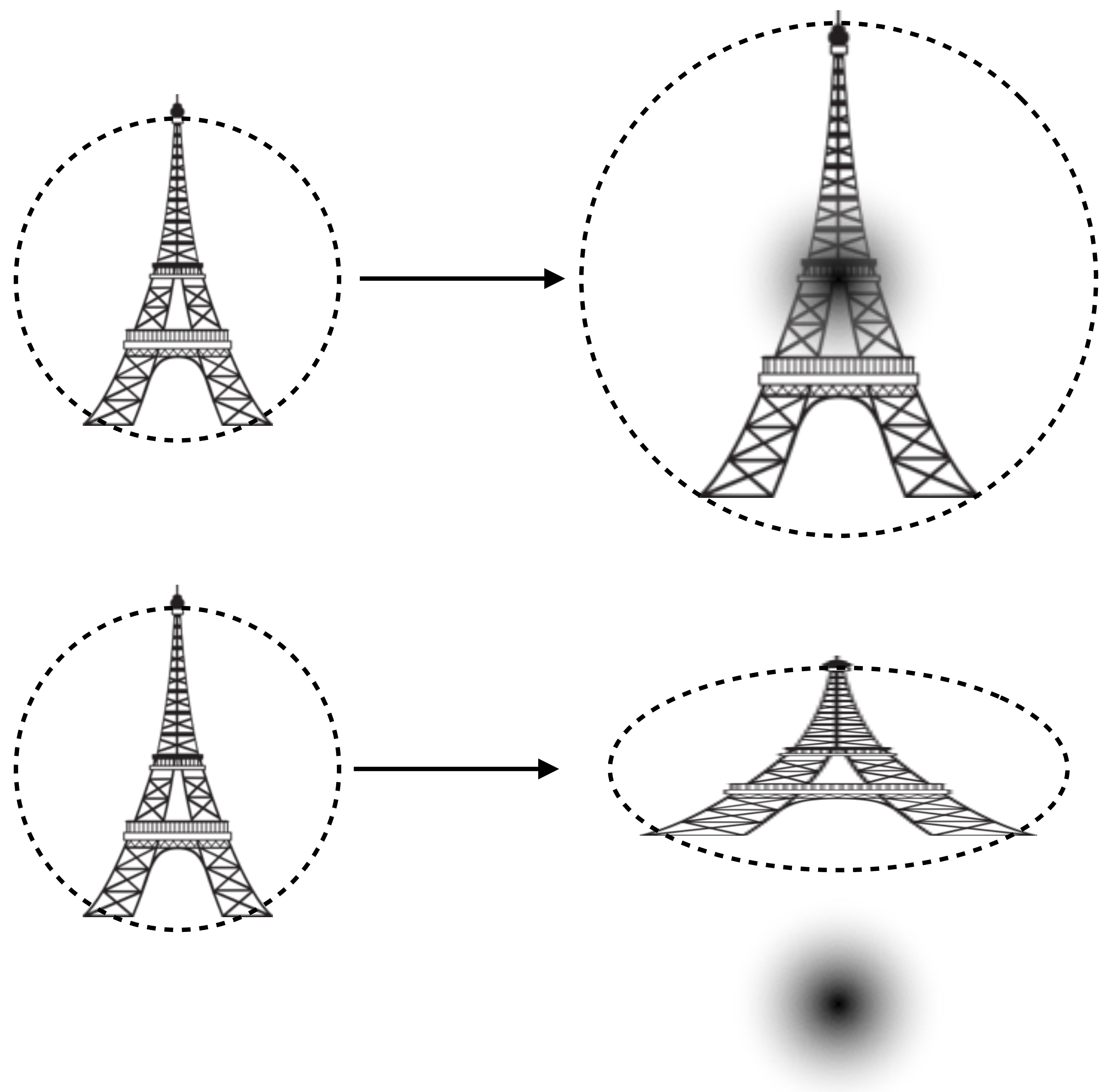
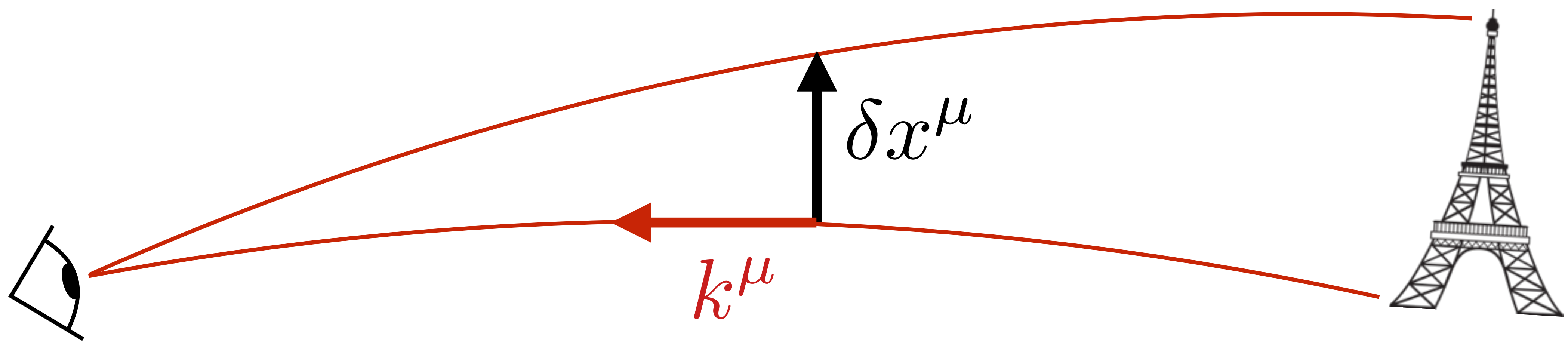


Theory: infinitesimal beams



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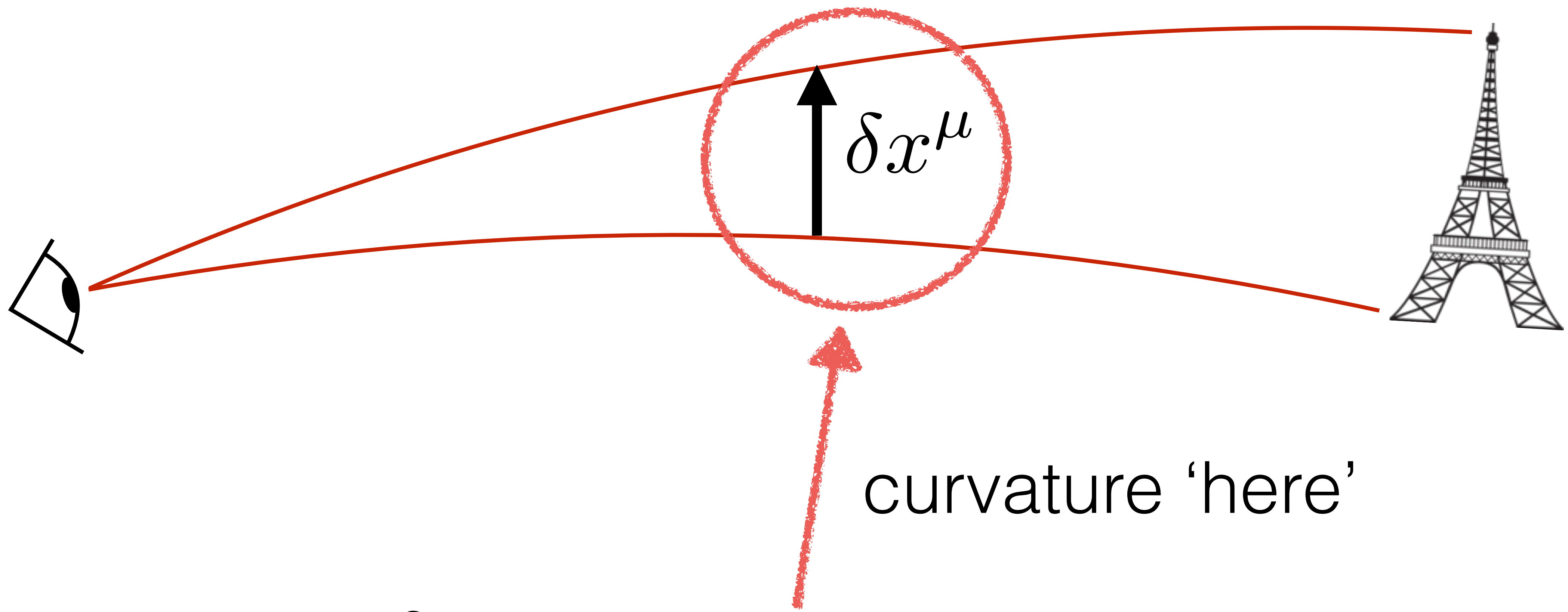

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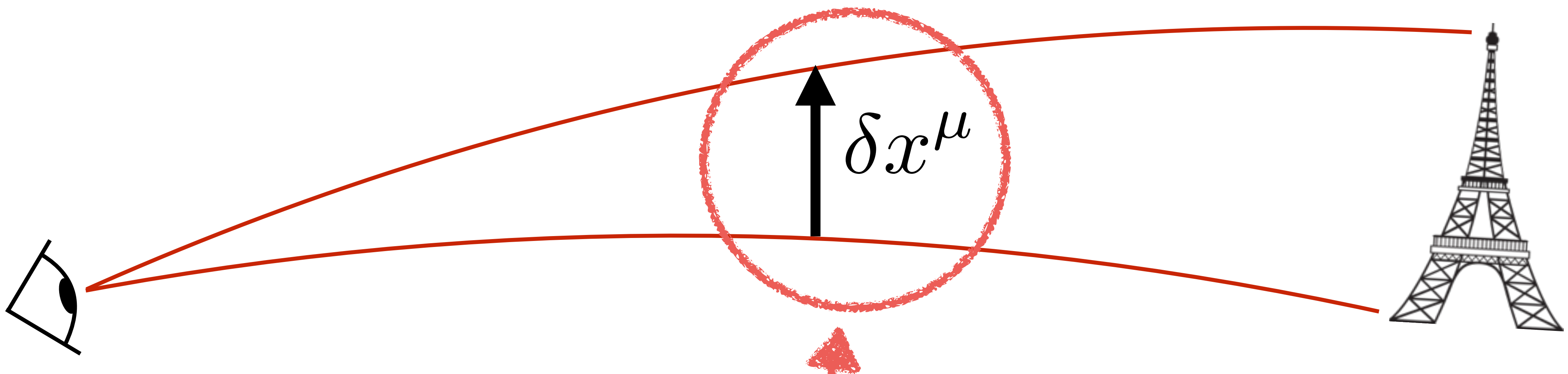

curvature 'here'

$$\frac{\mathrm{D}^2 \delta x^{\mu}}{\mathrm{d}\lambda^2} = R^{\mu}{}_{\nu\rho\sigma} k^{\nu} k^{\rho} \delta x^{\sigma}$$

Valid only if $|\delta x^{\mu}| \ll \rho / \partial \rho$

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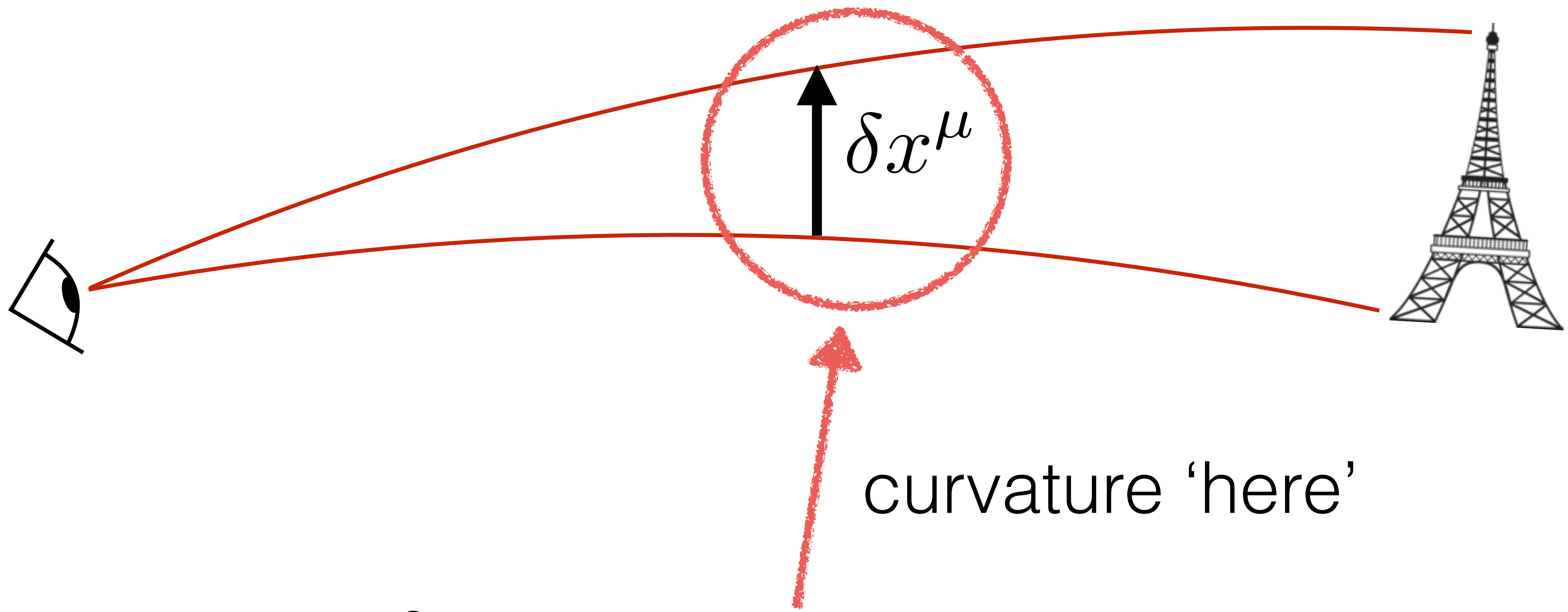

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Never true!

The Ricci/Weyl dichotomy

$$R_{\mu\nu\rho\sigma} = R_{[\mu\rho}g_{\nu\sigma]} + C_{\mu\nu\rho\sigma}$$

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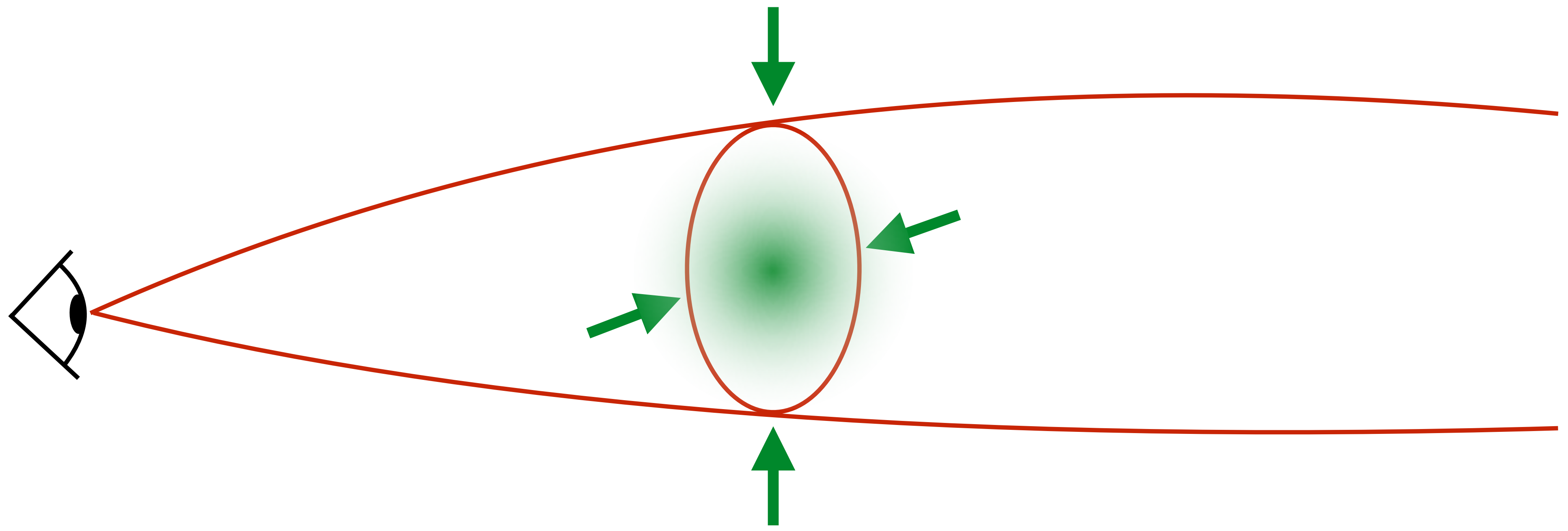
Ricci curvature
source of convergence



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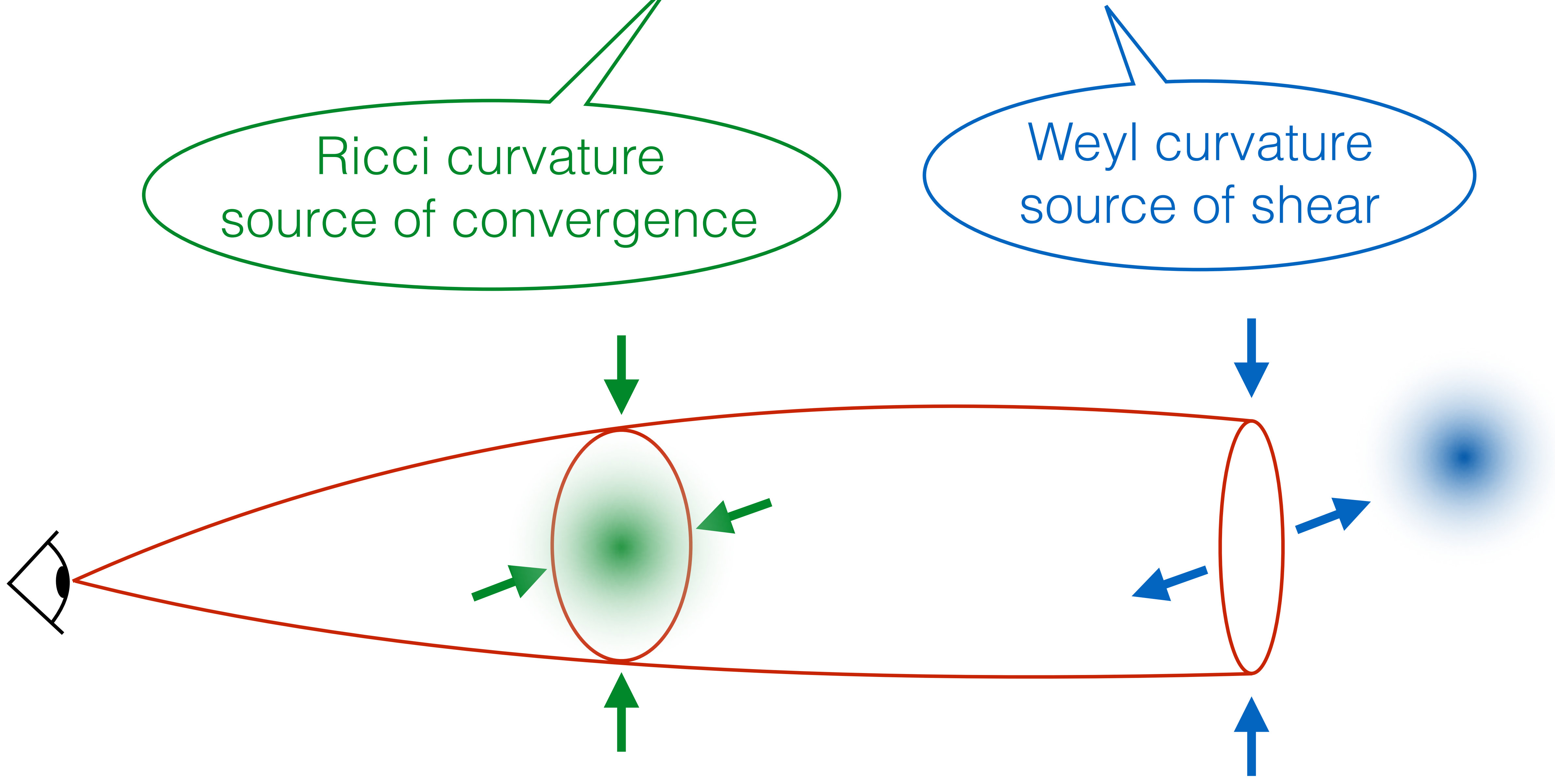

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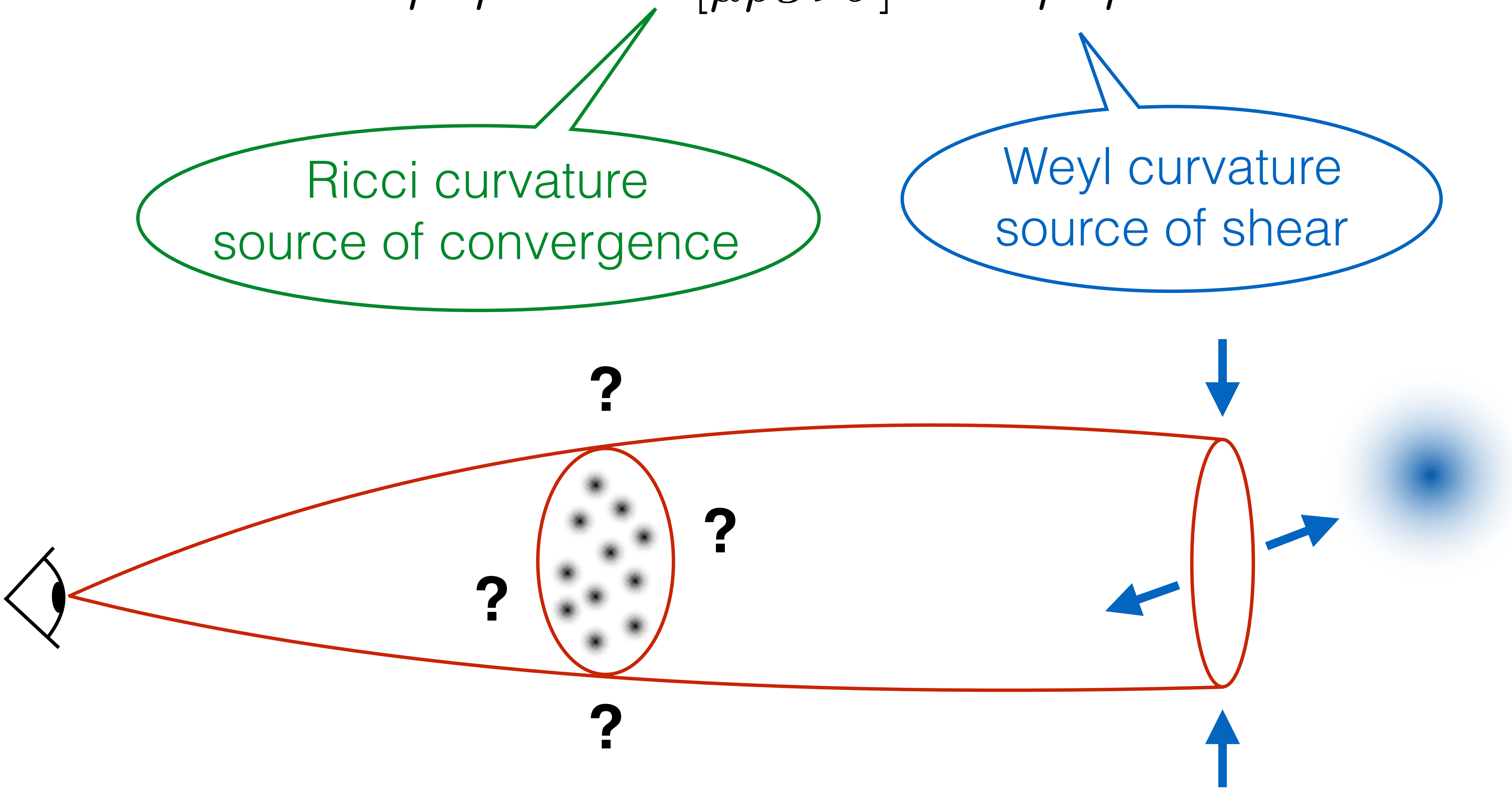

Questions

Pure theory

- How can spacetime curvature depend on the beam?
- Is there a Weyl to Ricci transition?

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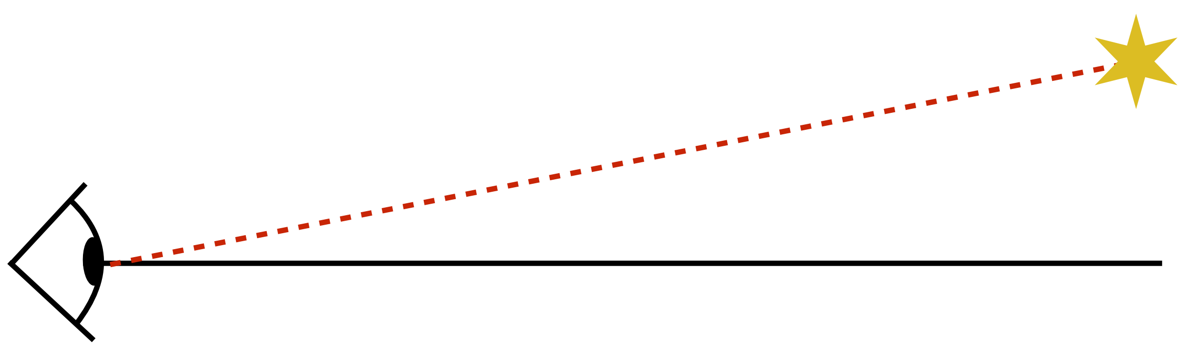
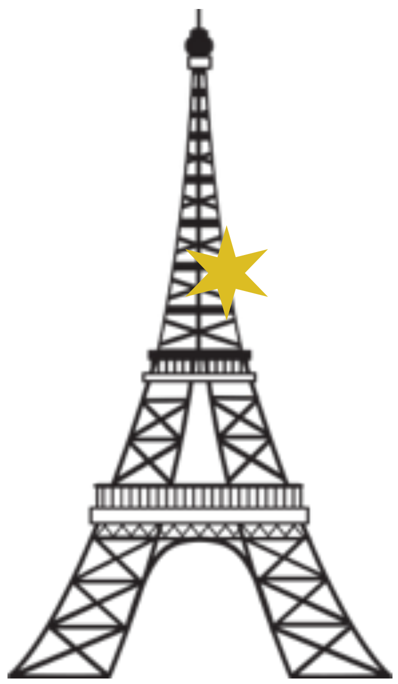
Observational cosmology

- To which extent can we trust the infinitesimal beam?
- Are there new observables for finite beams?

The strong rescuing the weak

Our method: deal with each ray of light individually





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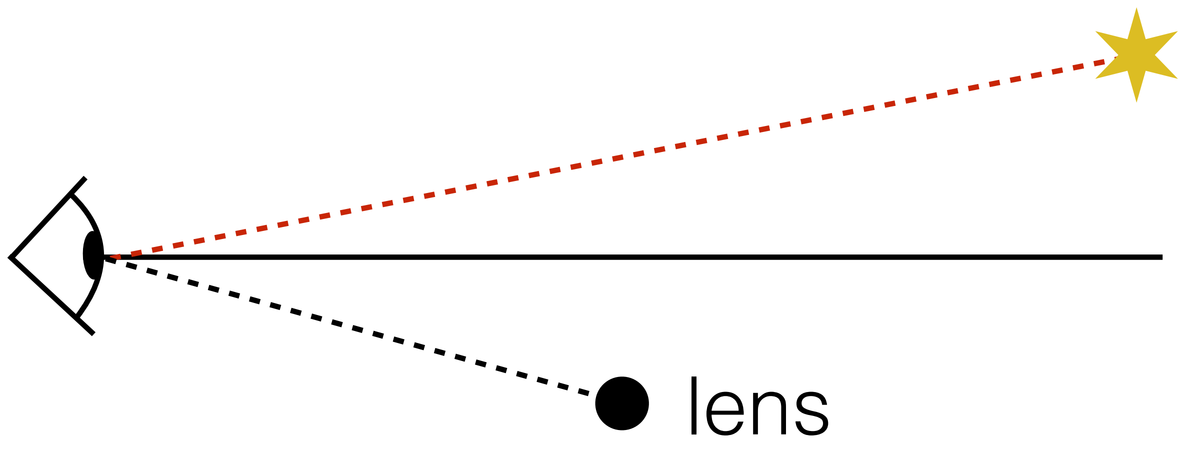

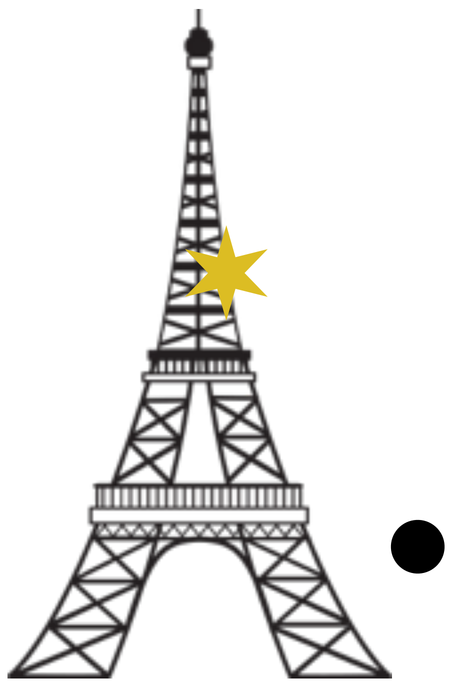

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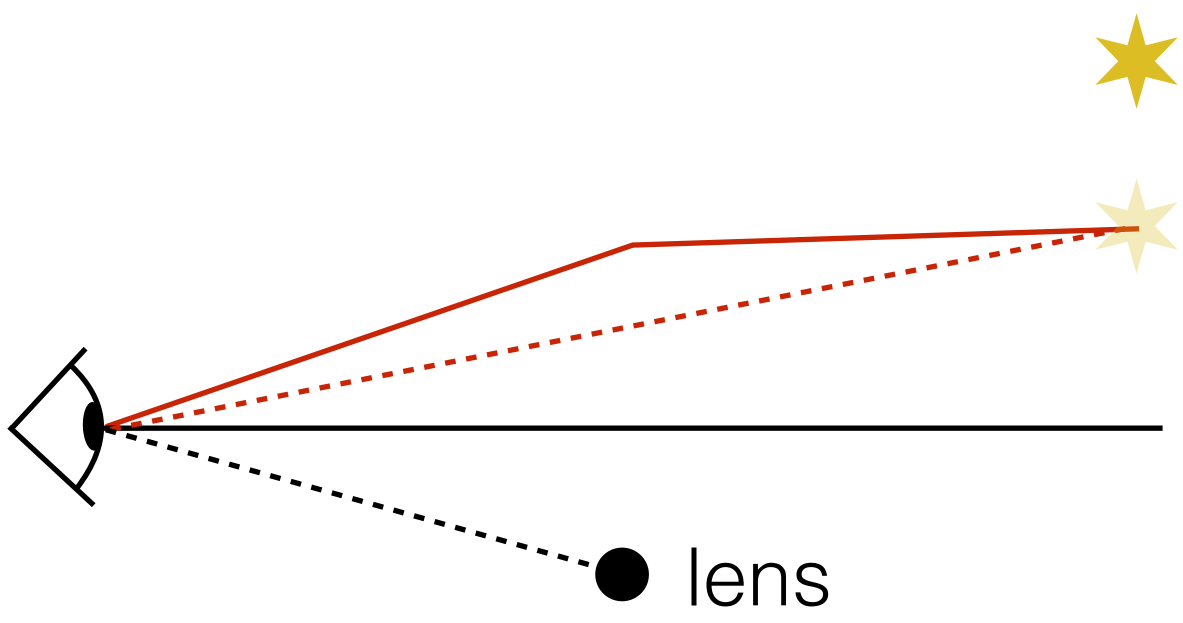

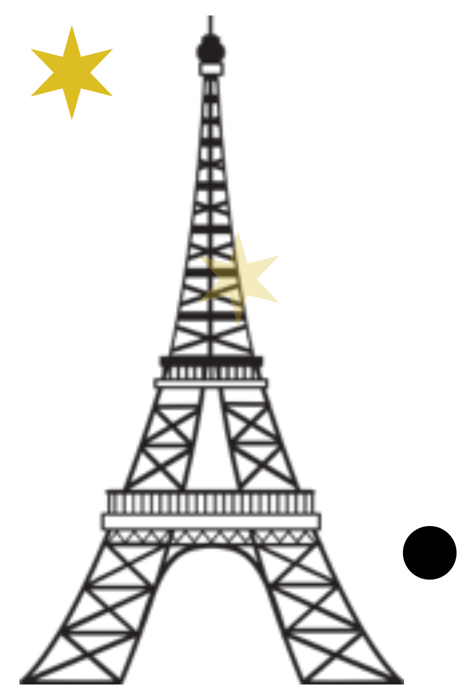

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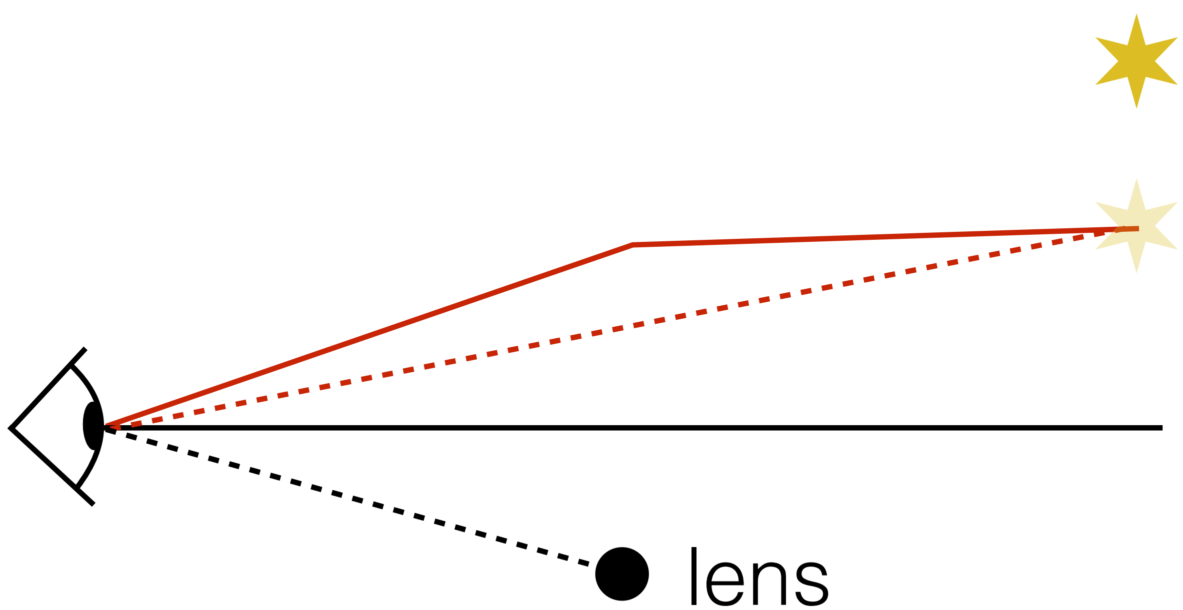

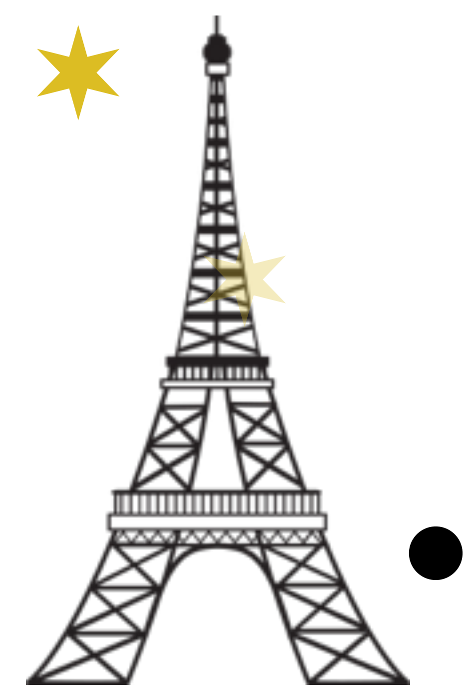

lens equation: $$\boldsymbol{\beta} = \boldsymbol{\theta} - \sum_{k=1}^{N} \varepsilon_k^2 \frac{\boldsymbol{\theta} - \boldsymbol{\theta}_k}{|\boldsymbol{\theta} - \boldsymbol{\theta}_k|^2}$$

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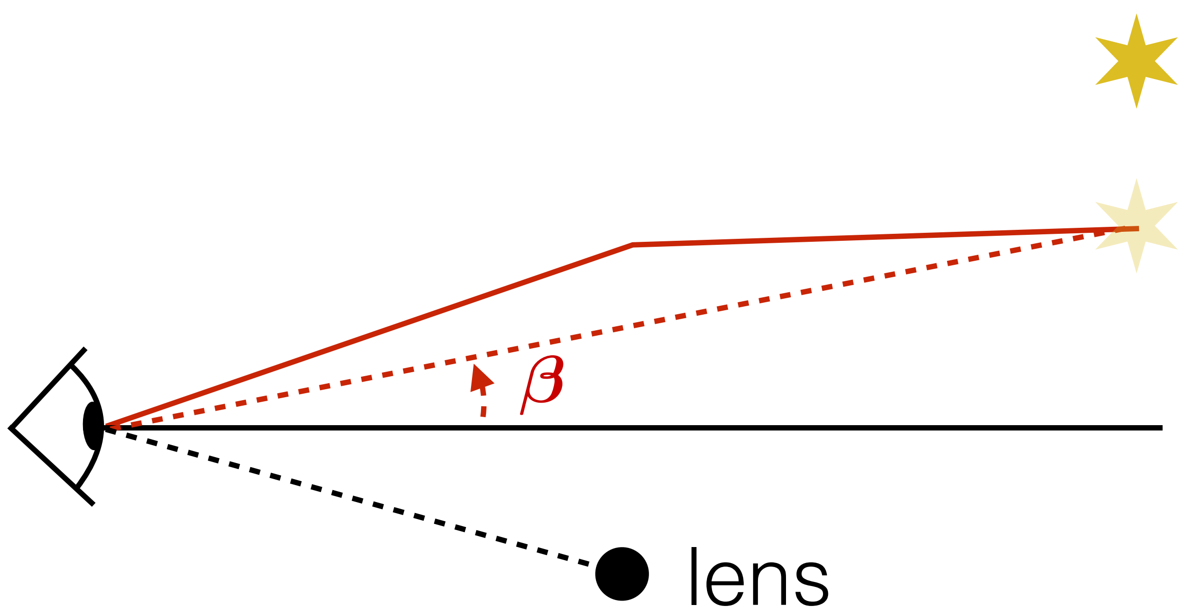

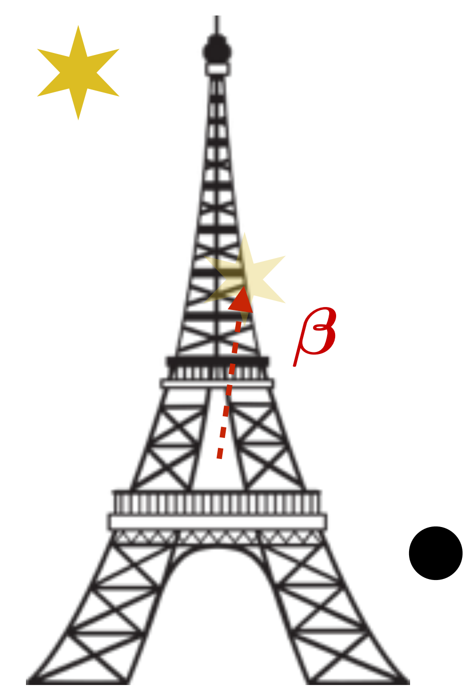

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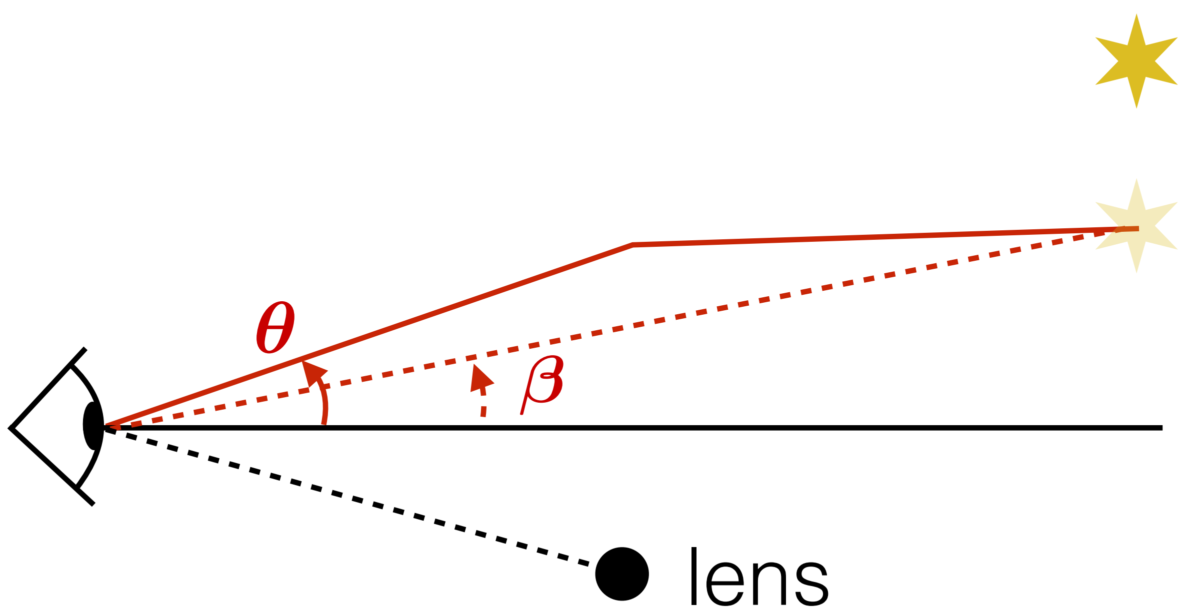

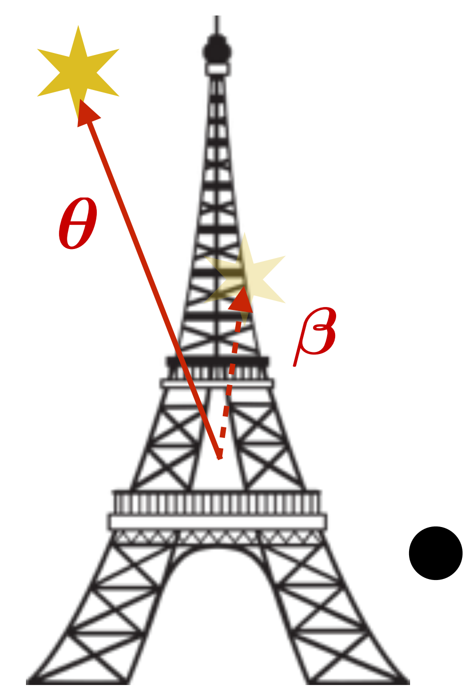

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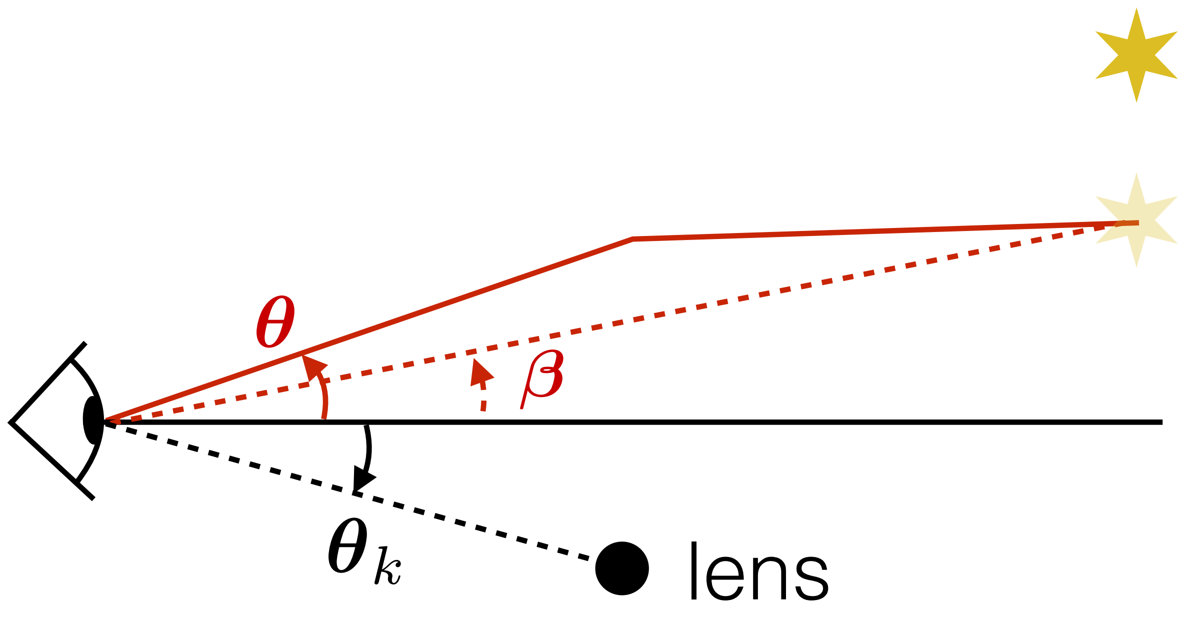

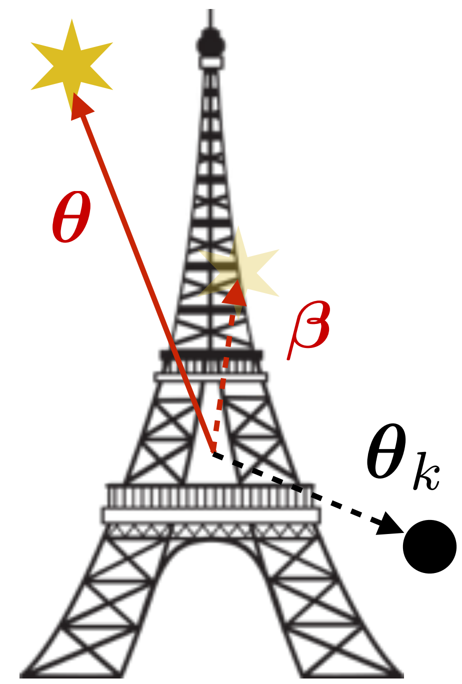

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Lensing can be complex

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Replace 2-vectors by complex numbers:

$$\boldsymbol{\beta} \to s$$

$$\boldsymbol{\theta} \to z$$

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↓

$$s = z - \sum_{k=1}^{N} \frac{\varepsilon_k^2}{z^* - w_k^*}$$

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$$\Omega = \frac{1}{2\mathrm{i}} \int_{\mathcal{I}} z^* \mathrm{d}z$$

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Only lenses **enclosed** by the beam contribute

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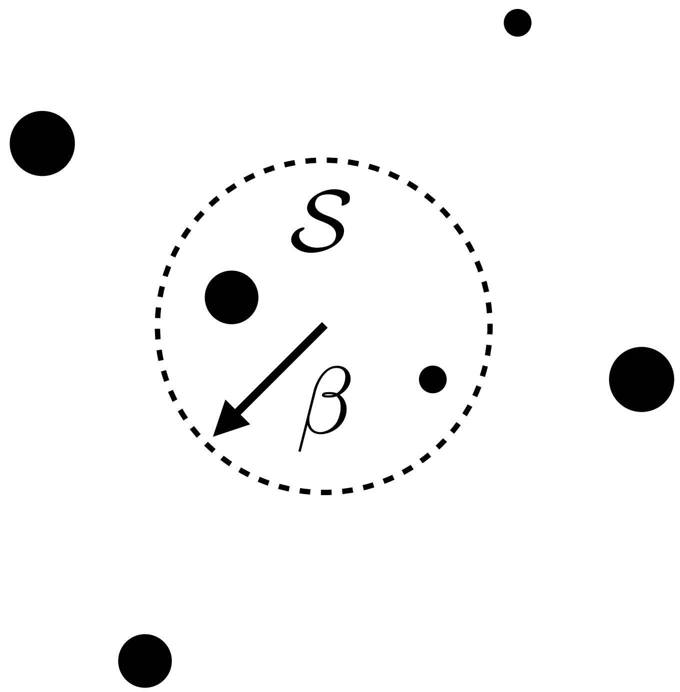
Residue theorem

Only lenses **enclosed** by the beam contribute

- Light beams smooth out the matter they encounter
- No Weyl to Ricci transition, just Ricci!

Shear



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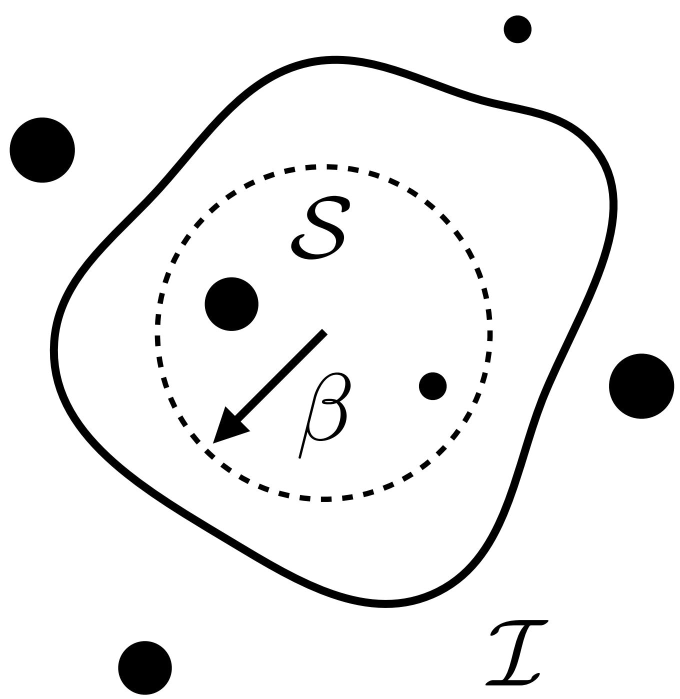

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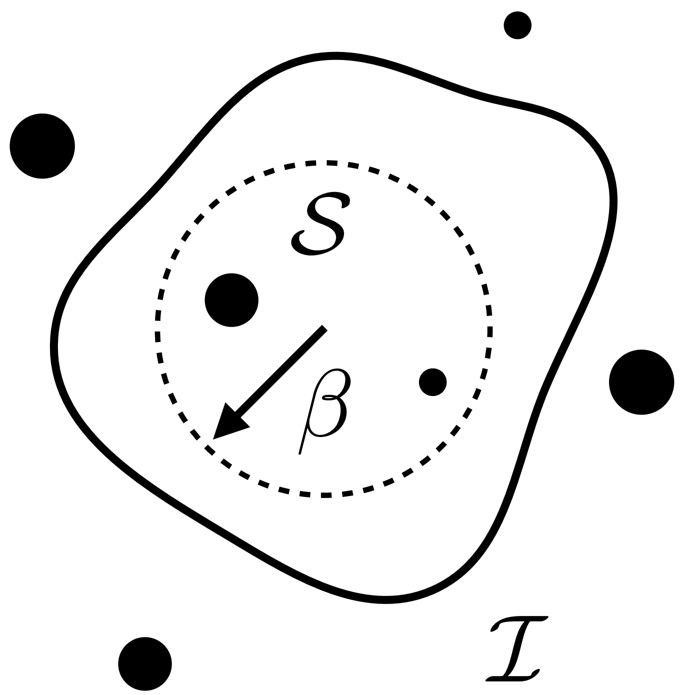

From the image quadrupole

$$\gamma = \sum_{k \in \mathcal{S}} \left(\frac{\varepsilon_k w_k}{\beta^2} \right)^2 - \sum_{k \notin \mathcal{S}} \left(\frac{\varepsilon_k}{w_k} \right)^2$$

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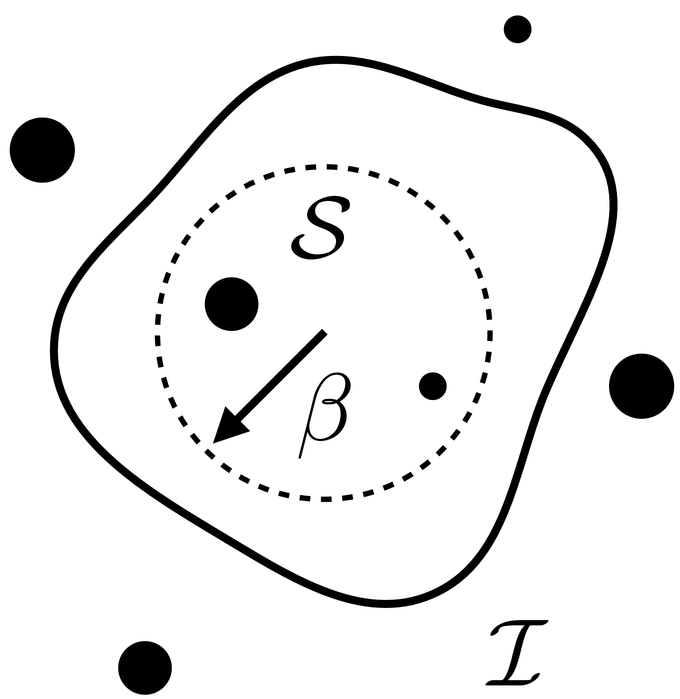

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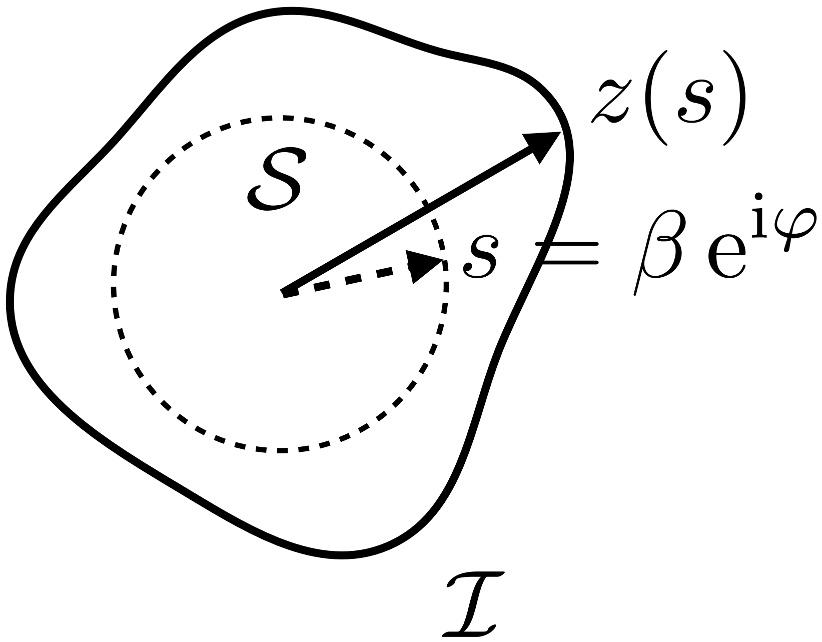
New contribution from interior lenses!

Higher deformation modes



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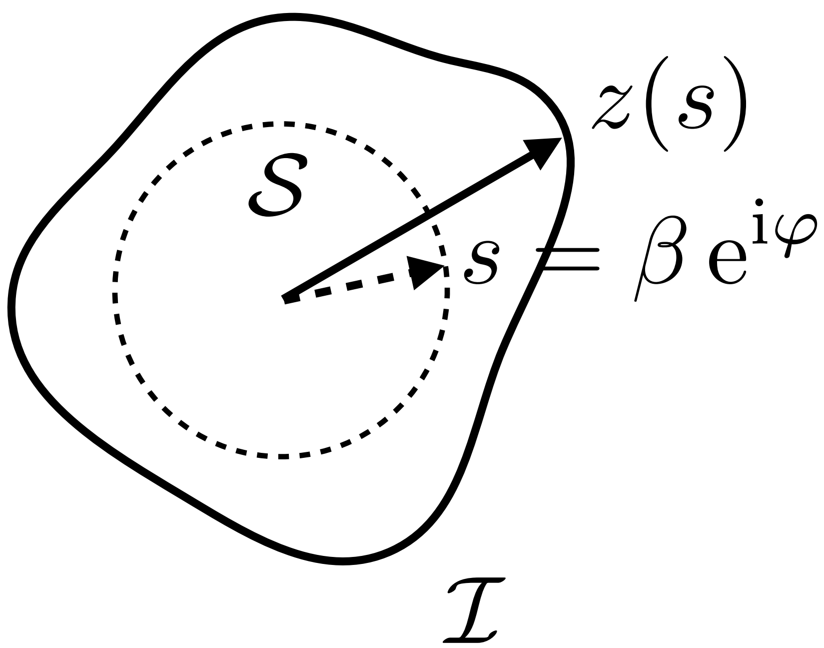

Fourier decomposition:

$$z = \sum_{\ell \in \mathbb{Z}} z_\ell\, \mathrm{e}^{\mathrm{i}(\ell+1)\varphi}$$

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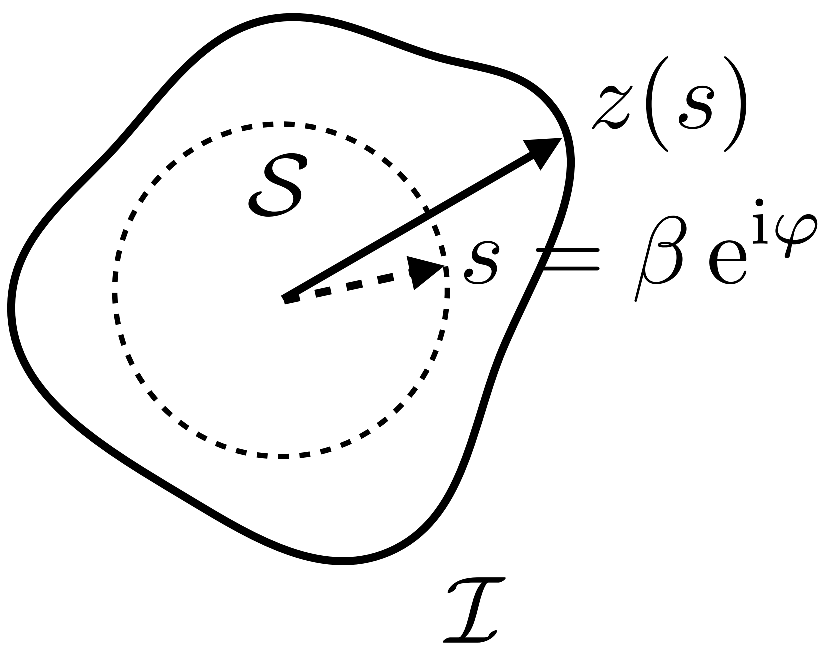

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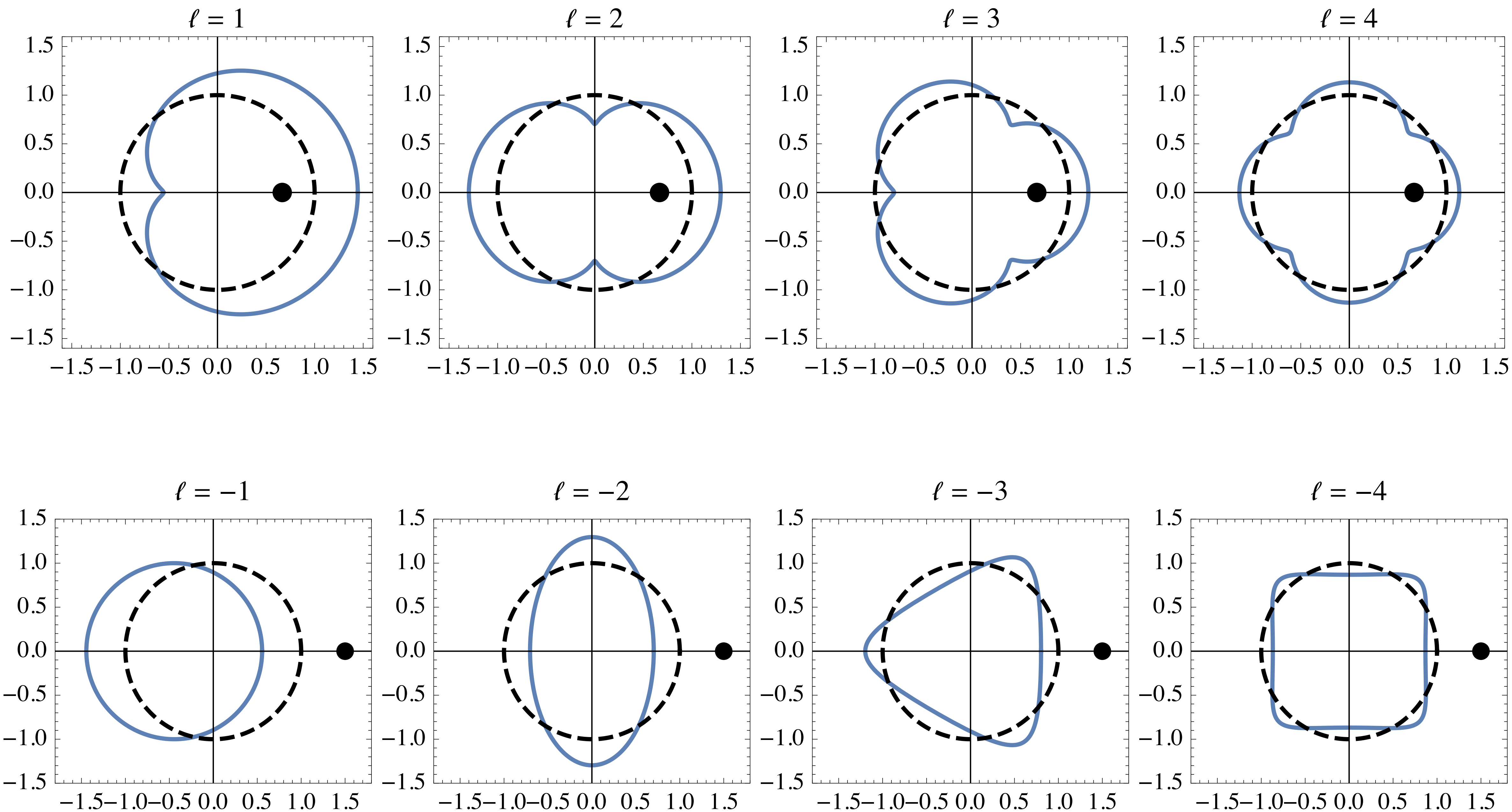
$$z_{\ell>0} = \frac{1}{\beta} \sum_{k \in \mathcal{S}} \varepsilon_k^2 \left(\frac{w_k^*}{\beta} \right)^\ell$$

$$z_{\ell<0} = -\frac{1}{\beta} \sum_{k \notin \mathcal{S}} \varepsilon_k^2 \left(\frac{w_k^*}{\beta} \right)^\ell$$

Higher deformation modes



Shear statistics

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But for with finite beams, in a Universe randomly filled with point masses, we find

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Conclusion

Context

- standard weak lensing assumes infinitesimal beams
- it raises theoretical and observational issues

Our work

used the strong-lensing language to deal with finite beams

Results

- understand the Ricci/Weyl dichotomy
- generic violation of shear-convergence relation
- new observables?