

# **Stable bouncing universe in Hořava-Lifshitz Gravity**



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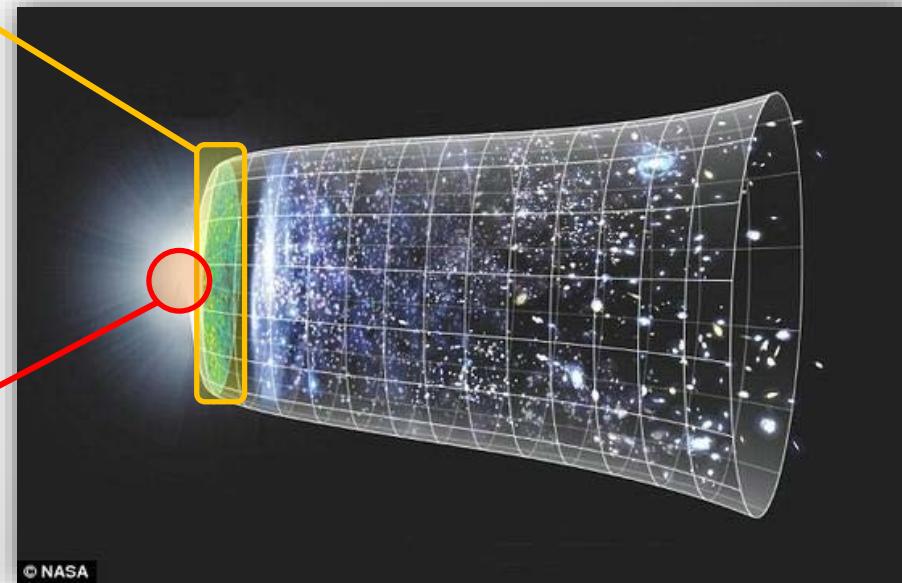
Based on "Phys. Rev. D95 044044 (2017)"

## Introduction: BIG BANG Singularity

### ✓ Inflation & BIG BANG Cosmology

$$w = -1$$

- “initial condition” of the Universe  
(flatness, horizon problem, monopole, ...)
- Seed of LSS by quantum fluctuations
- BIG BANG Nucleosynthesis
- Cosmic Microwave Background



### ✓ BIG BANG Singularity

Initial singularity must be appeared at finite past in the Standard Cosmology

To avoid singularity : **Violation of null energy condition (NEC)**

$$\rho + p = (1 + w)\rho < 0$$

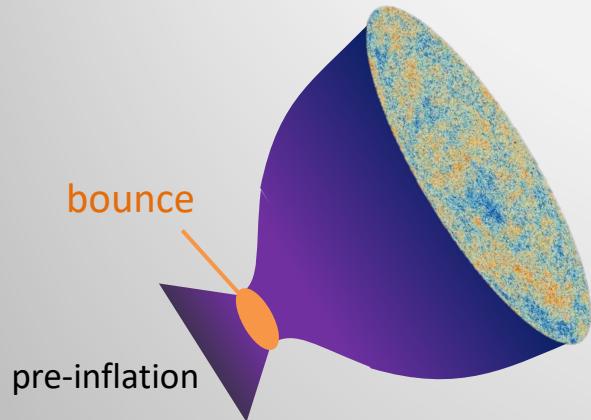
inflation cannot solve singularity problem

# Introduction: Singularity Avoidance in Hořava-Lifshitz Gravity

- ✓ Singularity-free background solution based on Hořava-Lifshitz (HL) Theory

[Brandenberger 2009 / Maeda, Misonoh, Kobayashi 2010 / Misonoh, Maeda, Kobayashi 2011]

→ Null energy condition is effectively violated by quantum correction of gravity



Singularity is avoided  
by **bouncing universe**

Spatially curved BG  
work as effective **exotic matter**  
“dark radiation” & “dark stiff matter”  
 $\rho < 0$

## Our Motivation

- To investigate the **stability** of singularity-free bouncing solutions in non-flat FLRW BG based on HL theory
- Can we obtain further information about bouncing solution ?

# Hořava-Lifshitz Theory

[Hořava 2009]

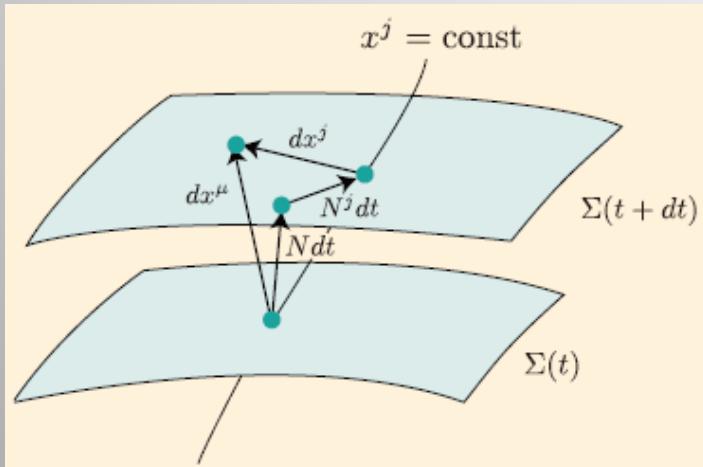
- ✓ Lorentz violated gravitational theory

$$\text{Lifshitz scaling} \quad x \rightarrow b^{-1}x, \quad t \rightarrow b^{-z}t \quad \longrightarrow \quad [x] = -1, \quad [t] = -z$$

- ✓ (Power counting) Renormalizable theory  $[\kappa^2] = z - 3$  gravitational coupling

- $z = 3$
- 2nd power of time derivative : No Ostrogradski instabilities
  - 6th power of spatial derivative : Reduce UV divergence

$$\longrightarrow S = \frac{1}{2\kappa^2} \int dt d^3x N \sqrt{\gamma} \left[ \mathcal{K}_{ij} \mathcal{K}^{ij} - \lambda \mathcal{K}^2 - 2\Lambda + \mathcal{R} + \mathcal{O}(\mathcal{R}^2) + \mathcal{O}(\mathcal{R}^3) \right]$$



ADM form

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

extrinsic curvature

$$\begin{aligned} \mathcal{K}_{ij} &= \frac{1}{2N} [\partial_t \gamma_{ij} - 2\nabla_{(i} N_{j)}] \\ &\sim \partial_t \end{aligned}$$

3d Ricci tensor

$$\mathcal{R}_{ij} \sim \partial_x^2$$

## (projectable) Hořava-Lifshitz Theory

[Hořava 2009 / Sotiriou, Visser, Weinfurtner 2009]

$$\mathcal{S} = \frac{m_{LV}^2}{2} \int dt d^3x (\mathcal{L}_K + \mathcal{L}_P) \quad N = N(t) \quad \text{projectability condition}$$

$$\mathcal{L}_K := N\sqrt{\gamma}(\mathcal{K}_{ij}\mathcal{K}^{ij} - \lambda\mathcal{K}^2) \quad \mathcal{L}_P := -N\sqrt{\gamma}\left[\mathcal{V}_{z=1} + m_{LV}^{-2}\mathcal{V}_{z=2} + m_{LV}^{-4}\mathcal{V}_{z=3}\right]$$

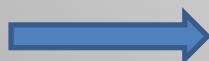
$$\mathcal{V}_{z=1} := 2\Lambda - \mathcal{R}$$

$$\mathcal{V}_{z=2} := g_2\mathcal{R}^2 + g_3\mathcal{R}^i{}_j\mathcal{R}^j{}_i$$

$$\mathcal{V}_{z=3} := g_4\mathcal{R}^3 + g_5\mathcal{R}\mathcal{R}^i{}_j\mathcal{R}^j{}_i + g_6\mathcal{R}^i{}_j\mathcal{R}^j{}_k\mathcal{R}^k{}_i + g_7\mathcal{R}\nabla^2\mathcal{R} + g_8\nabla_i\mathcal{R}_{jk}\nabla^i\mathcal{R}^{jk}$$

✓ Projectable HL Theory is renormalizable [Barvinsky, et. al. 2016]

- All kind of UV divergence can be canceled with finite counter terms
- Running coupling constants ( $\lambda, \Lambda, g_n$ ) are determined via  $\beta$ -functions



The values of coupling constants might be calculated in near future

## Bouncing solutions in HL Theory

[Brandenberger 2009 / Maeda, Misonoh, Kobayashi 2010 / Misonoh, Maeda, Kobayashi 2011]

✓ FLRW background

$$ds^2 = -dt^2 + \underbrace{a^2(t)}_{\text{scale factor}} \left[ d\chi^2 + f^2(\chi) (d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

spatial curvature

$$f(\chi) = \begin{cases} \chi & (K = 0) \\ \sin \chi & (K = 1) \\ \sinh \chi & (K = -1) \end{cases}$$

<b>flat</b>	<b>closed</b>
<b>open</b>	

Friedmann equation ( $m_{LV} = 1$ )

$$H^2 = \frac{2}{3(3\lambda - 1)} \left[ \Lambda - 3\frac{K}{a^2} + \boxed{g_r \frac{K^2}{a^4}} + \boxed{g_s \frac{K^3}{a^6}} \right]$$

$$g_r := 6(3g_2 + g_3)$$

$$g_s := 12(9g_4 + 3g_5 + g_6)$$

# Bouncing solutions in HL Theory

[Brandenberger 2009 / Maeda, Misonoh, Kobayashi 2010 / Misonoh, Maeda, Kobayashi 2011]

$$\begin{aligned}\mathcal{V}_{z=1} &:= 2\Lambda - \mathcal{R} \\ \mathcal{V}_{z=2} &:= g_2 \mathcal{R}^2 + g_3 \mathcal{R}^i_j \mathcal{R}^j_i \\ \mathcal{V}_{z=3} &:= g_4 \mathcal{R}^3 + g_5 \mathcal{R} \mathcal{R}^i_j \mathcal{R}^j_i + g_6 \mathcal{R}^i_j \mathcal{R}^j_k \mathcal{R}^k_i + g_7 \mathcal{R} \nabla^2 \mathcal{R} + g_8 \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk}\end{aligned}$$

## Friedmann equation

$$H^2 = \frac{2}{3(3\lambda - 1)} \left[ \Lambda - 3\frac{K}{a^2} + \boxed{g_r \frac{K^2}{a^4}} + \boxed{g_s \frac{K^3}{a^6}} \right]$$

“radiation”  
“stiff matter”



NEC is violated by higher curvature terms  
in non-flat FLRW background

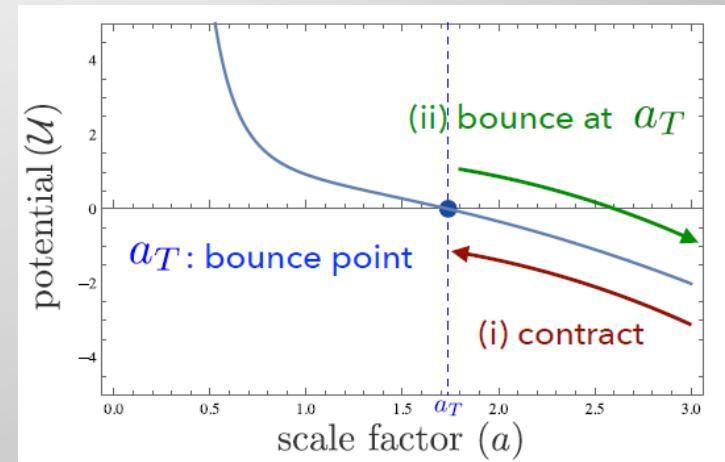
$$\frac{1}{2} \dot{a}^2 + \mathcal{U}(a) = 0 \longrightarrow \mathcal{U}(a) < 0$$

$$\mathcal{U}(a) = \frac{1}{3\lambda - 1} \left[ -\frac{\Lambda}{3} a^2 + K - \boxed{g_r \frac{K^2}{3a^2}} - \boxed{g_s \frac{K^3}{3a^4}} \right] \quad (\lambda > 1/3)$$

BG dynamics is determined only by  $(K, \Lambda, g_r, g_s)$  : degenerated

$$g_r := 6(3g_2 + g_3)$$

$$g_s := 12(9g_4 + 3g_5 + g_6)$$



# Perturbation analysis around non-flat FLRW BG

- ✓ Perturbed ADM variables

$$N = \bar{N} + \alpha \quad N_i = \beta_i \quad \gamma_{ij} = \bar{\gamma}_{ij} + h_{ij}$$

Expand them with 10 types of (pseudo-)spherical harmonics basis

[Lifshitz, Khalatnikov 1963 / Sandberg 1978 / Tomita 1982]

$$\mathbf{Y} = \left\{ Q, Q_i, Q_{ij}, P_{ij}, S_{(o)i}, S_{(e)i}, S_{(o)ij}, S_{(e)ij}, G_{(o)ij}, G_{(e)ij} \right\}$$

$$\alpha^{(\text{scalar})} = \cancel{\alpha(t)}$$

$$\beta_i^{(\text{scalar})} = \sum_{n,l,m} a^2 \left[ \beta_{(Q)}^{(n;lm)}(t) Q_i^{(n;lm)}(\chi, \theta, \phi) \right]$$

$$h_{ij}^{(\text{scalar})} = \sum_{n,l,m} a^2 \left[ h_{(Q)}^{(n;lm)}(t) Q_{ij}^{(n;lm)}(\chi, \theta, \phi) + h_{(P)}^{(n;lm)}(t) P_{ij}^{(n;lm)}(\chi, \theta, \phi) \right]$$

$$\beta_i^{(\text{vector})} = \sum_{n,l,m} a^2 \left[ \beta_{(S;o)}^{(n;lm)}(t) S_{(o)i}^{(n;lm)}(\chi, \theta, \phi) + \beta_{(S;e)}^{(n;lm)}(t) S_{(e)i}^{(n;lm)}(\chi, \theta, \phi) \right]$$

$$h_{ij}^{(\text{vector})} = \sum_{n,l,m} a^2 \left[ h_{(S;o)}^{(n;lm)}(t) S_{(o)ij}^{(n;lm)}(\chi, \theta, \phi) + h_{(S;e)}^{(n;lm)}(t) S_{(e)ij}^{(n;lm)}(\chi, \theta, \phi) \right]$$

$$h_{ij}^{(\text{tensor})} = \sum_{n,l,m} a^2 \left[ h_{(G;o)}^{(n;lm)}(t) G_{(o)ij}^{(n;lm)}(\chi, \theta, \phi) + h_{(G;e)}^{(n;lm)}(t) G_{(e)ij}^{(n;lm)}(\chi, \theta, \phi) \right]$$

$$n \geq 1, \quad 0 \leq l \leq n-1, \quad 0 \leq |m| \leq l$$

gauge fixing

$$\alpha, h_{(P)}, h_{(S;o)}, h_{(S;e)}$$

constraint equation

$$\beta_{(Q)}, \beta_{(S;o)}, \beta_{(S;e)}$$

remaining modes

$$h_{(Q)}, h_{(G;o)}, h_{(G;e)}$$

## Perturbation analysis around non-flat FLRW BG

remaining modes

$$h_{(Q)}, h_{(G;o)}, h_{(G;e)}$$

- ✓ Quadratic HL action in non-flat FLRW (**scalar** part)

$$\delta_{(2)}\mathcal{L}^{(s)} = \sum_{n,l,m} \frac{a^3}{2} \left[ \mathcal{F}_{(Q)} \dot{h}_{(Q)}^2 - \mathcal{G}_{(Q)} h_{(Q)}^2 \right]$$

$$\mathcal{F}_{(Q)} := \frac{2(3\lambda - 1)(\nu^2 - 3K)}{3[(\lambda - 1)\nu^2 + 2K]}$$

$$\begin{aligned} \mathcal{G}_{(Q)} := & -\frac{2}{3a^2}(\nu^2 - 3K) + \frac{2}{27a^4}(\nu^2 - 3K)[2g_r(2\nu^2 - 3K) + 3g_3\nu^2] \\ & + \frac{2}{9a^6}(\nu^2 - 3K)[g_s K(4\nu^2 - 9K) + 2(3g_5 + 3g_6 - 4g_7)K\nu^2 + (-8g_7 + 3g_8)\nu^2(3\nu^2 - 10K)] \end{aligned}$$

$$\nu^2 := \begin{cases} n^2 - 1, & n \in \mathbb{R} \quad \text{for } K = 0 \\ n^2 - 1, & n \in \mathbb{N} \quad \text{for } K = 1 \\ n^2 + 1, & n \in \mathbb{R} \quad \text{for } K = -1 \end{cases}$$

- ✓ Quadratic HL action in non-flat FLRW (**tensor** part)

$$\delta_{(2)}\mathcal{L}^{(t)} = \sum_{n,l,m} \frac{a^3}{2} \left[ \mathcal{F}_{(G)} \dot{h}_{(G)}^2 - \mathcal{G}_{(G)} h_{(G)}^2 \right] \quad \xleftarrow{\hspace{1cm}} \quad h_{(G;o)}, h_{(G;e)}$$

$$\mathcal{F}_{(G)} := 1$$

$$\mathcal{G}_{(G)} := \frac{\nu^2}{a^2} + \frac{\nu^2}{3a^4} \left[ -2g_r K + 3g_3\nu^2 \right] + \frac{\nu^2}{a^6} \left[ -g_s K^2 + 6(g_5 + g_6)K\nu^2 + g_8\nu^2(\nu^2 - 2K) \right]$$

## Ghost modes avoidance

$$\delta_{(2)}\mathcal{L} = \sum_{n,l,m} \frac{a^3}{2} \left[ \mathcal{F}\dot{h}^2 - \mathcal{G}h^2 \right] \longrightarrow \boxed{\mathcal{F} > 0} \quad \text{ghost avoidance}$$

### I. Tensor mode

$$\mathcal{F}_{(G)} := 1$$

$$\nu^2 := \begin{cases} n^2 - 1, & n \in \mathbb{R} \quad \text{for } K = 0 \\ n^2 - 1, & n \in \mathbb{N} \quad \text{for } K = 1 \\ n^2 + 1, & n \in \mathbb{R} \quad \text{for } K = -1 \end{cases} \quad n \geq 1$$

### II. Scalar mode

$$\mathcal{F}_{(Q)} := \frac{2(3\lambda - 1)(\nu^2 - 3K)}{3[(\lambda - 1)\nu^2 + 2K]}$$

(a) flat case  $K = 0$

$$\lambda > 1$$

(b) closed case  $K = 1$

$$\lambda \geq 1 \quad n \geq 3$$

(c) open case  $K = -1$

$$\lambda > 2$$

- $n = 1$  : correspond to shift of scale factor  $a(t) \rightarrow a(t) + \delta a(t)$
- $n = 2$  : automatically vanish  $\mathcal{F}_{(Q)}^{(2;lm)} = 0$

## Tachyon modes avoidance (tensor mode)

$$\delta_{(2)}\mathcal{L} = \sum_{n,l,m} \frac{a^3}{2} \left[ \mathcal{F}\dot{h}^2 - \mathcal{G}h^2 \right] \longrightarrow \boxed{\mathcal{G} \geq 0} \quad \text{tachyon avoidance}$$

### I. Tensor mode

$$\mathcal{G}_{(G)} := \frac{\nu^2}{a^2} + \frac{\nu^2}{3a^4} \left[ -2g_rK + 3g_3\nu^2 \right] + \frac{\nu^2}{a^6} \left[ -g_sK^2 + 6(g_5 + g_6)K\nu^2 + g_8\nu^2(\nu^2 - 2K) \right]$$

# Tachyon modes avoidance (tensor mode)

$$\mathcal{G} \geq 0$$

$(\tilde{g}_r, \tilde{g}_s)$  plane

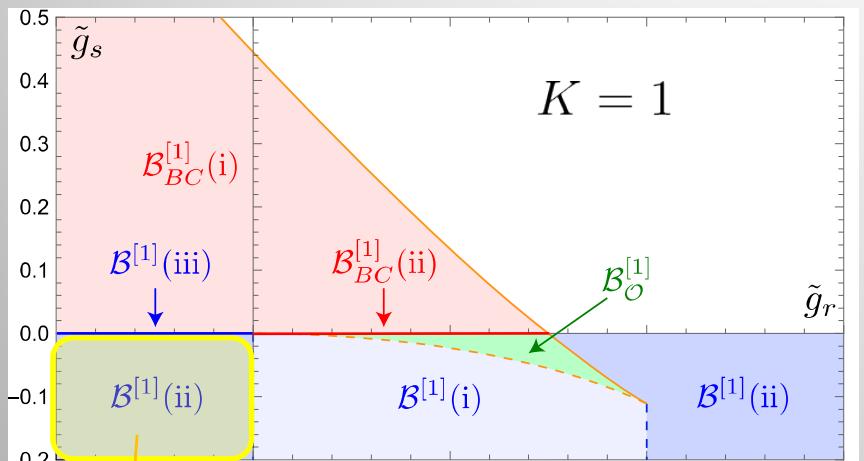
$$\tilde{g}_r := g_r K^2 (\Lambda/3)$$

$$\tilde{g}_s := g_s K^3 (\Lambda/3)^2$$

## I. Tensor mode

$$\mathcal{G}_{(G)} := \frac{\nu^2}{a^2} + \frac{\nu^2}{3a^4} \left[ -2g_r K + 3g_3 \nu^2 \right] + \frac{\nu^2}{a^6} \left[ -g_s K^2 + 6(g_5 + g_6)K\nu^2 + g_8 \nu^2(\nu^2 - 2K) \right]$$

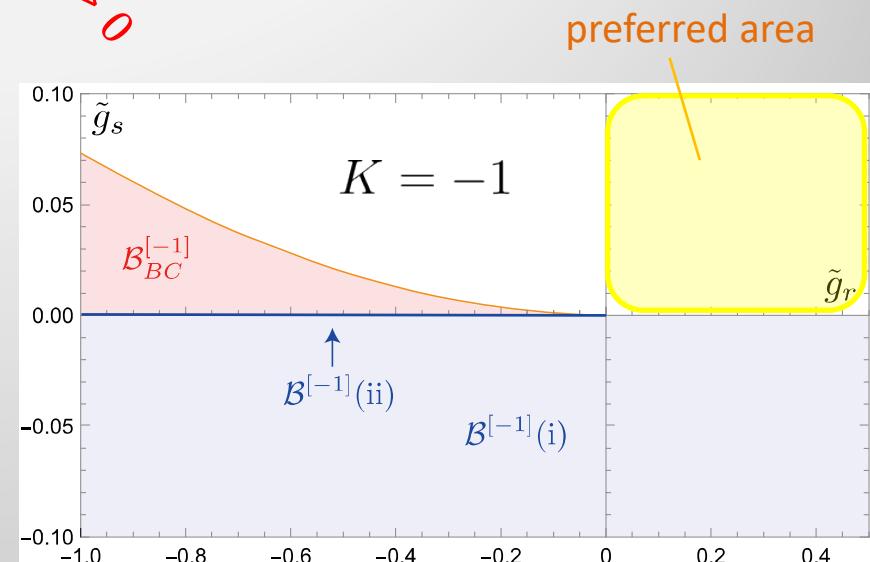
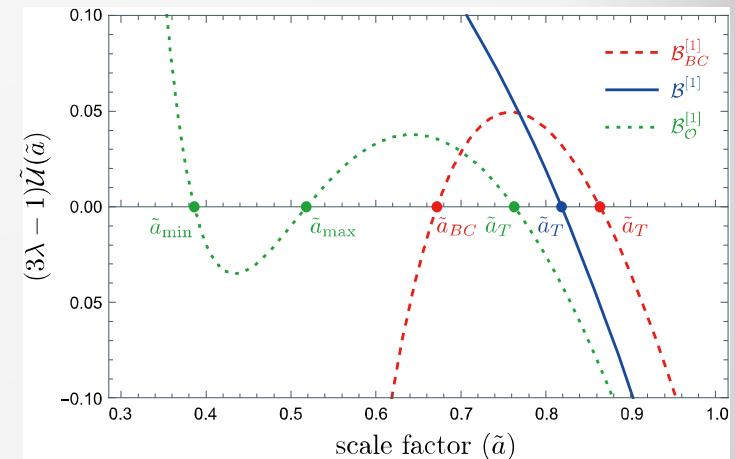
$$\Lambda > 0$$



preferred area



The bouncing solutions in open FLRW spacetime tend to be unstable



## Tachyon modes avoidance (scalar mode)

### II. Scalar mode

$$\mathcal{G} \geq 0$$

$$\begin{aligned}
 \mathcal{G}_{(Q)} &:= -\frac{2}{3a^2}(\nu^2 - 3K) + \frac{2}{27a^4}(\nu^2 - 3K)[2g_r(2\nu^2 - 3K) + 3g_3\nu^2] \\
 &\quad + \frac{2}{9a^6}(\nu^2 - 3K)[g_s K(4\nu^2 - 9K) + 2(3g_5 + 3g_6 - 4g_7)K\nu^2 + (-8g_7 + 3g_8)\nu^2(3\nu^2 - 10K)] \\
 &\longrightarrow -\frac{2}{3a^2}(\nu^2 - 3K) \quad (\text{IR limit})
 \end{aligned}$$

$\nwarrow_O$

$$\begin{aligned}
 \mathcal{V}_{z=1} &:= 2\Lambda - \mathcal{R} \\
 \mathcal{V}_{z=2} &:= g_2 \mathcal{R}^2 + g_3 \mathcal{R}^i{}_j \mathcal{R}^j{}_i \\
 \mathcal{V}_{z=3} &:= g_4 \mathcal{R}^3 + g_5 \mathcal{R} \mathcal{R}^i{}_j \mathcal{R}^j{}_i + g_6 \mathcal{R}^i{}_j \mathcal{R}^j{}_k \mathcal{R}^k{}_i + g_7 \mathcal{R} \nabla^2 \mathcal{R} + g_8 \nabla_i \mathcal{R}_{jk} \nabla^i \mathcal{R}^{jk}
 \end{aligned}$$

$$\mathcal{L}_P := -N\sqrt{\gamma} \left[ \mathcal{V}_{z=1} + m_{LV}^{-2} \mathcal{V}_{z=2} + m_{LV}^{-4} \mathcal{V}_{z=3} \right]$$



In IR region, scalar perturbation modes have negative sign of mass

Tachyon instability ? → check dynamics

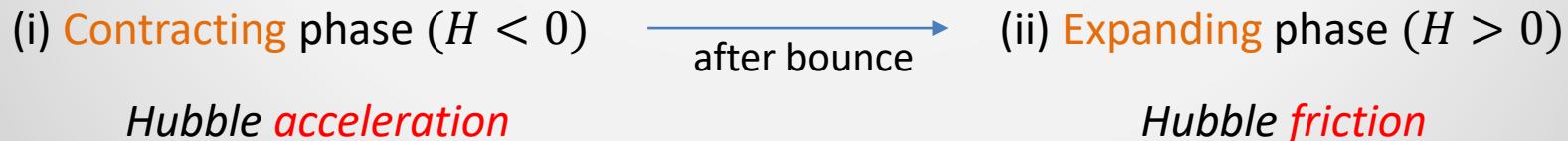
## Evolution of scalar perturbation

$$\delta_{(2)}\mathcal{L}^{(s)} = \sum_{n,l,m} \frac{a^3}{2} \left[ \mathcal{F}_{(Q)} \dot{h}_{(Q)}^2 - \mathcal{G}_{(Q)} h_{(Q)}^2 \right]$$

- ✓ Equation of motion for scalar perturbation

$$\ddot{h}_{(Q)} + \boxed{3H\dot{h}_{(Q)}} + \mathcal{M}_{(Q)}^2 h_{(Q)} = 0$$

squared effective mass  $\mathcal{M}_{(Q)}^2 := \frac{\mathcal{G}_{(Q)}}{\mathcal{F}_{(Q)}}$



- ✓ Stabilization condition

$$H^2 = \frac{2}{3(3\lambda - 1)} \left[ \Lambda - 3\frac{K}{a^2} + g_r \frac{K^2}{a^4} + g_s \frac{K^3}{a^6} \right] \rightarrow \frac{2\Lambda}{3(3\lambda - 1)}$$

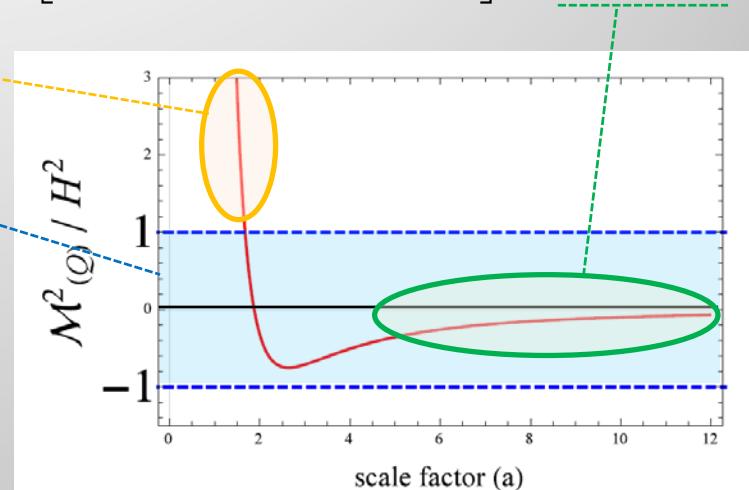
(i) Contracting phase :  $|\mathcal{M}^2/H^2| \gtrsim 1$

(ii) Expanding phase :  $|\mathcal{M}^2/H^2| \lesssim 1$

$$\mathcal{M}^2 < 0$$

$$\mathcal{M}_{(Q)}^2 \rightarrow -\frac{(\lambda - 1)\nu^2 + 2K}{3\lambda - 1} \frac{1}{a^2} < 0$$

Positive cosmological constant  $\Lambda$  is necessary in order to stabilize scalar perturbation in IR region



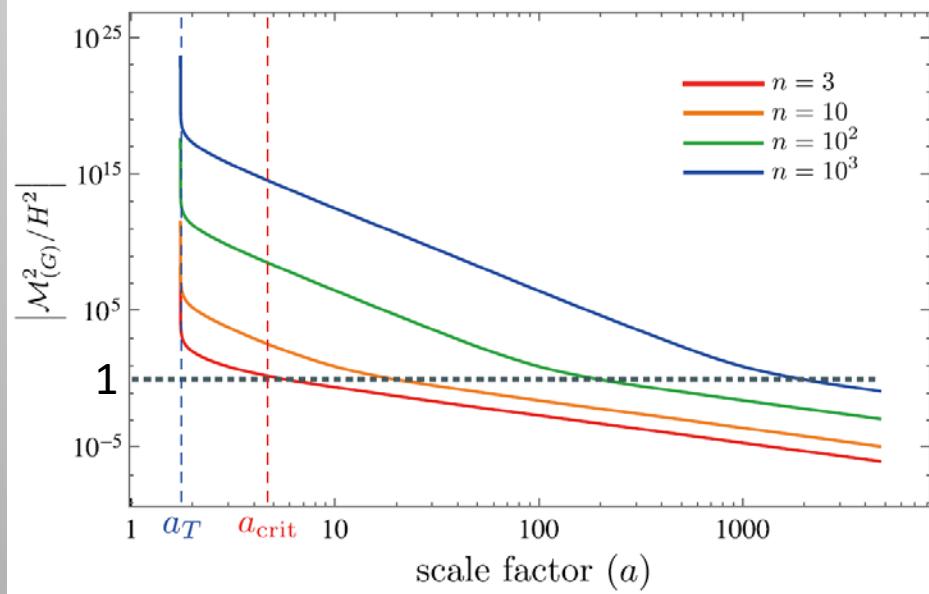
## Example of stable bouncing solution

type	$K$	$\Lambda$	$\lambda$	$g_2$	$g_3$	$g_4$	$g_5$	$g_6$	$g_7$	$g_8$	$g_r$	$g_s$	$a_T$	$a_{\text{crit}}$
$\mathcal{B}^{[1]}$	+1	1	1	$-\frac{29}{90}$	1	$-\frac{7}{108}$	$\frac{1}{2}$	-1	-1	1	$\frac{1}{5}$	-1	1.744	4.699

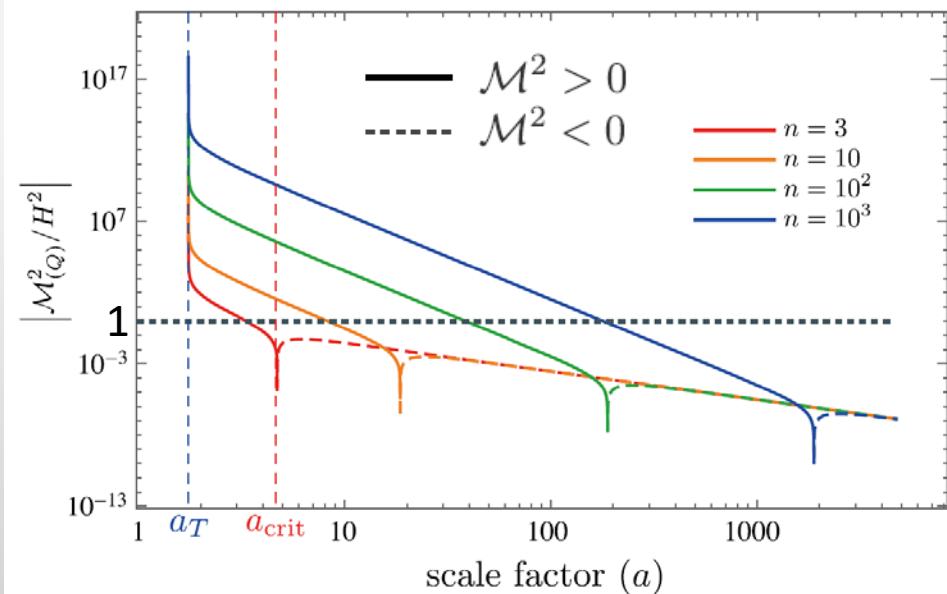
$$\mathcal{M}^2 = 0$$

✓ Effective mass to Hubble ratio

tensor perturbation



scalar perturbation



All perturbation modes satisfy the stabilization condition

Tachyonic instabilities are avoided

## Discussion : non-projectable HL case

$$N = N(t) \longrightarrow N = N(t, x^i)$$

remove “projectability condition”

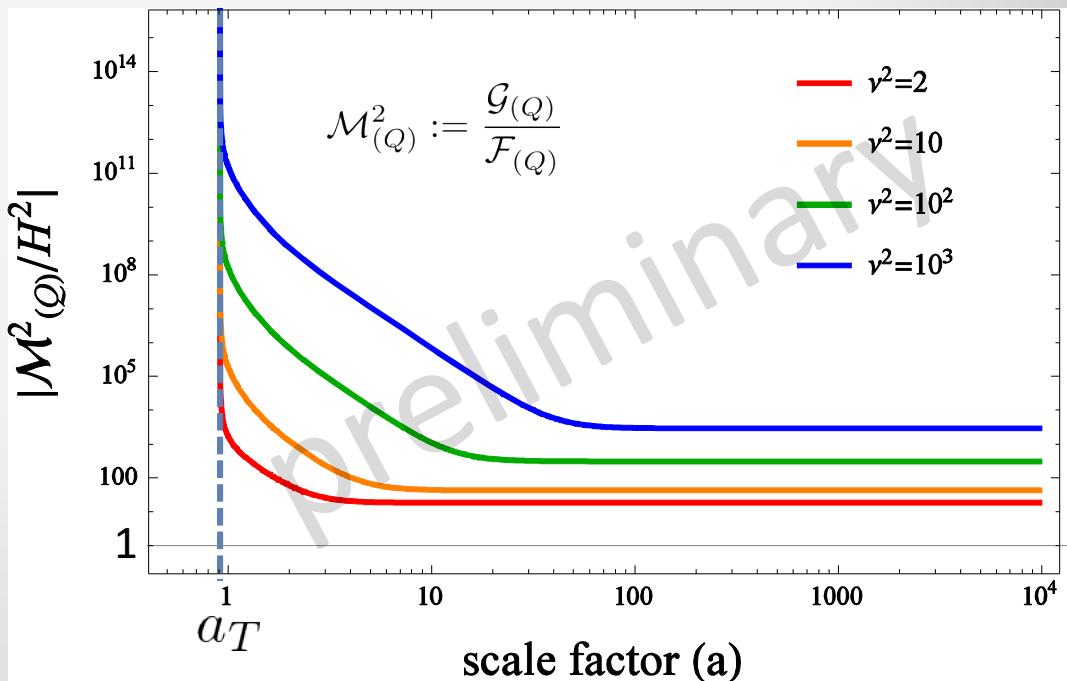
$$\mathcal{S} = \frac{m_{LV}^2}{2} \int dt d^3x (\mathcal{L}_K + \mathcal{L}_P + \mathcal{L}_{NP})$$

$$\mathcal{L}_{NP} := N\sqrt{g} (\zeta \Phi^i \Phi_i) + \dots$$

$$\Phi_i := \nabla_i \ln N$$



scalar perturbation is changed



$$\mathcal{G}_{(Q)} \Big|_{IR} \approx \frac{2\nu^2(\nu^2 - 3K)}{3a^2} \times \left[ \frac{(2-\varsigma)(\lambda-1)\nu^2 - 2K(\varsigma+3\lambda-3)}{\varsigma(\lambda-1)\nu^4 - 2K(2-\varsigma)\nu^2 + 12K^2} \right]$$

can be **positive** in IR region

$$\Lambda = 0$$

$$0 < \varsigma < 2$$

Cosmological constant is **not necessary** for stable solutions in **non-projectable HL theory**

## Summary

- ✓ We investigated the **stability** of bouncing universe in non-flat FLRW background based on projectable Hořava-Lifshitz theory which was shown as renormalizable gravitational theory

### Ghost modes avoidance (scalar perturbation)

$\lambda \geq 1$  for closed universe

$\lambda > 2$  for open universe

### Tachyon modes avoidance

- Tensor perturbation

The bouncing solutions in **open** FLRW universe tend to be **unstable**

- Scalar perturbation

In **IR** region, the squared effective **mass** of scalar perturbation must be **negative**



**Positive cosmological constant  $\Lambda$**  is necessary to strong **Hubble friction** for avoiding tachyon instability