Galileon p-form theories

Sebastian Garcia-Saenz

Institut d'Astrophysique de Paris

Based on **1704.02980** with C. Deffayet, S. Mukohyama and V. Sivanesan

Galileon scalars in modified gravity

A lot of effort has been put in the past years to classify the most general scalar-tensor theories:

- Horndeski theory

  G. Horndeski (1974)

  C. Deffayet et al. (2009)

- Beyond Horndeski

  J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi (2015)

- DHOST/EST

  M. Crisostomi, K. Koyama and G. Tasinato (2016)

  J. Ben Achour, D. Langlois and K. Noui (2016)

These are all theories that modify GR by the addition of a **single** degree of freedom. This is good because

- it is minimal (we don't want to mess up GR too much)
- allows for more theoretical control

Galileon scalars in modified gravity

Can we consider more than just one scalar? Some examples:

- Spin-1: Generalized Proca theories

  G. Tassinato (2016)

  L. Heisenberg (2016)

- Spin-2: Massive gravity, bi-gravity, multi-gravity

It would be nice to have a **full classification** of these modified gravity theories.

This is a hard problem, so a natural first step is to simply ignore gravity. Why?

- this proved helpful in the case of scalars
- even without gravity the theories may have interesting properties

Galileon scalars in modified gravity

Horndeski theory (and beyond) with gravity turned off:

$$\mathcal{L} = \sum_{m=2}^{5} \alpha_m \mathcal{L}_{H,m}$$

$$\begin{aligned}
\mathcal{L}_{H,2} &= f_2(X,\pi) \\
\mathcal{L}_{H,3} &= f_3(X,\pi)\,[\Pi] \\
\mathcal{L}_{H,4} &= f_4(X,\pi)\left([\Pi]^2 - [\Pi^2]\right) \\
\mathcal{L}_{H,5} &= f_5(X,\pi)\left([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^3]\right)
\end{aligned}$$

where

$\pi \to$ scalar field

$X = (\partial\pi)^2$

$\Pi_{ab} = \partial_a\partial_b\pi$

$[\ldots] = \mathrm{tr}(\ldots)$

$\alpha_m = \mathrm{constants}$

Galileon scalars in modified gravity

In the special case where the functions $f_{2,3,4,5}(X,\pi) = X$ the resulting theory is known as **Galileon**

It is invariant under

$$\pi \to \pi + b_a x^a + c \qquad \to \qquad \textbf{Galileon symmetry}$$

Nicolis et al. (2008)

Basic properties:

- Second order equation of motion and hence no ghosts
- Robust implementation of Vainshtein mechanism

Galileon scalars in modified gravity

Galileons are not just a particular theory of modified gravity but actually appear as effective DoF's in various models

Examples:

- DGP model:

$$S = M^3 \int d^5X \sqrt{-g_{(5)}}\, R_{(5)} + M_P^2 \int d^4x \sqrt{-g_{(4)}}\, R_{(4)}$$

$$\rightarrow \quad S_{\mathrm{eff}} = M_P^2 \int d^4x \sqrt{-g}\, R + \int d^4x \Big[-\frac{1}{2}(\partial\pi)^2 - \frac{M_P^3}{M^6}(\partial\pi)^2 \Box\pi \Big]$$

- dRGT massive gravity:

$$S = M_P^2 \int d^4x \sqrt{-g} \Big(R + m^2 U(h, \eta) \Big)$$

$$\rightarrow \quad S_{\mathrm{dec}} = \int d^4x \sum_{m=2}^{4} \alpha_m \mathcal{L}_{G,m}(\pi) + S(h, \pi)$$

Galileon scalars in field theory

Galileons are also interesting field theories in their own right

- Galileons as Goldstone bosons: Goon et al. (2012)

  The Galileon symmetry $\pi \to \pi + b_a x^a + c$ is nonlinearly realized, and so we can interpret π as a Goldstone boson

  Two types of Lagrangians:

  - Strictly invariant: $\mathcal{L} = F(\partial\partial\pi)$
  - Wess–Zumino terms: $\mathcal{L}_{G,m}$

- Enhanced soft limits: Cheung et al. (2015)

  Consider the soft limit (external momentum $k \to 0$) of a scattering amplitude $\mathcal{A}$

  - Shift-symmetric scalar: $\mathcal{A} \sim k(\ldots)$
  - Galileon: $\mathcal{A} \sim k^2(\ldots)$
  - Special Galileon: $\mathcal{A} \sim k^3(\ldots)$

Galileon scalars — multiple fields

How can we generalize Galileons?

- **Multi-Galileons** extend the original model to multiple scalars, with or without internal symmetries

  Hinterbichler et al. (2010), Padilla et al. (2011), SGS (2013)

- Move on from scalars, i.e. spin-0 fields, to consider **spin-1 fields**

- The simplest case is a vector A_a, but (generically) a spin-1 particle can also be described by a totally antisymmetric tensor, or **p-form**:

$$A_{a_1 \cdots a_p}$$

- p-form gauge theories naturally generalize Maxwell's theory:

$$S = -\frac{1}{4} \int d^D x \, F^{a_1 a_2} F_{a_1 a_2} \quad \rightarrow \quad -\frac{1}{2(p+1)} \int d^D x \, F^{a_1 \cdots a_{p+1}} F_{a_1 \cdots a_{p+1}}$$

  where

  $F_{a_1 \cdots a_{p+1}} \equiv (p+1) \partial_{[a_1} A_{a_2 \cdots a_{p+1}]} \rightarrow$ field strength

  $D \rightarrow$ dimension of spacetime

Galileon p-forms — motivation

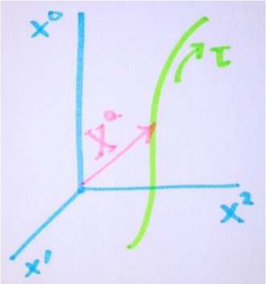
Properties of p-form gauge fields:

- Abelian gauge symmetry:

$$\delta A_{a_1 \cdots a_p} = \partial_{[a_1} \Lambda_{a_2 \cdots a_p]}$$

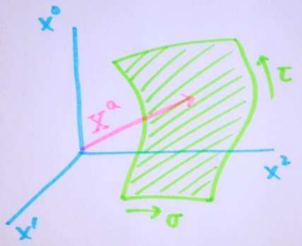
- They are important ingredients in string theory and supergravity
- They can be coupled in a natural way to extended objects

$$\begin{aligned} S_{p=1} &= q \int A_a(X) dX^a \\ &= q \int d\tau \, A_a(X) \partial_\tau X^a \end{aligned}$$



Galileon p-forms — motivation

$$S_{p=2} = q_2 \int A_{a_1 a_2}(X) dX^{a_1} dX^{a_2}$$
$$= q_2 \int d\tau d\sigma \, A_{a_1 a_2}(X) \partial_\tau X^{a_1} \partial_\sigma X^{a_2}$$



Note that these extended objects don't need to be fundamental!

For example a 2-form field can be coupled to a thin fluid vortex

SGS, E. Mitsou and A. Nicolis (to appear)

Galileon p-forms — classification

The first Galileon p-form theories were found by mimicking the example of scalars

Deffayet et al. (2010)

To this end it's useful to rewrite the Galileon Lagrangians using the Levi-Civita tensor, for example

$$S_{0,4} = \int d^3x\, \epsilon^{a_1 a_2 a_3} \epsilon^{b_1 b_2 b_3} (\partial_{a_1}\pi)(\partial_{b_1}\pi)(\partial_{a_2}\partial_{b_2}\pi)(\partial_{a_3}\partial_{b_3}\pi)$$

Suppose we replace $\partial_a \pi \to F_{a_1 a_2 a_3}$ (field strength of a 2-form) in $S_{0,4}$:

$$S_{2,4} = \int d^7x\, \epsilon^{a_1 \cdots a_7} \epsilon^{b_1 \cdots b_7} F_{a_1 a_2 a_3} F_{b_1 b_2 b_3} \partial_{a_4} F_{b_5 b_6 b_7} \partial_{b_4} F_{a_5 a_6 a_7}$$

$\to S_{2,4}$ is gauge invariant and leads to second order eqs. of motion, thanks to the Bianchi identity $\partial_{[a_1} F_{a_2 \cdots a_{p+2}]} = 0$

Galileon p-forms — classification

Remarks:

- In general the minimal allowed dimension for Galileon p-forms is $D > 4$
- The above method doesn't work for odd vertices (cubic, quintic, etc.):

$$S_{p,3} \stackrel{?}{=} \int d^D x \, \epsilon^{a_1 \cdots} \epsilon^{b_1 \cdots} F_{a_1 \ldots} F_{b_1 \ldots} \partial_{a_2} F_{b_2 \ldots}$$

  $\rightarrow$ indices cannot even be matched

- The above method doesn't work for odd p values:

$$S_{3,4} \stackrel{?}{=} \int d^9 x \, \epsilon^{a_1 \cdots a_9} \epsilon^{b_1 \cdots b_9} F_{a_1 \cdots a_4} F_{b_1 \cdots b_4} \partial_{a_5} F_{b_6 \cdots b_9} \partial_{b_5} F_{a_6 \cdots a_9}$$

  $\rightarrow$ $S_{3,4}$ vanishes identically

- There are obviously many other ways to contract indices!

Galileon p-forms — classification

We would like a more general classification scheme for Galileon p-forms

Deffayet et al. (2016)

The eq. of motion $\mathcal{E}^1$ of a Galileon theory can only depend on $\partial\partial A$. Consider taking derivatives of $\mathcal{E}^1$ with respect to $\partial\partial A$:

$$(\mathcal{E}^2)^{\cdots} \equiv \frac{\partial(\mathcal{E}^1)^{\cdots}}{\partial(\partial\partial A\ldots)}\,, \qquad (\mathcal{E}^3)^{\cdots} \quad \equiv \frac{\partial(\mathcal{E}^2)^{\cdots}}{\partial(\partial\partial A\ldots)}$$

and similarly for all the tensors $\mathcal{E}^m$ up to $m_{\rm max}$

We can constrain the index symmetries of the tensors $\mathcal{E}^m$ from the following properties:

(a) The eq. of motion $\mathcal{E}^1$ should derive from an action

(b) The eq. of motion $\mathcal{E}^1$ should be gauge invariant

Galileon p-forms — classification

From these one can show that the tensors $\mathcal{E}^m$ satisfy certain algebraic conditions:

- They belong to the "plethysm" $\mathrm{Sym}^m(\wedge^p) \otimes \mathrm{Sym}^{m-1}(\mathrm{Sym}^2)$

  $\mathrm{Sym}^m(\wedge^p) \rightarrow$ symmetrized tensor product of m p-forms

  $\mathrm{Sym}^{m-1}(\mathrm{Sym}^2) \rightarrow$ symmetrized tensor product of $m-1$ symmetric 2-tensors

- They can be decomposed into irreducible tensors with symmetries corresponding to **two-column Young tableaux**

$$S_{[ab]} \rightarrow \ytableaushort{a,b} \qquad T_{(ab)} \rightarrow \ytableaushort{ab}$$

$$R_{abcd} \rightarrow \ytableaushort{ac,bd} \qquad (\mathcal{E}^m)^{\cdots} \rightarrow \ydiagram{2,2,1}$$

Galileon p-forms — classification

Applications:

- One can show that $\mathcal{E}^m$ can only exist if

$$m \leq p + 1 \qquad \text{(odd } p\text{)}$$

  Now, for the theory to be interacting we need $m \geq 3$, which is impossible if $p = 1$

  $\rightarrow$ Gauge invariant vector Galileons do not exist

  Deffayet et al. (2014)

- The second derivative $\partial\partial A$ enters in the tensors $\mathcal{E}^m$ only in the form ∂F

  $\rightarrow$ The eq. of motion $\mathcal{E}^1$ depends on the p-form gauge field only through the field strength $F_{a_1 \cdots a_{p+1}}$

  Henneaux and Knaepen (1997)

Galileon p-forms — classification

- For a given dimension, there is only a finite number of **admissible** two-column Young diagrams, each corresponding to a candidate Galileon theory
  - Why is it a "candidate"?
    The above properties only deal with index symmetries—they don't guarantee that we can actually construct the tensors $\mathcal{E}^m$ from the basic building blocks at hand, namely

$$\eta_{ab}\,, \qquad \epsilon^{a_1\cdots a_D}\,, \qquad \partial_{b_1}\partial_{b_2}A_{a_1\cdots a_p}$$

  From this we can infer a **uniqueness argument**: whenever a Galileon interaction vertex can be constructed, it is guaranteed to be unique

Galileon p-forms — construction

Main result:

Construction of *all* Galileon p-form vertices that exist up to $D = 11$

Notation:

$S_{p,m} \rightarrow$ vertex of order m for forms of rank p

$F_{a_1 \cdots a_{p+1}} \equiv (p+1)\partial_{[a_1} A_{a_2 \cdots a_{p+1}]} \rightarrow$ field strength

- **2-form vertices**

$$S_{2,4} = \int d^7 x \, \epsilon^{a_1 \cdots a_7} \epsilon^{b_1 \cdots b_7} F_{a_1 a_2 a_3} F_{b_1 b_2 b_3} \partial_{a_4} F_{b_4 b_5 b_6} \partial_{b_7} F_{a_5 a_6 a_7}$$

$$S_{2,6} = \int d^{11} x \, \epsilon^{a_1 \cdots a_{11}} \epsilon^{b_1 \cdots b_{11}} F_{a_1 a_2 a_3} F_{b_1 b_2 b_3} \times \partial_{a_4} F_{b_4 b_5 b_6} \partial_{a_5} F_{b_7 b_8 b_9} \partial_{b_{10}} F_{a_6 a_7 a_8} \partial_{b_{11}} F_{a_9 a_{10} a_{11}}$$

Galileon p-forms — construction

Notation:

$S_{p,m} \rightarrow$ vertex of order m for forms of rank p

$F_{a_1 \cdots a_{p+1}} \equiv (p+1)\partial_{[a_1} A_{a_2 \cdots a_{p+1}]} \rightarrow$ field strength

- **3-form vertex**

$$S_{3,4} = \int d^9x\, \epsilon^{a_1 \cdots a_9} \epsilon^{b_1 \cdots b_9} F_{a_1 a_2 a_3 a_4} F_{b_1 b_2 b_3 b_4} \partial_{a_5} F_{b_6 b_7 b_8 a_9} \partial_{b_5} F_{a_6 a_7 a_8 b_9}$$

- **4-form vertices**

$$S_{4,3} = \int d^8x\, \epsilon^{a_1 \cdots a_8} \epsilon^{b_1 \cdots b_8} F_{a_1 a_2 a_3 a_4 a_5} F_{b_1 b_2 b_3 b_4 b_5} \partial_{a_6} F_{a_7 a_8 b_6 b_7 b_8}$$

$$S_{4,4} = \int d^{11}x\, \epsilon^{a_1 \cdots a_{11}} \epsilon^{b_1 \cdots b_{11}} F_{a_1 \cdots a_5} F_{b_1 \cdots b_5} \partial_{a_6} F_{b_7 \cdots b_{11}} \partial_{b_6} F_{a_7 \cdots a_{11}}$$

Outlook

- Summary/take-home:
  - Scalar Galileons are not only relevant in modified gravity but are interesting from a field theoretic point of view
  - This motivates to study Galileon analogues for other types of fields, for instance p-forms $A_{a_1 \cdots a_p}$
  - We focused on abelian gauge invariant p-forms:
    - By analyzing the tower of tensors $\mathcal{E}^1 \to \cdots \to \mathcal{E}^{m_{\max}}$ one can derive a set of conditions on their index symmetries
    - The bottomline is that $\mathcal{E}^m$ must be a combination of tensors with symmetries characterized by certain Young diagrams
    - We constructed all such vertices up to $D = 11$

Outlook

- Future prospects:
  - Relax the assumption of Galileon symmetry: $S[A, \partial A, \partial\partial A]$
  - Relax the assumption of gauge symmetry or look at *other* gauge symmetries
  - Consider more than one field, for example "colored" p-forms $A^a_{(p)}$ or vertices with different forms $A_{(p)} - A_{(q)} - A_{(r)}$
  - Other spins?

    A. Chatzistavrakidis, F. S. Khoo, D. Roest and P. Schupp (2016)
  - Interaction with gravity: can all Galileon theories be covariantized?

Thank you!