Fully Relativistic Higher Order Effects in Weak Lensing using the Post-Friedmann Approximation Scheme

Hedda Gressel

Institute of Cosmology and Gravitation (ICG), University of Portsmouth
Supervisors: Marco Bruni, David Bacon
in collaboration with Camille Bonvin

hedda.gressel@port.ac.uk

21st annual International Conference on Particle Physics and Cosmology (COSMO-17)

Overview

Motivation for the PF Approach I

we study the growth of large-scale structure in two different ways:

- larger, linear scales: **fully relativistic perturbation schemes**
- smaller, non-linear scales: **Newtonian methods; N-body simulations**

Motivation for the PF Approach II

Current and future surveys will provide a great amount of *precise data*, which largely will come from *non-linear scales* of the Universe.

$\Rightarrow$ **Will the Newtonian Approximation be good enough for non-linear structure formation?**

- e.g. Euclid target: N-body simulations with 1% accuracy
  what if *relativistic corrections are of order* $\mathcal{O}(1\%)$?

PF Approximation includes all scales: bridging the fully relativistic perturbative scheme on **large, linear scales** with the Newtonian approximation on **small, non-linear scales.**

What is the Post-Friedmann Approach?

- *post-Newtonian (PN)* type approximation for **cosmology**
  $\rightarrow$ expansion in $1/c^2$
- instead of flat background $\rightarrow$ **FLRW background**
- only **peculiar velocities** are assumed to be **small**:
  $\dot{\mathbf{r}} = H\mathbf{r} + \mathbf{v}$ with $\mathbf{v} \ll c$ and $\mathbf{r} = a\mathbf{x}$
  $\rightarrow$ no restriction over scale via velocity
- **density contrast** δ can be > 1

(Milillo et al. 2015)

The Scalar, Vector, and Tensor Perturbations in PF

We assume *Poisson gauge*, $B^i{}_{,i} = 0$, and $h^i_{j,i} = h^i_i = 0$.

$$g_{00} = -\left[1 - \frac{2U_N}{c^2} + \frac{1}{c^4}\left(2U_N^2 - 4U_P\right)\right] + \mathcal{O}\left(\frac{1}{c^6}\right)$$

$$g_{0i} = -\frac{a}{c^3}B_i^N - \frac{a}{c^5}B_i^P + \mathcal{O}\left(\frac{1}{c^7}\right)$$

$$g_{ij} = a^2\left[\left(1 + \frac{2V_N}{c^2} + \frac{1}{c^4}\left(2V_N^2 + 4V_P\right)\right)\delta_{ij} + \frac{1}{c^4}h_{ij}\right] + \mathcal{O}\left(\frac{1}{c^6}\right)$$

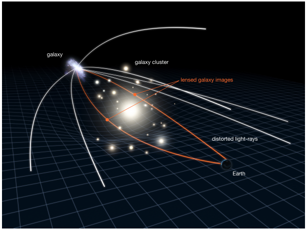
- the *scalar potentials and vector potential* are split into leading order **Newtonian components** (U_N, V_N, and B_i^N) and **post-Friedmann components** (U_P, V_P, and B_i^P)

(Milillo et al. 2015)

Newtonian and relativistic limit

- *at leading order in Einstein's field equations and conservation equations:* reduces to **fully non-linear Newtonian cosmology**
- *defining resummed variables (such as* $\phi_P := -U_N - \frac{2}{c^2} U_P$*) and linearising Einstein's field equations:* reduces to **standard first-order perturbation theory**

Gravitational Lensing (GL)



(a) Copyright: NASA, ESA and L. Calcada


(b) Galaxy cluster Abell 370, Copyright: NASA, ESA, the Hubble SM4 ERO Team and ST-ECF

Condition for GL: δ can be > 1, bg: FLRW, $\mathbf{v}_{\text{pec}} \ll 1$

Weak Lensing (WL)

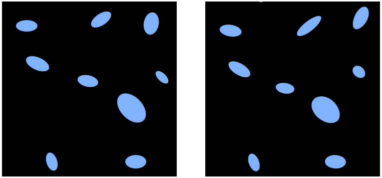


Figure: Credit: Matthew Withers

cosmic shear: lensing by the large-scale structure in the universe

- possible **constraints** on the equation of state of **dark energy** or **modified gravity models**
- study of distribution of **dark matter**

My goal is to calculate the shear and the convergence in a PF context.

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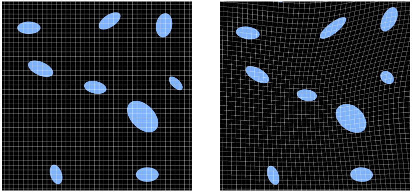

Figure: Credit: Matthew Withers

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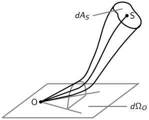
Convergence and Shear

$$\underbrace{\delta x^i(\lambda_S)}_{\text{deviation vector at source}} = \overbrace{\mathcal{D}^i_j(\lambda_S)}^{\text{Jacobi mapping}} \underbrace{\delta\theta^j(\lambda_O)}_{\text{angular vector at observer}} \quad (1)$$

with



$$\mathcal{D}^i_j \propto \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2 \\ -\gamma_2 & 1-\kappa+\gamma_1 \end{pmatrix} \quad (2)$$

and κ and
γ being the **convergence** and **shear**, respectively.

Evolution equation for Jacobi mapping

This evolution equation is derived from the geodesic deviation equation mapped on the screen-space.

$$\boxed{\frac{d^2}{d\chi^2}\mathcal{D}_{ab} + \frac{1}{k^0}\frac{dk^0}{d\chi}\frac{d}{d\chi}\mathcal{D}_{ab} = \frac{1}{(k^0)^2}\mathcal{R}_a^{\ c}\mathcal{D}_{cb}} \qquad (3)$$

with $\chi = c(\eta_0 - \eta)$, $k^\mu = \frac{dx^\mu}{d\lambda}$ and λ being a affine parameter, and $\mathcal{R}_{ab} = R_{\alpha\gamma\delta\beta}k^\gamma k^\delta n_a^\alpha n_b^\beta$

(Bernardeau et al, 2010)

Solution for the Jacobi mapping up to $\mathcal{O}\left(1/c^3\right)$

$$\mathcal{D}_{ab} = \chi_S \left[1 + \frac{V_N}{c^2} + \frac{2}{c^2 \chi_S} \int_0^{\chi_S} d\chi \left(-2W_N + (\chi_S - \chi)\, \dot{W}_N\right)\right] \delta_{ab} + \\ + \frac{2}{c^2} \int_0^{\chi_S} d\chi\, (\chi_S - \chi)\, \chi e_a^i e_b^j W_{N,ij} - \frac{1}{c^3} \int_0^{\chi_S} B_{i,j} \bar{k}^i \bar{k}^j \delta_{ab} + \\ + \frac{1}{c^3} \int_0^{\chi_S} d\chi\, (\chi_S - \chi)\, \chi e_a^i e_b^j \left(\frac{dB_{i,(j}}{d\chi} + \frac{dB_{j,i)}}{d\chi} - \left(k^\alpha B_\alpha\right)_{,ij}\right)$$

with $W_X = \frac{1}{2}\left(U_X + V_X\right)$ and $X = N,\ P$.

resembles the outcome of standard perturbation theory,
($\rightarrow$ derived from the purely geometric geodesic deviation equation)
but the physical meaning differs $\rightarrow$ valitiy on small scales

(cf. Bernardeau et al, 2010)

$\mathcal{D}_{ab}$ in terms of the redshift:

At this order, only the convergence is afffected by redshift pertrubations

$$\kappa = \frac{V_N}{c^2} + \frac{2}{c^2}\int_0^{\chi_S} d\chi \left(-2W_N + (\chi_S - \chi)\dot{W}_N\right) + \\ + \frac{2}{c^2}\int_0^{\chi_S} d\chi\,(\chi_S - \chi)\,\chi n^i n^j W_{N,ij} - \frac{1}{c^3}\int_0^{\chi_S} B_{i,j}\bar{k}^i\bar{k}^j + \\ + \frac{1}{c^3}\int_0^{\chi_S} d\chi\,(\chi_S - \chi)\,\chi n^i n^j \left(\frac{dB_{i,(j}}{d\chi} + \frac{dB_{j,i)}}{d\chi} - (k^\alpha B_\alpha)_{,ij}\right) + \\ + \left(1 + \frac{1}{\mathcal{H}\chi_S}\right)\left[\frac{2}{c^2}\int_0^{\chi_S} \dot{W} d\chi + \frac{1}{c^3}\left(B_i\bar{k}^i - \int_0^{\chi_S} B_{i,j}\bar{k}^i\bar{k}^j\right)\right]$$

(cf. Bonvin, 2014)

Contributions from Frame-Dragging Potential $B_{N,i}$

- $B_{N\,i}$ doesn't influence the matter dynamics, but affects photon geodesics
- $B_{N\,i}$ contributes to the convergence and shear
- $B_{N\,i}$ is sourced by the Newtonian quantities $\rho\, v_i$ (see Einstein field equations with G^0_i)

(cf Thomas et al. 2015 and Bruni et al, 2013)

Order $\mathcal{O}\left(\frac{1}{c^4}\right)$

Stay tuned!

Conclusion

Post-Friedmann approximation:

- the PF approximation is valid on **linear and non-linear scales** :
  - at leading order: reduces to fully non-linear Newtonian cosmology
  - if linearised: reduces to standard first-order perturbation theory
  - $\rightarrow$ *favourable approximation scheme for a weak lensing analysis*

Weak Lensing with Post-Friedmann approximation:

- convergence and shear are resemble the convergence and shear in SPT, but **differ in the physical interpretation:**
  - validity on **all scales**
  - WL: **coupling of small scales to large scales**, e.g. correlation for the shear for two galaxies that are far apart but almost aligned w.r.t. the line of sight
- **frame dragging effect** sourced by Newtonian quantities

Future Work

- *shear and converegene at higher orders*
- *calculation of the two-point function*
- *comparing with numerical results*

Thank you!