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Stochastic spectator field dynamics

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Based on: arxiv:1705.05746, ‘A Quantum Window Onto Early Inflation’
RJH, Vincent Vennin, David Wands
arxiv:1701.06473, ‘The stochastic spectator’
RJH, Vincent Vennin, Christian T. Byrnes, Jesús Torrado, David Wands

In this talk...

- Inflation (skim over) and ‘spectator’ field
- Stochastic formalism
- Calculating field displacements
- Summary and applications

Inflation

Defined as an early phase of accelerated expansion.

HUBBLE PARAMETER

$$\frac{\dot{a}}{a} = H \sim \text{const.}$$

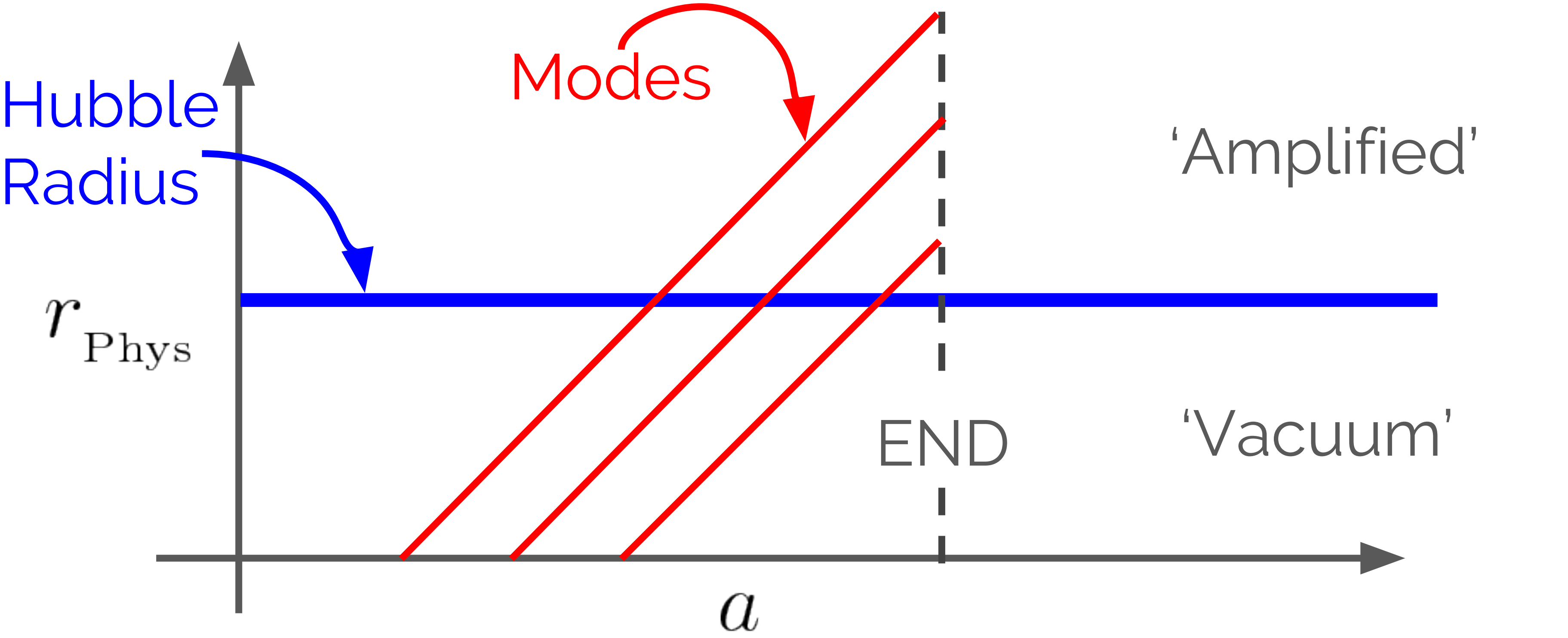
SCALE FACTOR

$$a \propto \exp(Ht)$$

'Old problems' in Cosmology are solved, but emphasise more that perturbations are sourced… seeds for structure

Guth, Linde, Sato, Starobinsky,,… (80's)

Qualitative picture



The scalar fields present

1. Inflaton ϕ - drives expansion for inflation
2. Higgs field - may be light w./w.o. BSM couplings
3. Additional fields?
   -> e.g. Curvatons? Broader class of observables

‘Spectator’ field σ

‘Test field’ $\rho_\sigma \ll 3M_{\rm Pl}^2 H^2$

We assume it is also ‘Light’ $V_{,\sigma\sigma} \ll H^2$

Curvatons, Modulons, Light Axions, Higgs-like, DE

PHENOMENOLOGY DEPENDS ON INIT. CONDS.

Lyth & Wands, Enqvist & Sloth (‘01), Ringeval et al (‘10), Figueroa & Byrnes (‘16)

Stochastic calculation

Using the Stochastic Inflation formalism we derive a distribution over this field's values by the end of Inflation.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}N} = -\frac{V_{,\sigma}(\sigma)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Drift

Super-Hubble Diffusion

Starobinsky ('86)

Corresponding equation

Fokker-Planck equation:

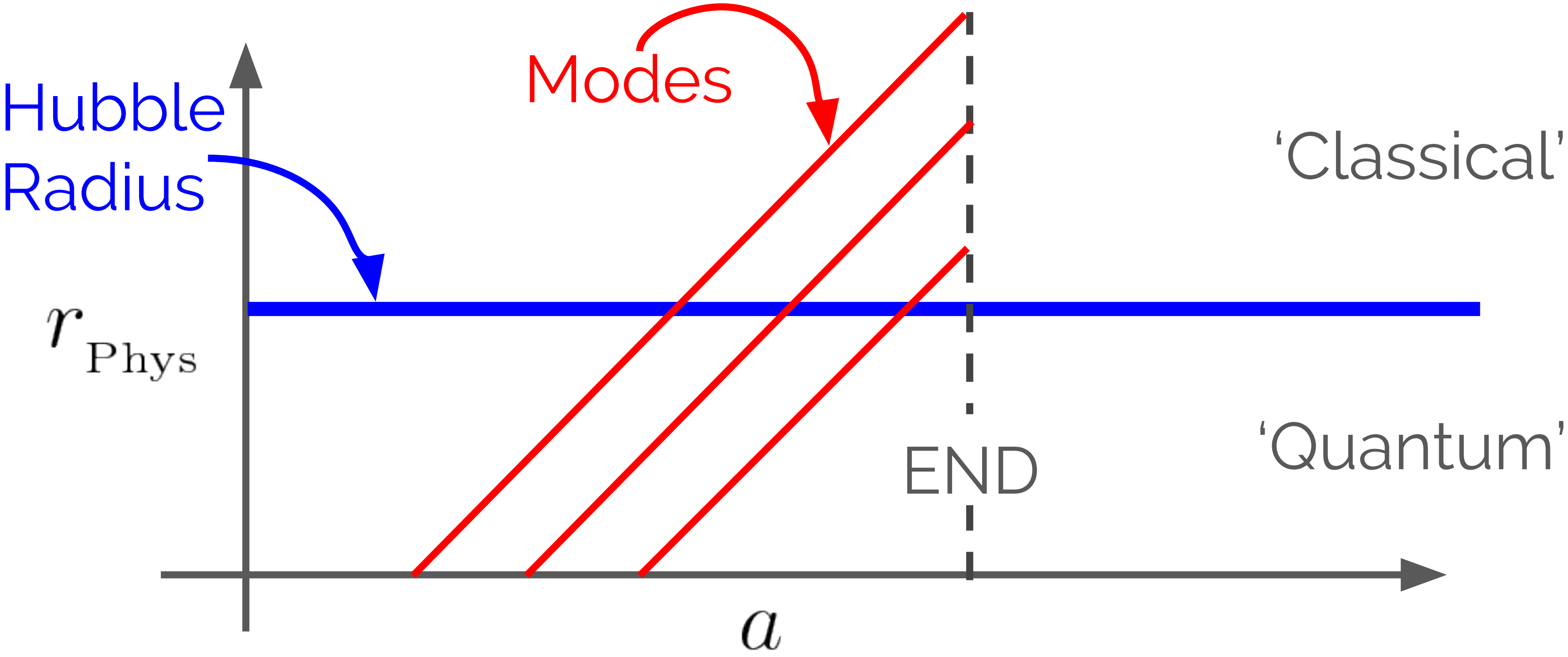
$$\frac{\partial P(\sigma,N)}{\partial N} = \frac{\partial}{\partial \sigma}\left[\frac{V_{,\sigma}}{3H^2}P(\sigma, N)\right] + \frac{H^2}{8\pi^2}\frac{\partial^2}{\partial \sigma^2}\left[P(\sigma, N)\right]$$

Drift

Super-Hubble
Diffusion

Starobinsky ('86)

Where does ξ come from?



Stochastic calculation

In pure de Sitter there is an equilibrium (stationary) attractor solution:

$$P_{\rm stat}(\sigma) \propto \exp\left[-\frac{8\pi V(\sigma)}{3H^4}\right]$$

E.g. the $\frac{1}{2}m^2\sigma^2$ case: A Gaussian with $\sqrt{\langle\sigma^2\rangle} \sim \frac{H^2}{m}$

Starobinsky ('86), Starobinsky and Yokoyama ('94)

Timescales

$$N_H = \frac{1}{\epsilon}$$

The number of e-folds associated to time variation $(\Delta t_H \, H = \frac{H}{\dot{H}} H)$ in Hubble parameter (slow-roll).

$$N_{\mathrm{relax}}$$

The number of e-folds required by the spectator in order to relax back to equilibrium distribution.

Timescales

E.g. Inflaton potential: $V \propto \phi^p$

Expansion: $N_{\mathrm{H}} \sim \left(\frac{H}{H_{\mathrm{end}}}\right)^{\frac{4}{p}}$

Relaxation in the $\frac{1}{2}m^2\sigma^2$ case: $N_{\mathrm{relax}} \sim \frac{H^2}{m^2}$

Relaxation in the $\lambda\sigma^4$ case: $N_{\mathrm{relax}} \sim \frac{1}{\sqrt{\lambda}}$

Enqvist et al ('12)

The $\frac{1}{2}m^2\sigma^2$ case

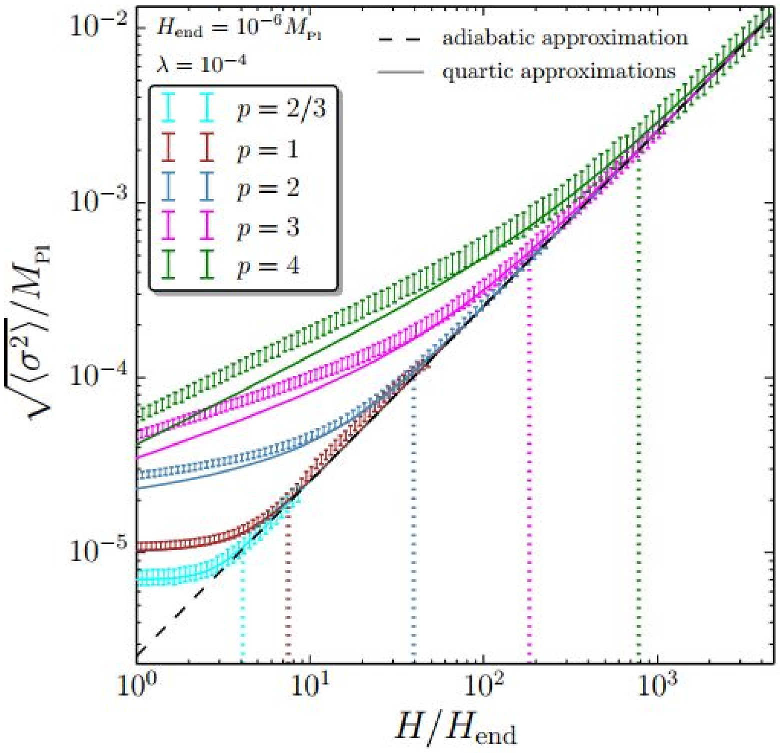
Always $N_{\rm H} \ll N_{\rm relax}$

So no dS eq:

$$P_{\rm stat}(\sigma) \propto \exp\left[-\frac{8\pi V(\sigma)}{3H^4}\right]$$

$P_{\rm eternal}$?

The $\lambda\sigma^4$ case



Earlier times:

$$N_{\rm H} \gg N_{\rm relax}$$

Later times:

$$N_{\rm H} \ll N_{\rm relax}$$

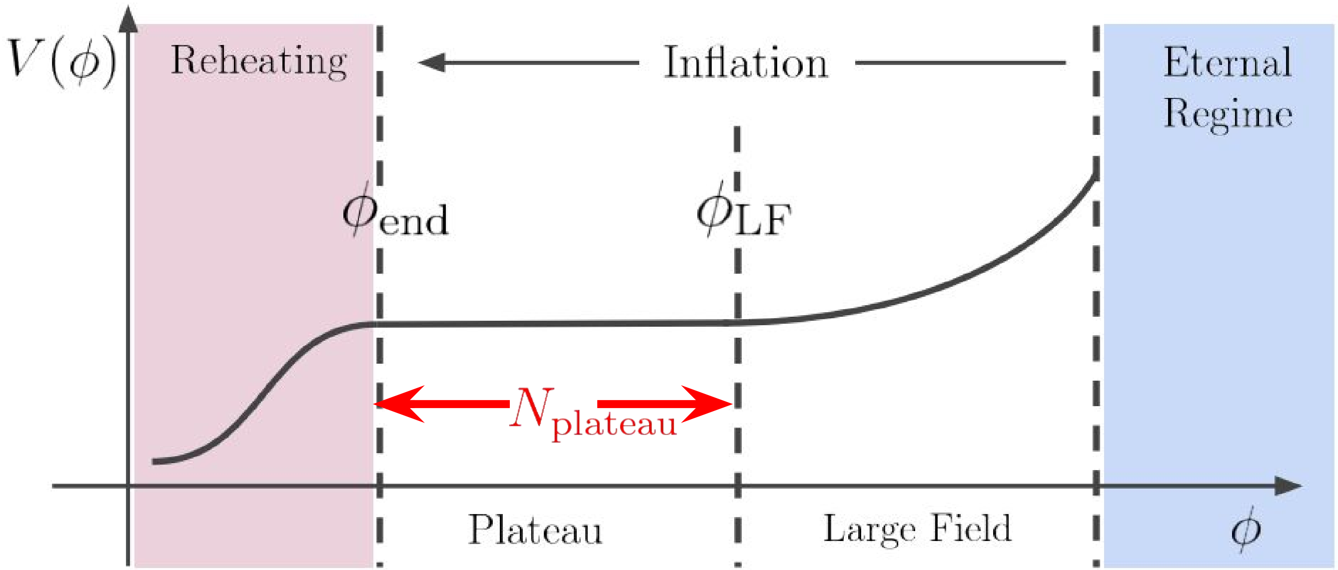
Recap

- We have shown that spectator field displacements @ end of inflation may strongly depend on the whole inflaton potential.

- In a particular, we find that $\frac{1}{2}m^2\sigma^2$ -> no dS-eq. So typically acquires super-Planckian field values but self-interactions ($\lambda\sigma^4$) can avoid this.

One consequence...

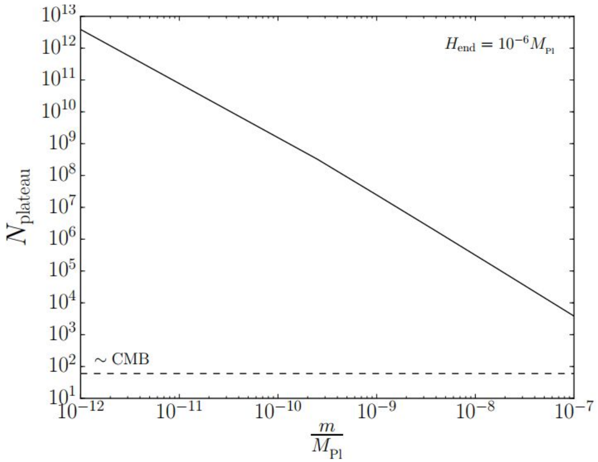
Consider the following inflaton potential:



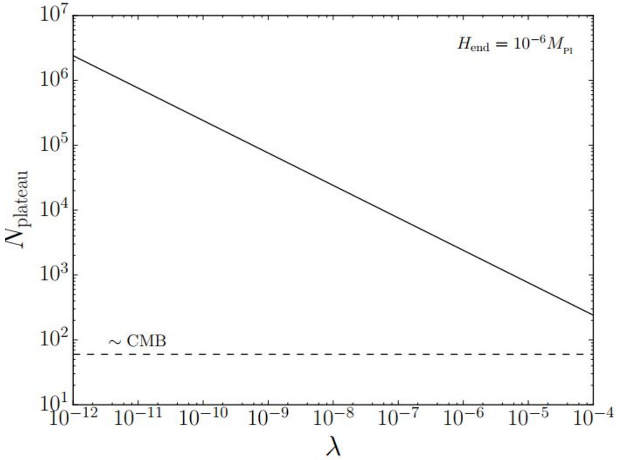
One consequence...

Define N_{plateau} as the number of e-folds on the plateau required to return to dS equilibrium.

Quadratic



Quartic



During inflation, any light test fields present could be very sensitive to the whole inflaton potential...

This is also very important for the theoretical ‘prior’ distribution of field values…

For the (still observationally viable) curvaton scenarios ->

Jesús Torrado, Christian T. Byrnes, RJH, Vincent Vennin, David Wands (in preparation)

Thanks for listening!

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