Field-theoretic simulations of colliding superconducting strings

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Collaboration with Daisuke Yamauchi (Kanagawa), Daniele Steer (APC)

Introduction

Kibble mechanism

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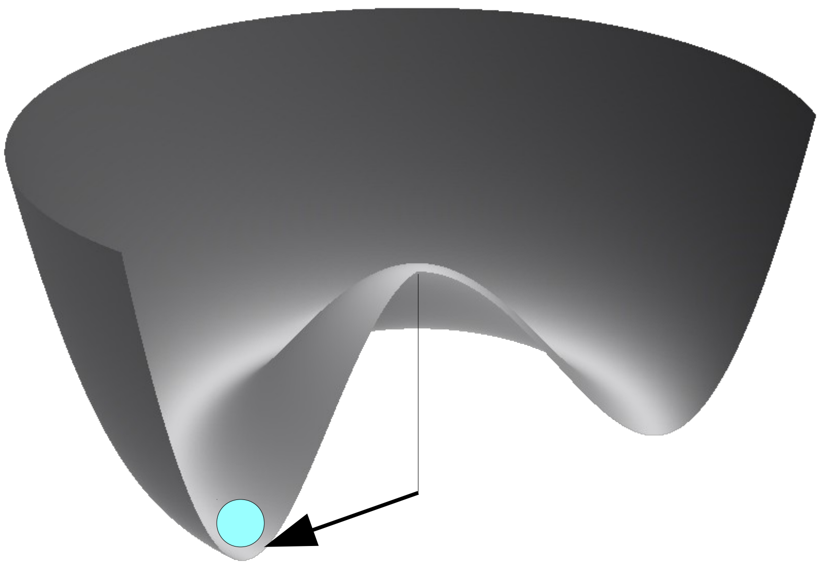
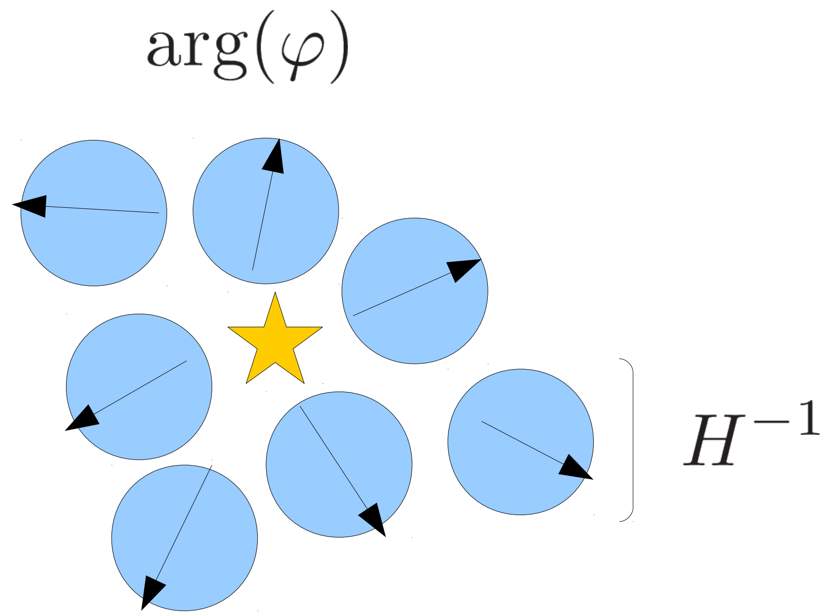
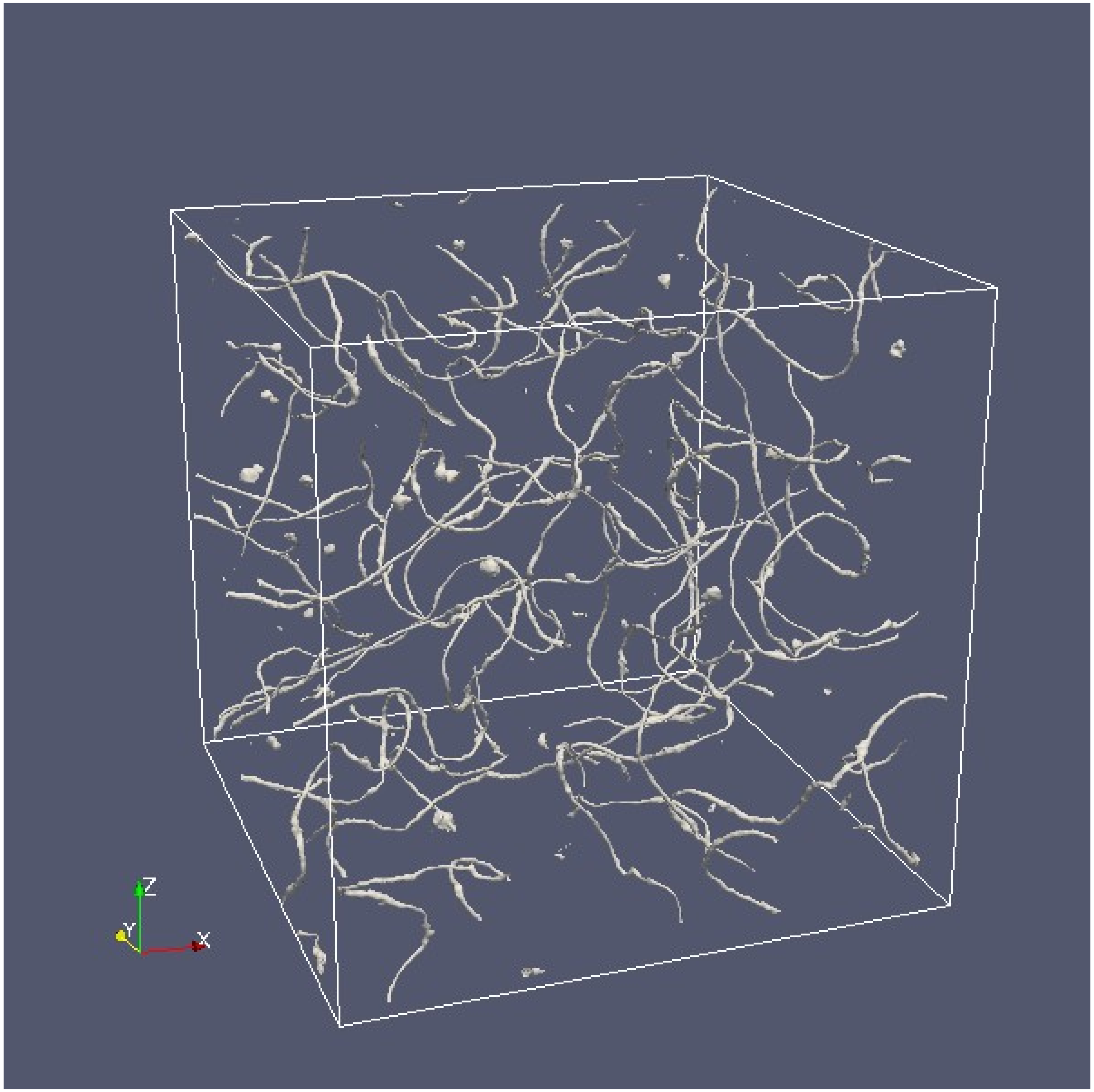




φ is aligned in a causal volume (=Hubble),
but its argument is different in each volume.

If the above situation is realised,
a cosmic string appears at  .

Kibble mechanism



Decay rate of energy density

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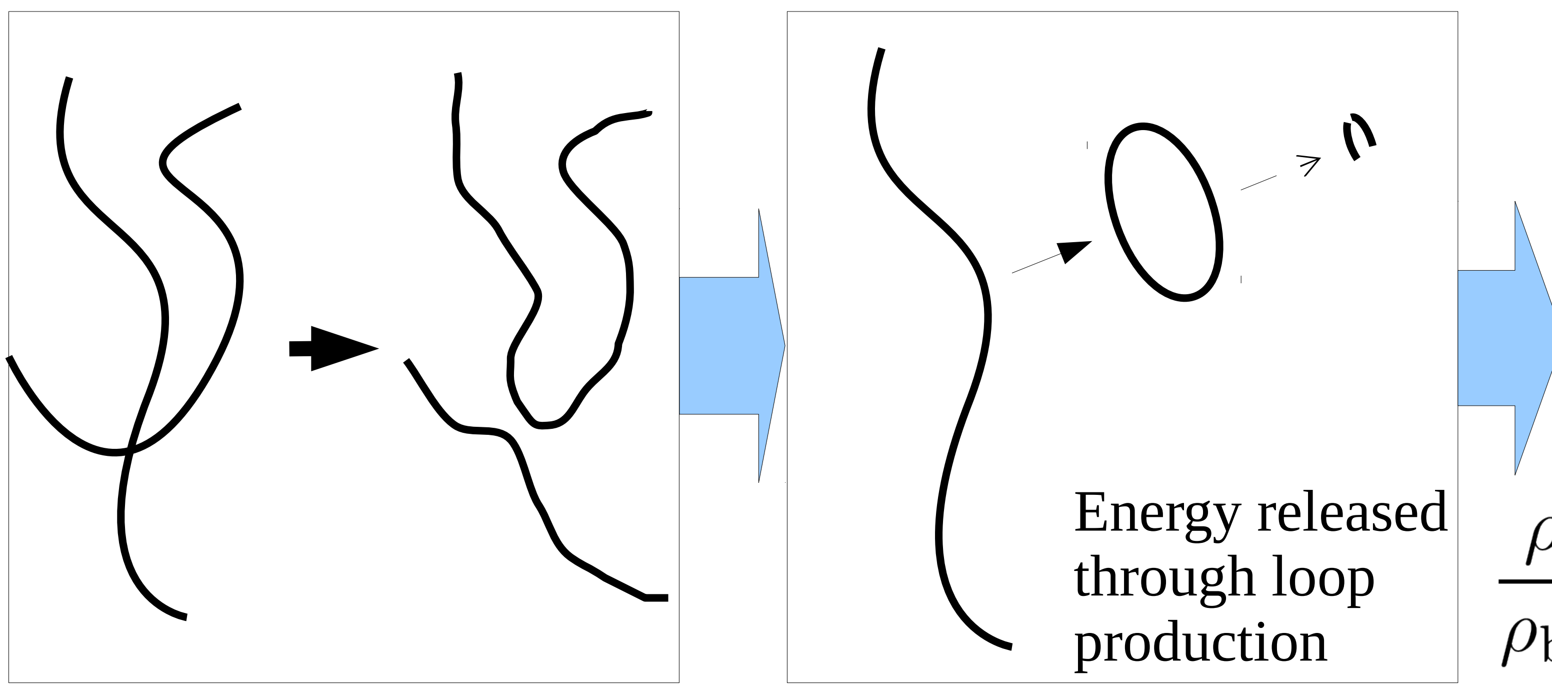
Naively, $\rho_{\rm str} \propto a^{-2}$ ⟶ Eventually dominated, so highly suppressed.

But,



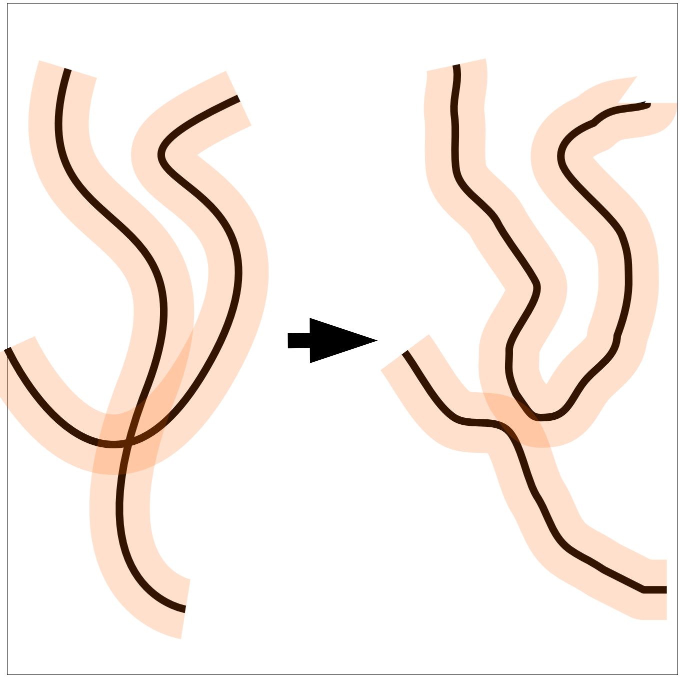
$\propto a^{-4}$ (RD)

$\propto a^{-3}$ (MD)

$\dfrac{\rho_{\rm str}}{\rho_{\rm back}}$ is always constant.

Whether cosmic strings have to be constrained highly depends on the efficiency of reconnection process.

What we'd like to study



Safely reconnected ?

Reconnection process works even if strings couple with matter ?

Extend a past numerical study by Laguna and Matzner. **Laguna and Matzner, PRD 41 (1990) 1751**

Action and vortex solution

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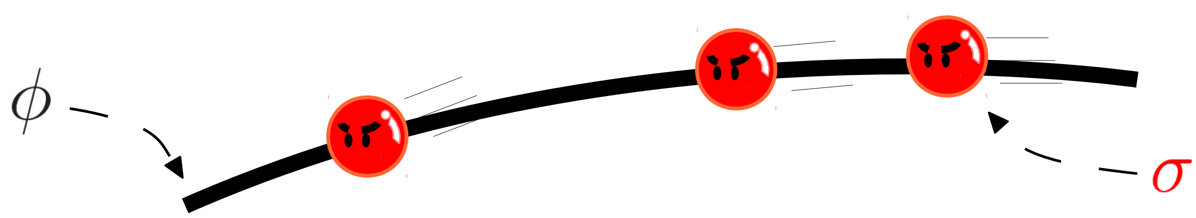
Model Lagrangian

Abelian-Higgs model ($U(1)$ gauge theory) + additional scalar field

$$S = -\int dx^4 \sqrt{-g}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) + (\partial_\mu\sigma)^*(\partial^\mu\sigma) + V(\phi,\sigma)\right)$$

$$V(\phi,\sigma) = \frac{\lambda_\phi}{4}(|\phi|^2-\eta^2)^2 + \boxed{\lambda_{\phi\sigma}(|\phi|^2-\eta^2)|\sigma|^2} + \frac{\lambda_\sigma}{4}|\sigma|^4 + \frac{m_\sigma^2}{2}|\sigma|^2$$

$$D_\mu \equiv \partial_\mu - ieA_\mu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



Conserved corrent : $j_\mu = 2\mathrm{Im}(\sigma^*\partial_\mu\sigma)$

* Its realisability and observability in cosmological context is discussed by Witten.

Witten, NPB 249 (1985) 557

Preparation

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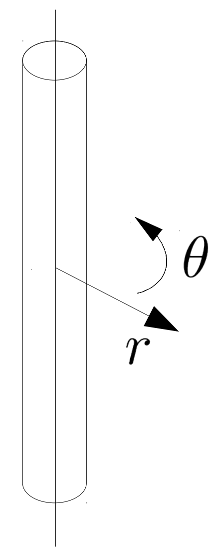
Field equations

$$D_\mu D^\mu \phi = \frac{\partial V}{\partial \phi^*} \qquad D_\mu F^{\mu\nu} = -2e\eta^{\mu\nu}\mathrm{Im}(\phi^* D_\mu \phi)$$

$$D_\mu D^\mu \sigma = \frac{\partial V}{\partial \sigma^*} \qquad (D_\mu = \partial_\mu - ieA_\mu)$$



Axially-symmetric ansatz

$$\phi(\mathbf{r}) = \eta f(r) e^{in\theta} \qquad A_\theta(\mathbf{r}) = \frac{n}{e}\alpha(r) \qquad \sigma(\mathbf{r}) = \eta g(r) e^{i(kz-\omega t)}$$

Solve them as 1-dim boundary-value problem with CGS+SOR

Preparation

Ansatz and potential

$$V(\phi,\sigma) = \frac{\lambda_\phi}{4}(|\phi|^2-\eta^2)^2 + \lambda_{\phi\sigma}(|\phi|^2-\eta^2)|\sigma|^2 + \frac{\lambda_\sigma}{4}|\sigma|^4 + \frac{m_\sigma^2}{2}|\sigma|^2$$

$$D_\mu = \partial_\mu - ieA_\mu$$

$$\phi(\mathbf{r}) = \eta f(r) e^{in\theta} \qquad A_\theta(\mathbf{r}) = \frac{n}{e}\alpha(r) \qquad \sigma(\mathbf{r}) = \eta g(r) e^{i(kz-\omega t)}$$

Model parameters

winding number : $n = 1$

self-coupling of ϕ : $\beta_\varphi \equiv \frac{\lambda_\varphi}{2e^2} = 1$

self-coupling of σ : $\beta_\sigma \equiv \frac{\lambda_\sigma}{2e^2} = 1$

coupling between ϕ & σ : $\beta_{\varphi\sigma} \equiv \frac{\lambda_{\varphi\sigma}}{2e^2}$

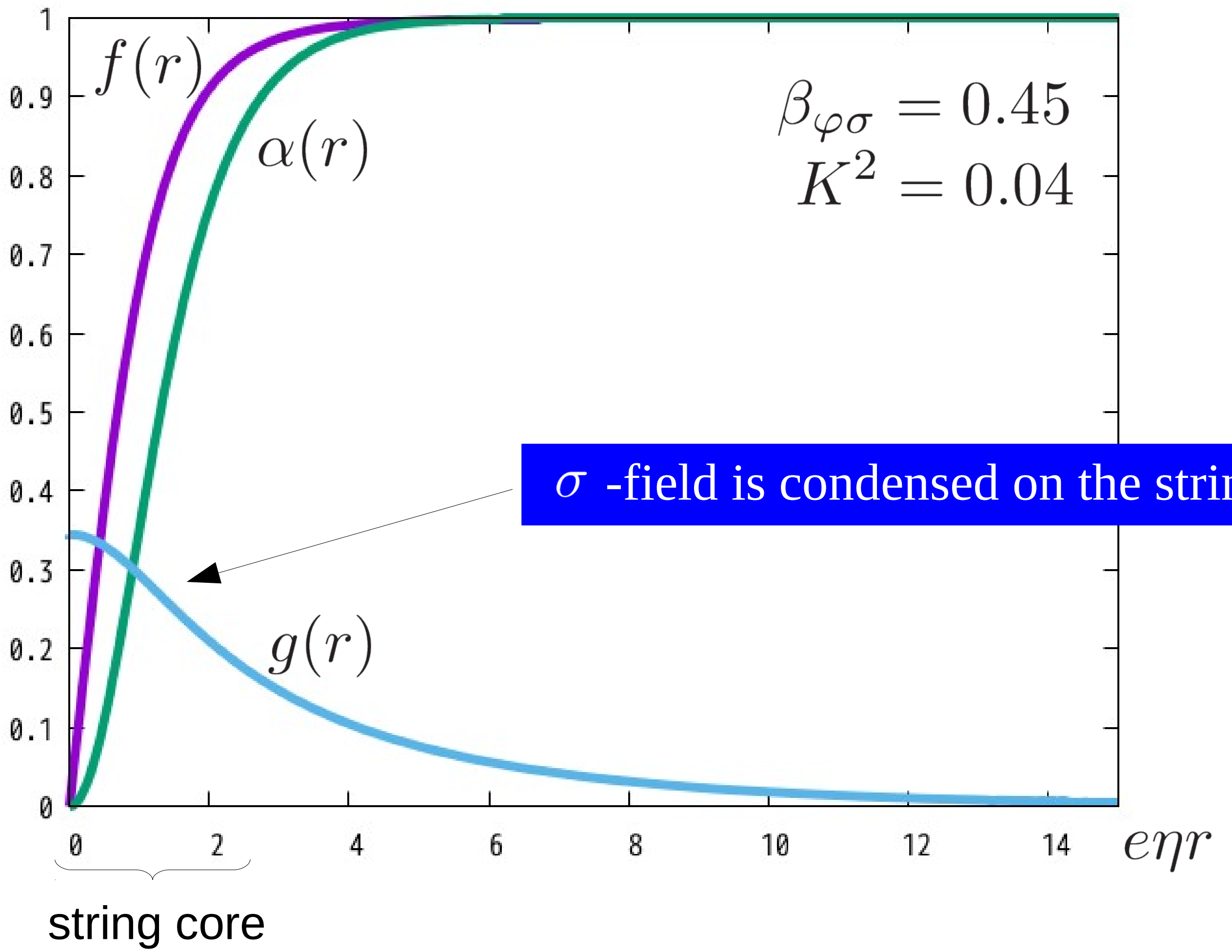
bear mass of σ : $\mu^2 \equiv \frac{m_\sigma^2}{2e^2\eta^2} = 0.01$

charge of σ : $\Omega^2 \equiv \frac{\omega^2}{e^2\eta^2} = 0$

current of σ : $K^2 \equiv \frac{k^2}{e^2\eta^2}$

Find straight vortex solutions

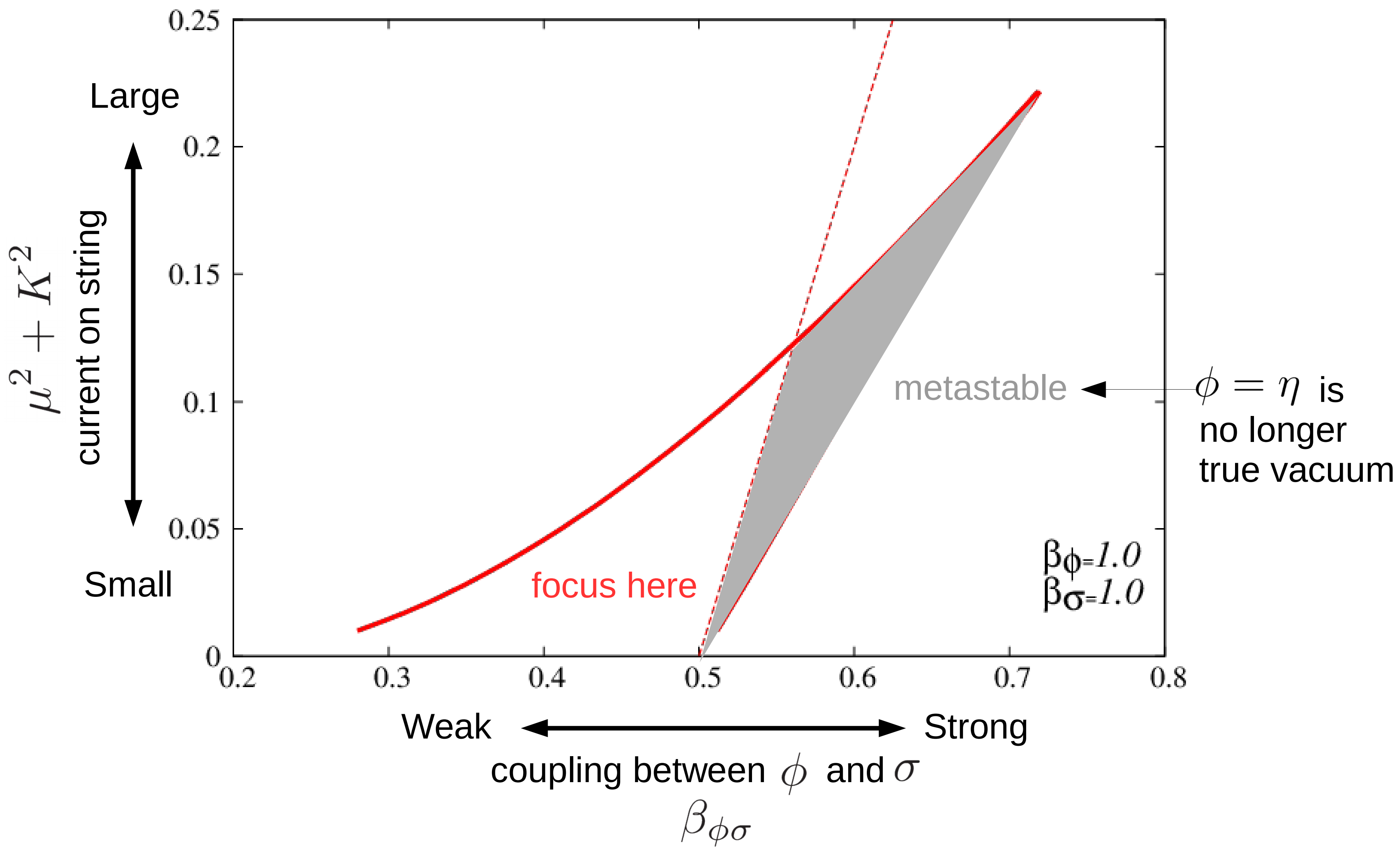
$$\phi(\mathbf{r}) = \eta f(r) e^{i\theta} \qquad A_\theta(\mathbf{r}) = \frac{1}{e}\alpha(r) \qquad \sigma(\mathbf{r}) = \eta g(r) e^{ikz}$$



σ -field is condensed on the string.

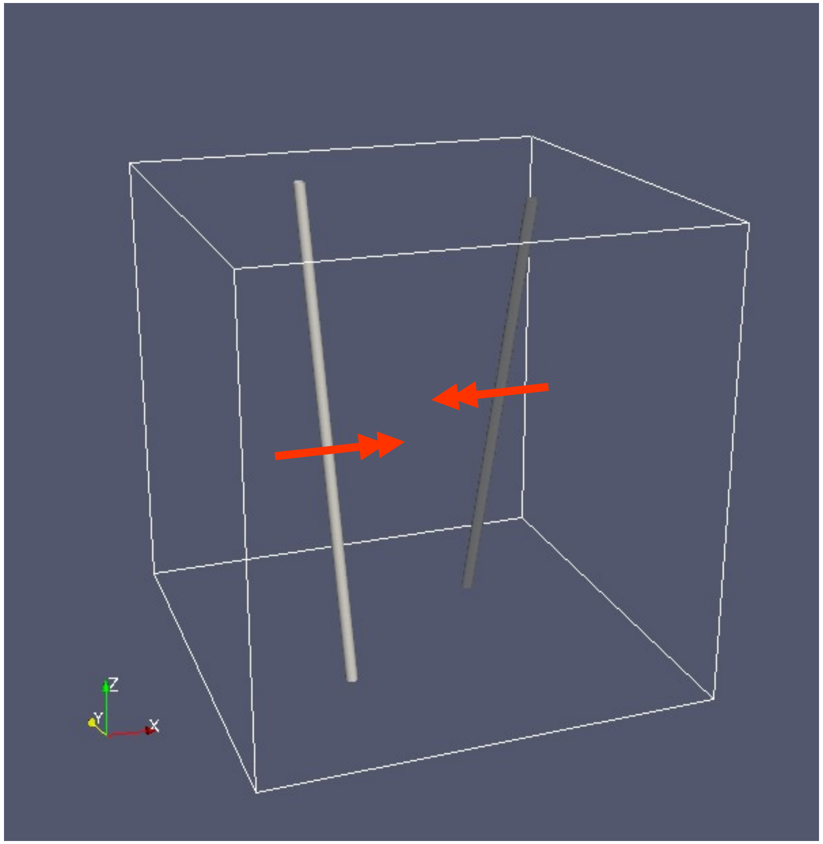
Result I : viable parameter region

Superconducting string configuration is available only in the triangle region.



Simulations of colliding strings

Setup of colliding simulations



Strategy

- Prepare 2 stable straight strings.
- Lorentz boost (velocity+rotation)

$$x^{\mu\prime} = \Lambda^{\mu}{}_{\nu} x^{\nu}$$

- Superposition

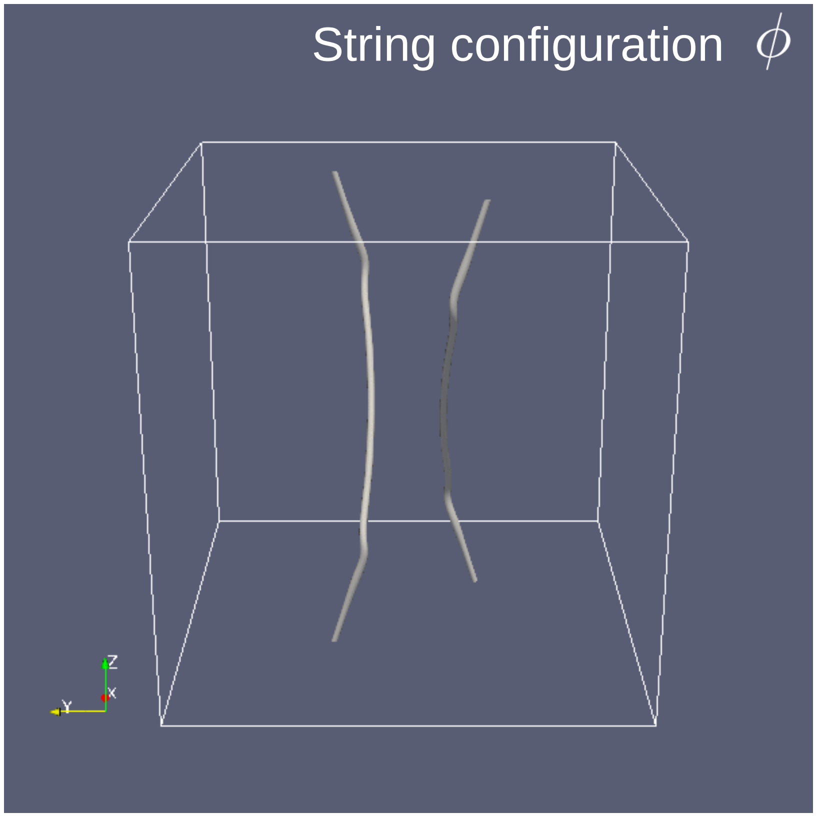
$$\phi = \frac{1}{\eta}\phi^{(1)}\phi^{(2)}$$
$$A_\mu = A_\mu^{(1)} + A_\mu^{(2)}$$
$$\sigma = \sigma^{(1)} + \sigma^{(2)}$$

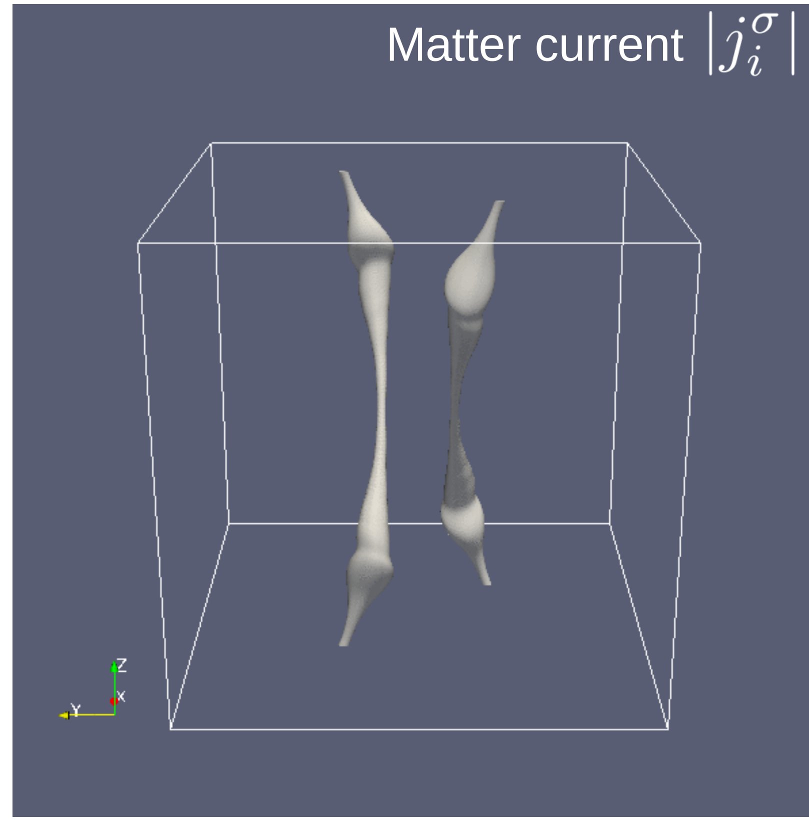
Numerical methods

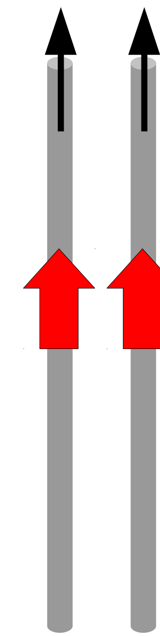
- Leap-Frog scheme
- 2nd-order finite difference
- Adaptive box size depending on velocity and angle, roughly $200^3 \sim 800^3$

Result II (1/3) : safe reconnection

Parallel – Parallel pair







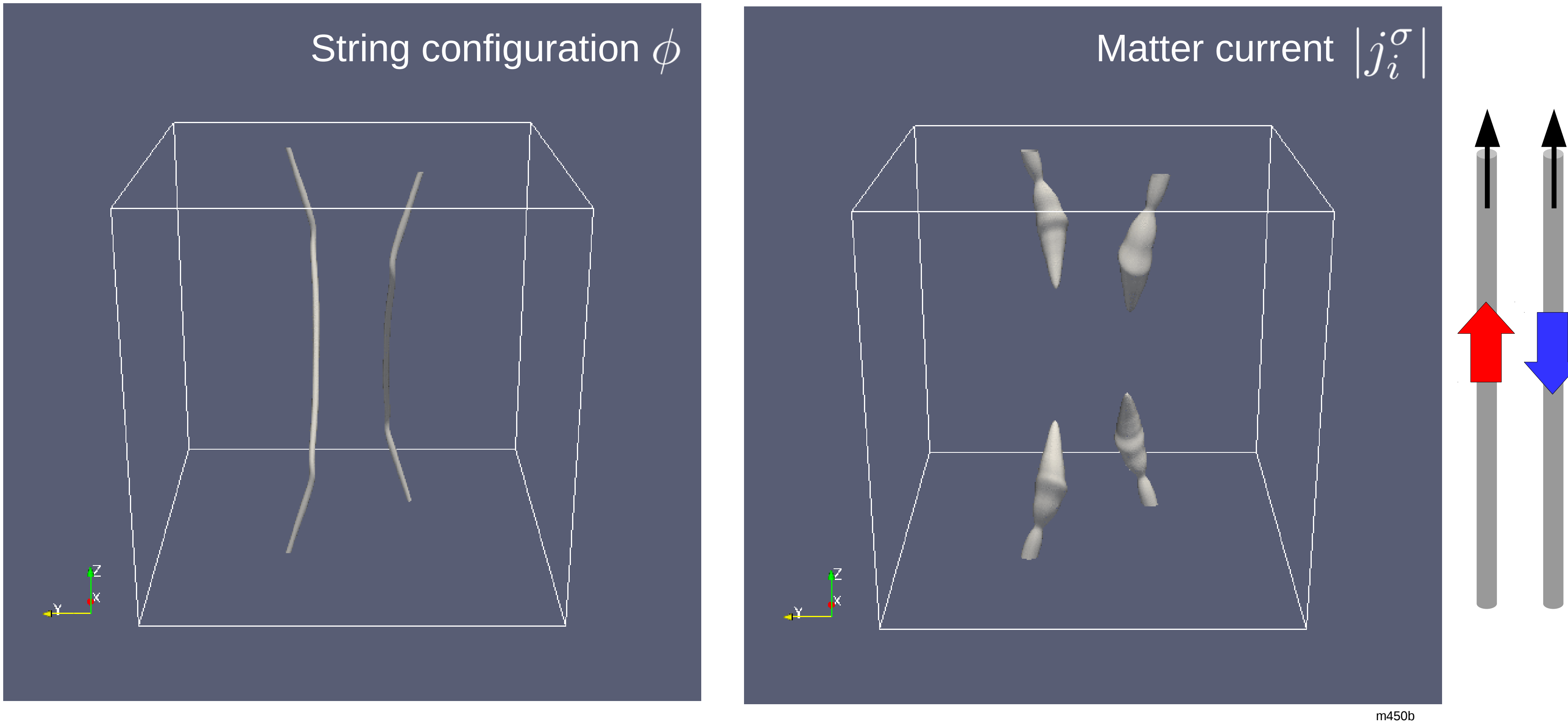
m470b

$$(v, \alpha) = (0.5, 0.2\pi)$$

Reproduced the Laguna-Matzner's simulations.
Matter current runs along the string circuit.

Result II (2/3) : current disappearance

Parallel – Antiparallel pair



$$(v, \alpha) = (0.5, 0.2\pi)$$

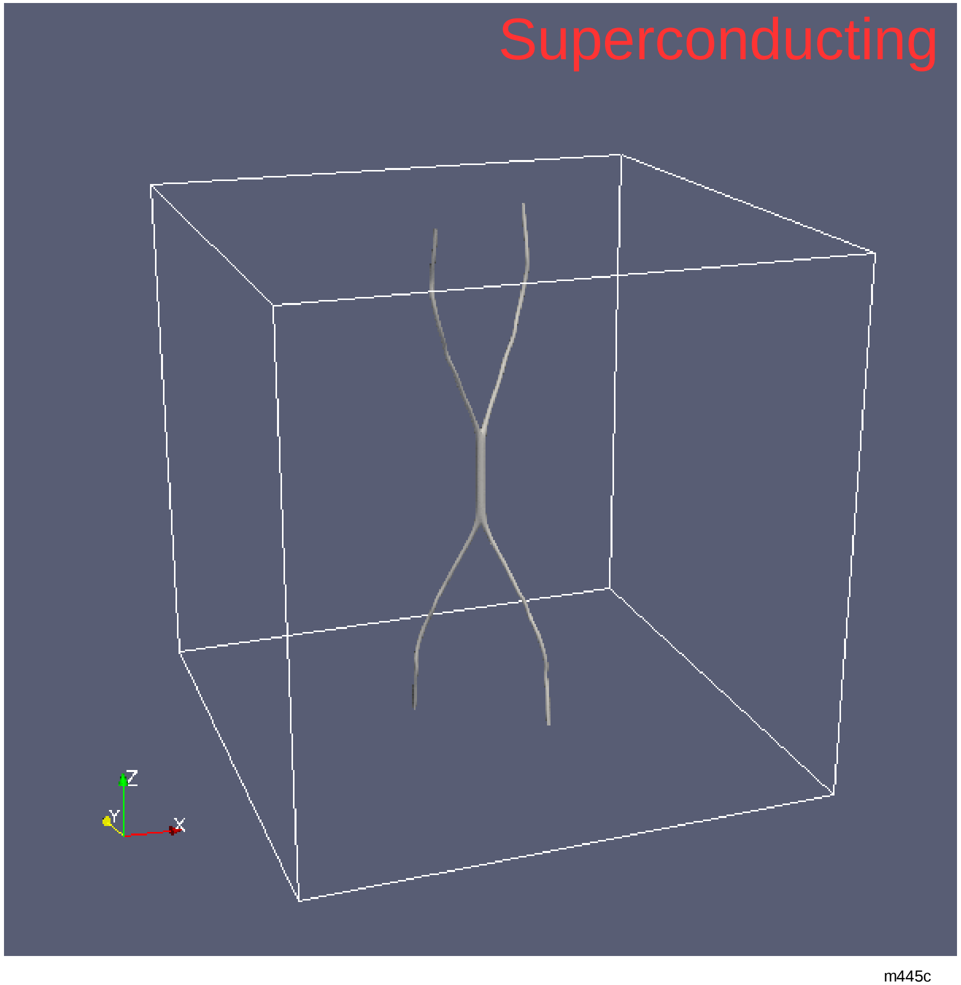
'current-nocurrent-current' structure appears in (p,a) pair. (reproduced Laguna-Matzner)
Current tends to revive after reconnection. (not well discussed in Laguna-Matzner)

Laguna and Matzner, PRD 41 (1990) 1751

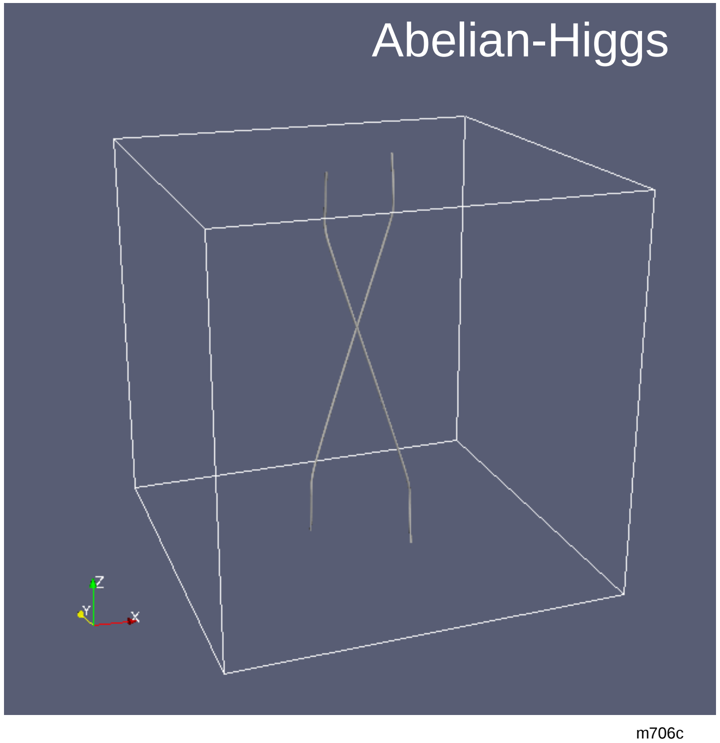
Result II (3/3) : bound state

Strings colliding with small angle and velocity are bounded after collision.

$$\beta_\varphi = 1 \quad \beta_{\varphi\sigma} > 0$$



$$\beta_\varphi = 1 \quad \beta_{\varphi\sigma} = 0$$



$$(\beta_{\varphi\sigma}, \gamma) = (0.45, 0.05) \qquad (v, \alpha) = (0.3, 0.1\pi)$$

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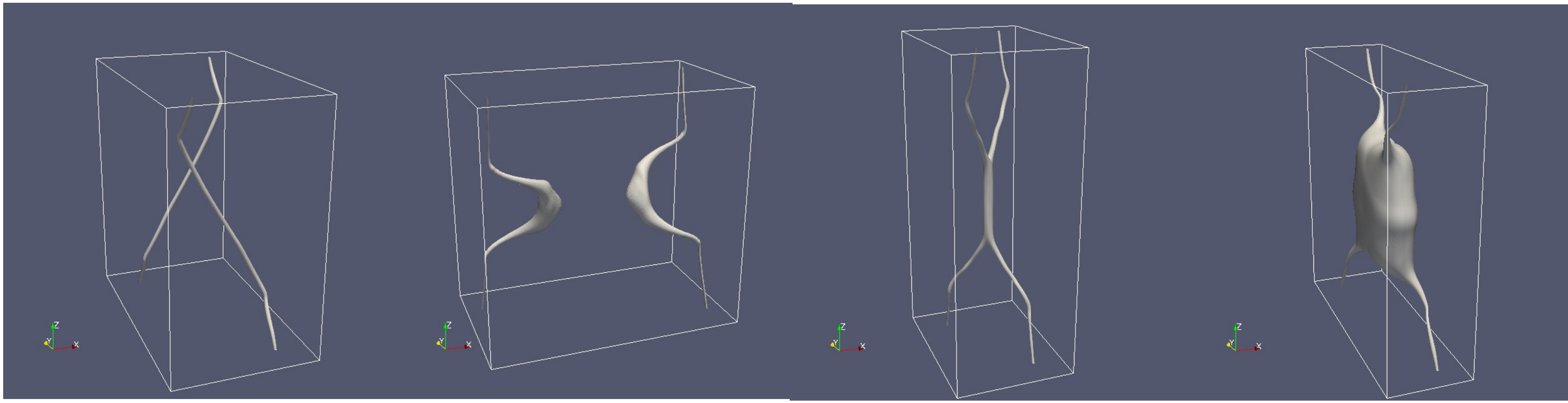
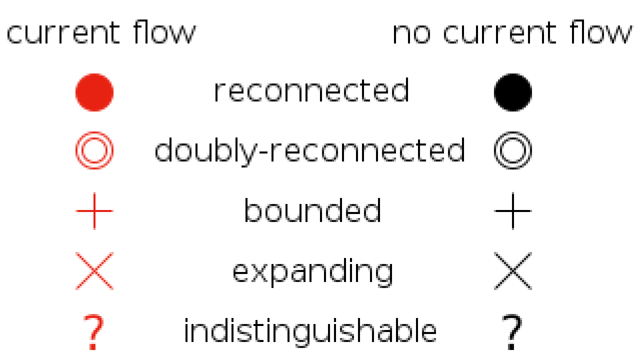
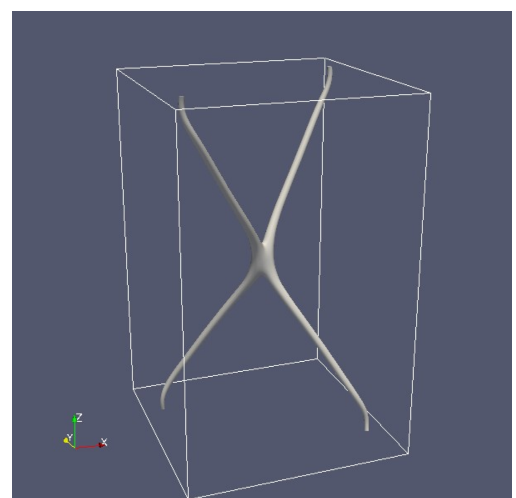
Define 4 kinds of final states



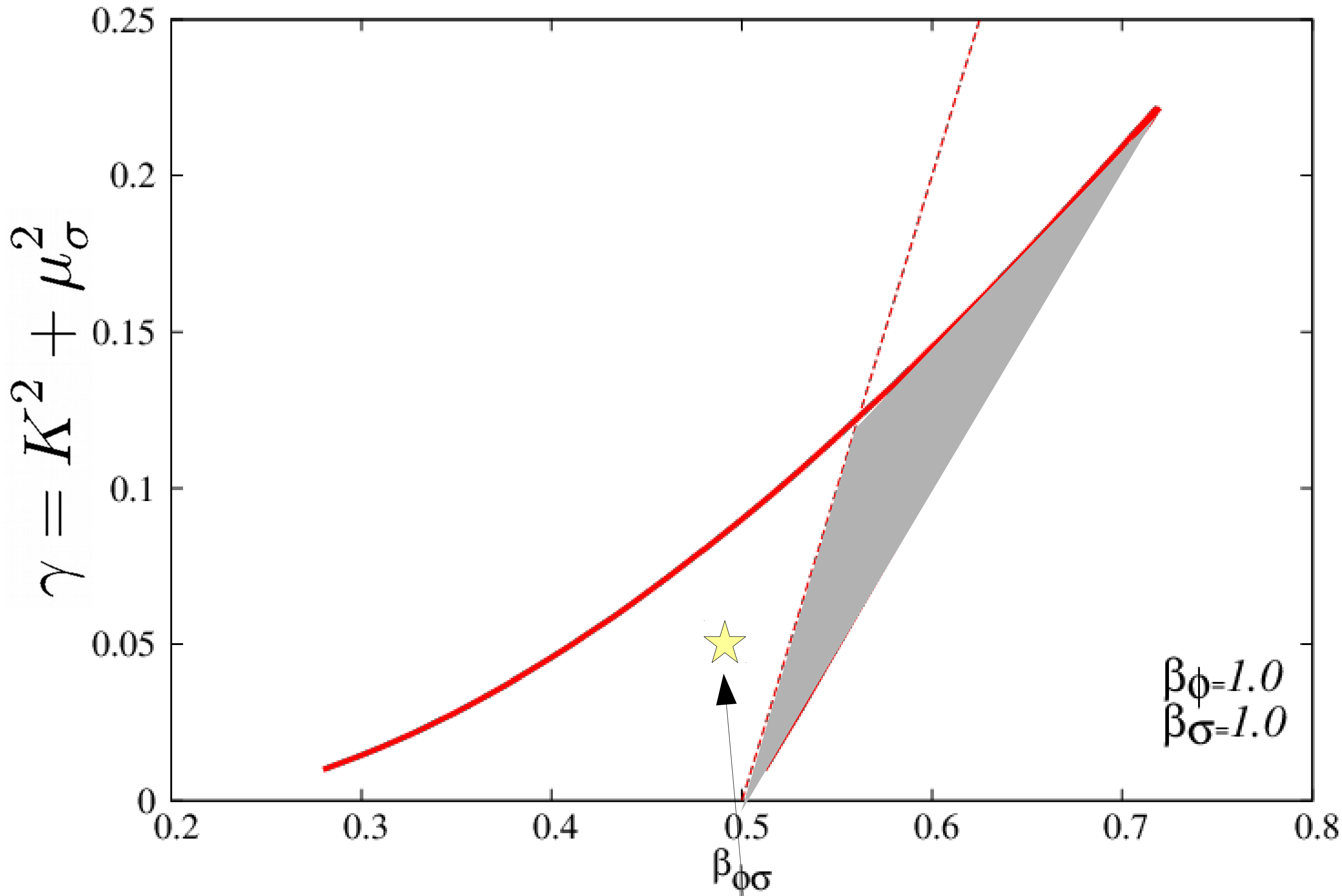
Reconnected — Doubly-reconnected (passing through) — Bounded — Expanding





Indistinguishable

‘Phase diagram’ : fiducial setup



Stable superconducting string
(fiducial parameter)

‘Phase diagram’ : fiducial setup

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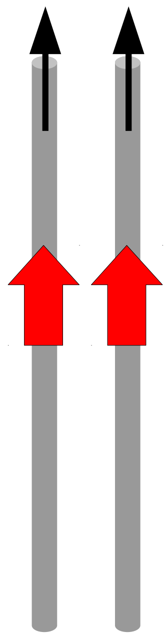
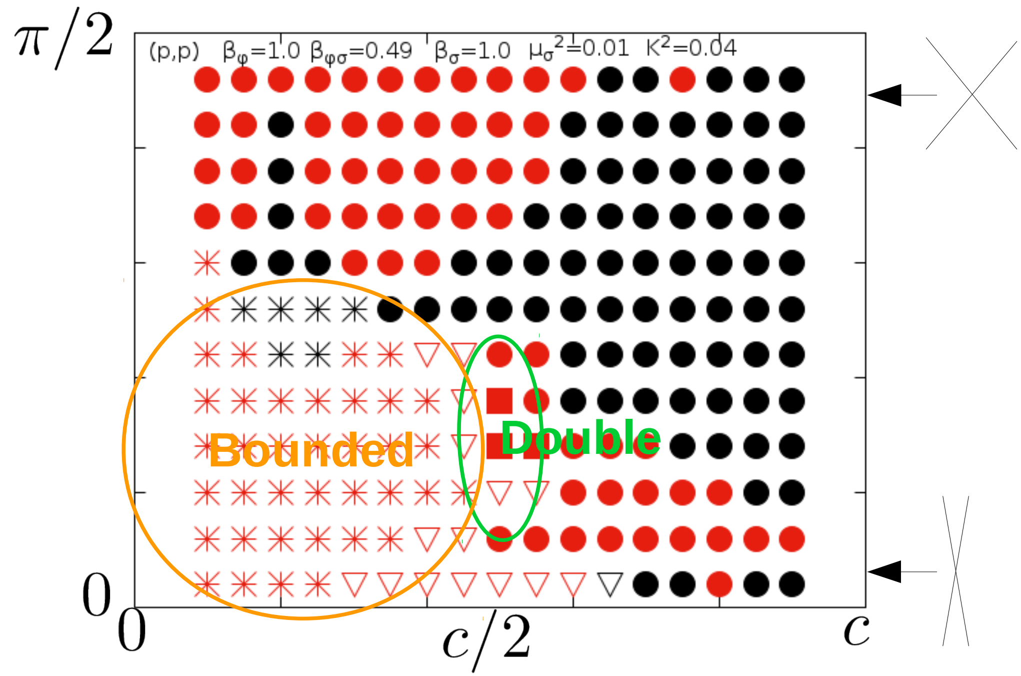
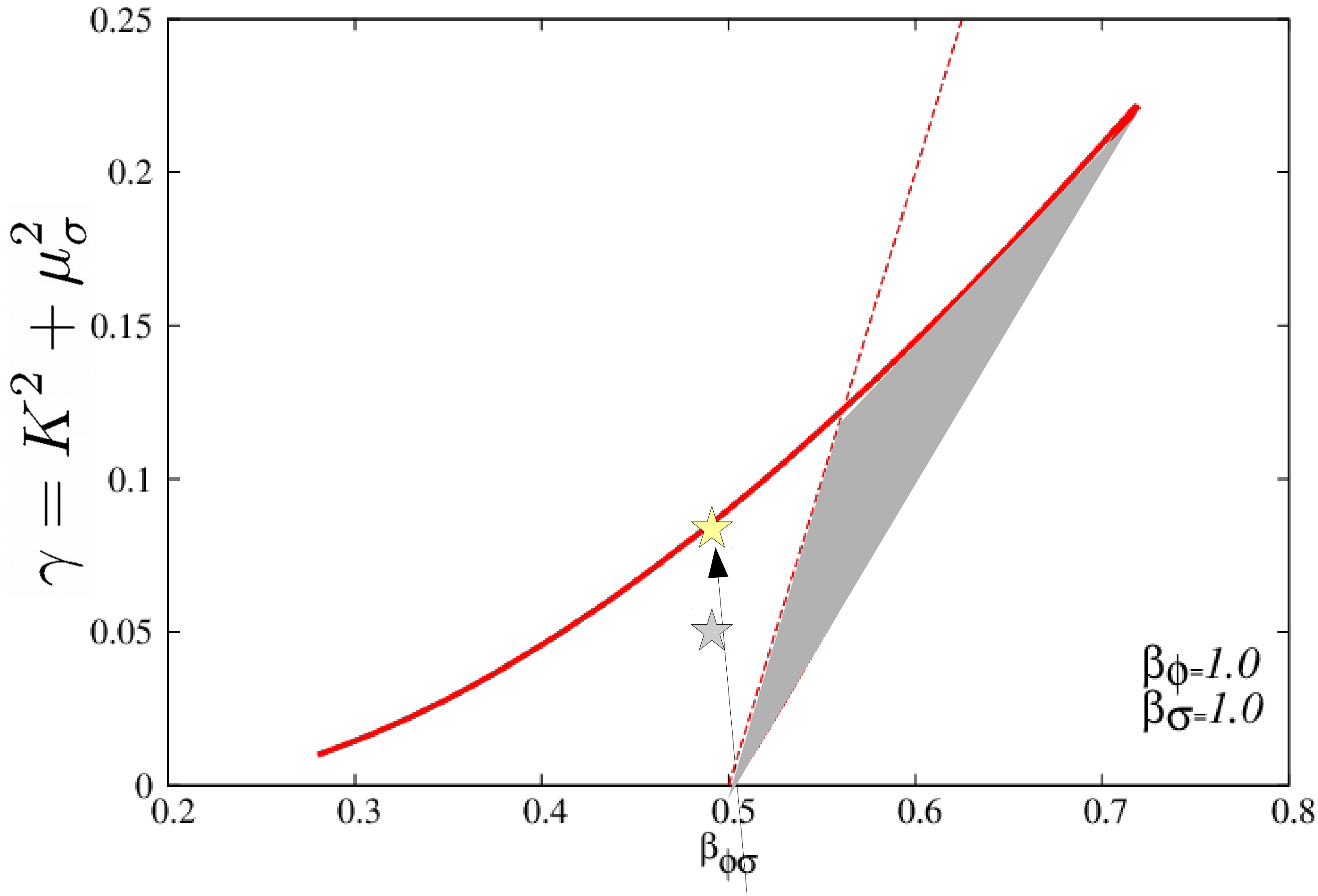




- Almost all configurations result in the usual reconnection.
- Low-angle and low-velocity collisions result in bound states.
- The double reconnection takes place for intermediate-velocity collisions.
- The current tends to disappear for high-speed collisions.

→ Attempting to understand this in analytic way (Yamauchi's talk)

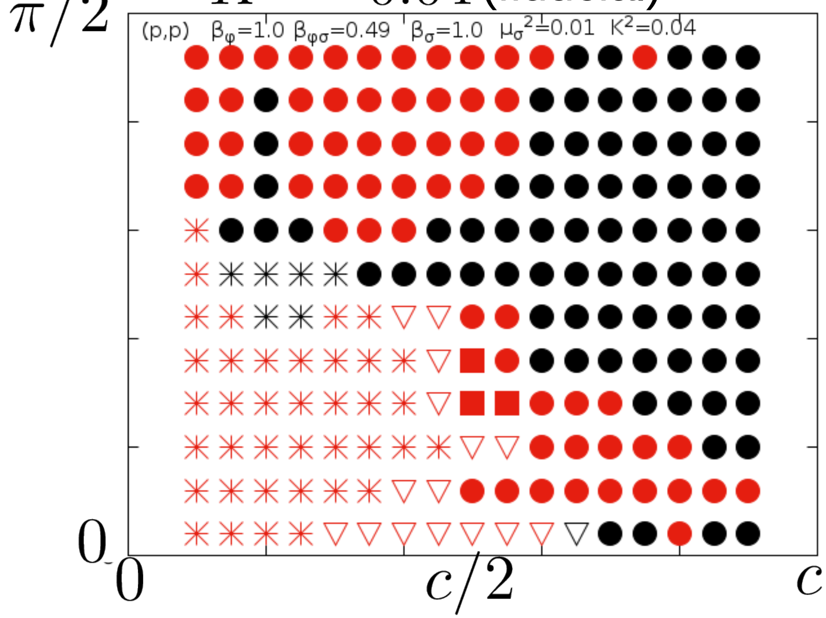
‘Phase diagram’ : K^2 dependence



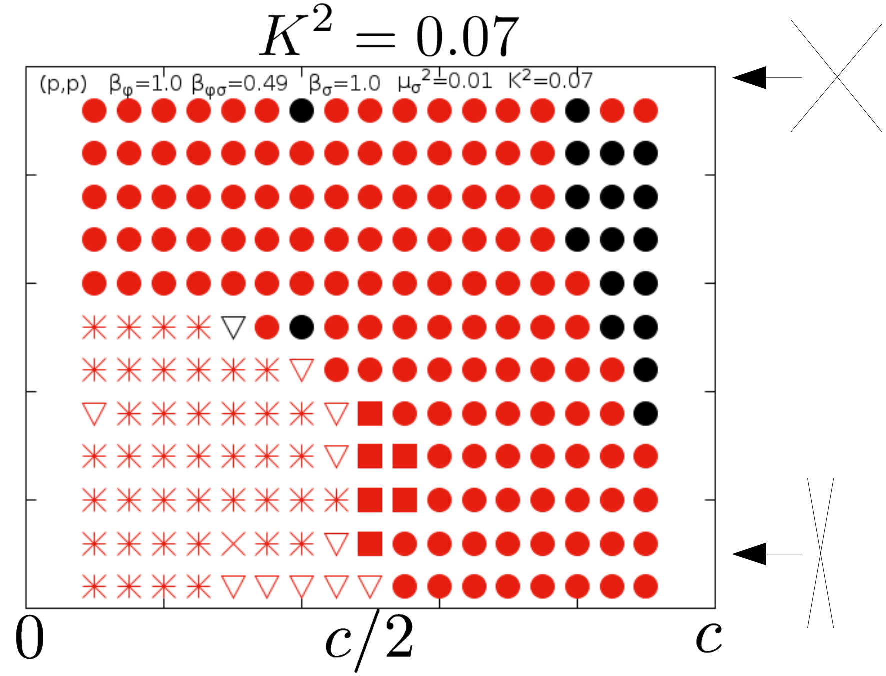
Stable superconducting string

‘Phase diagram’ : K^2 dependence

$K^2 = 0.04$ (fiducial)

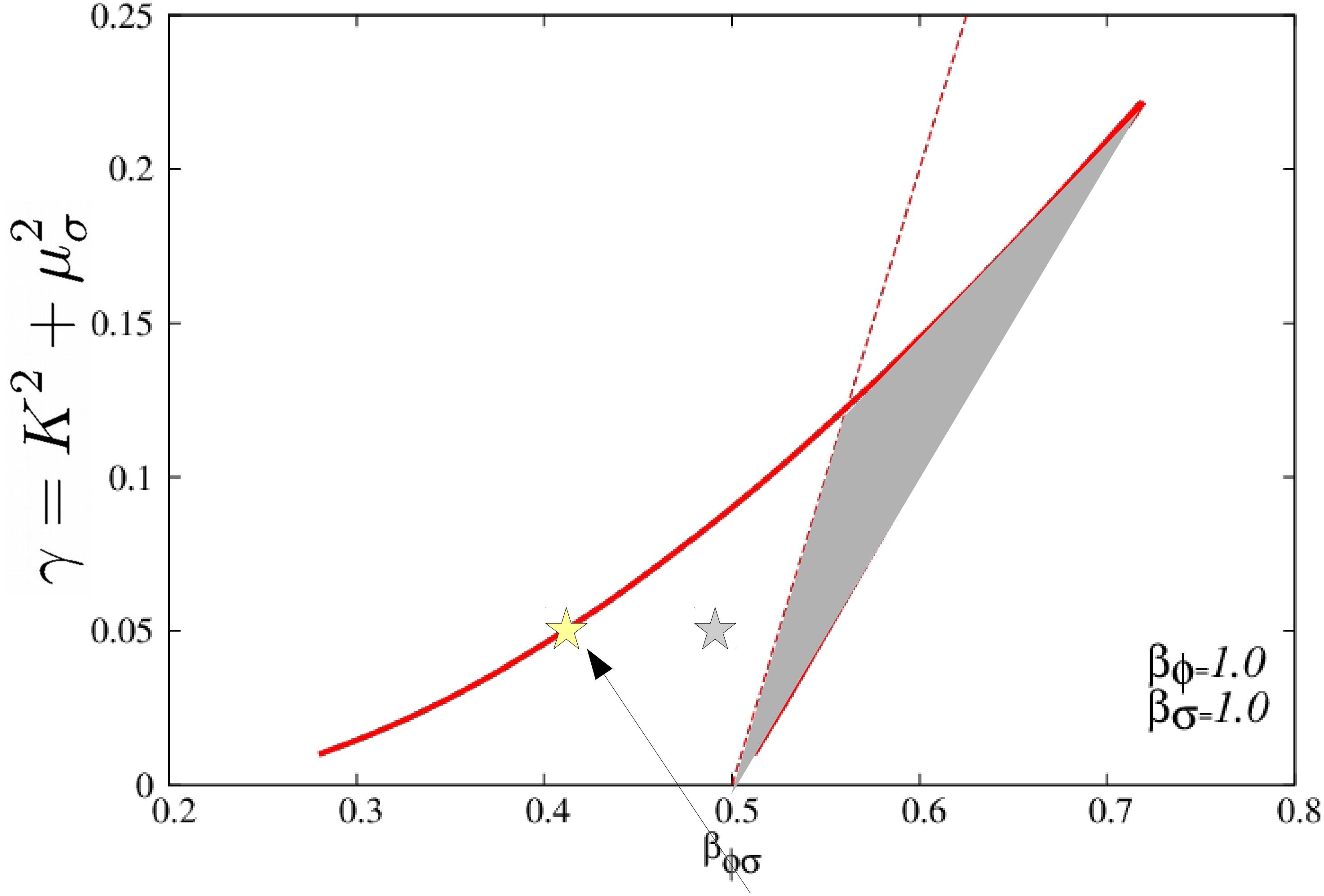


$K^2 = 0.07$



- The current after collisions is easy to survive for large K^2
- The initial current strength K^2 does not affect the final configuration.

‘Phase diagram’ : $\beta_{\varphi\sigma}$ dependence



Stable superconducting string

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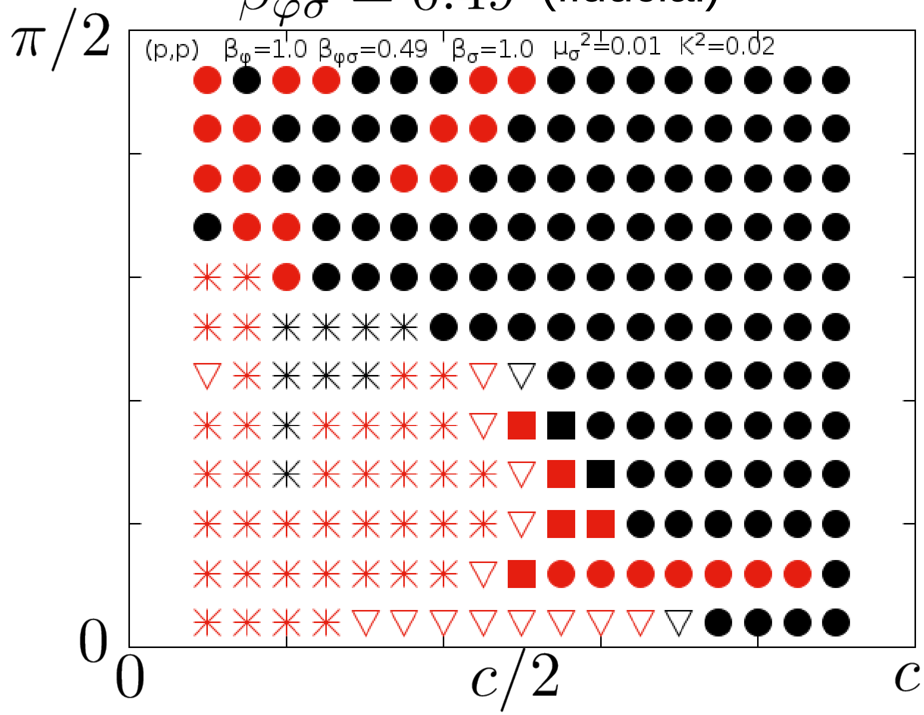
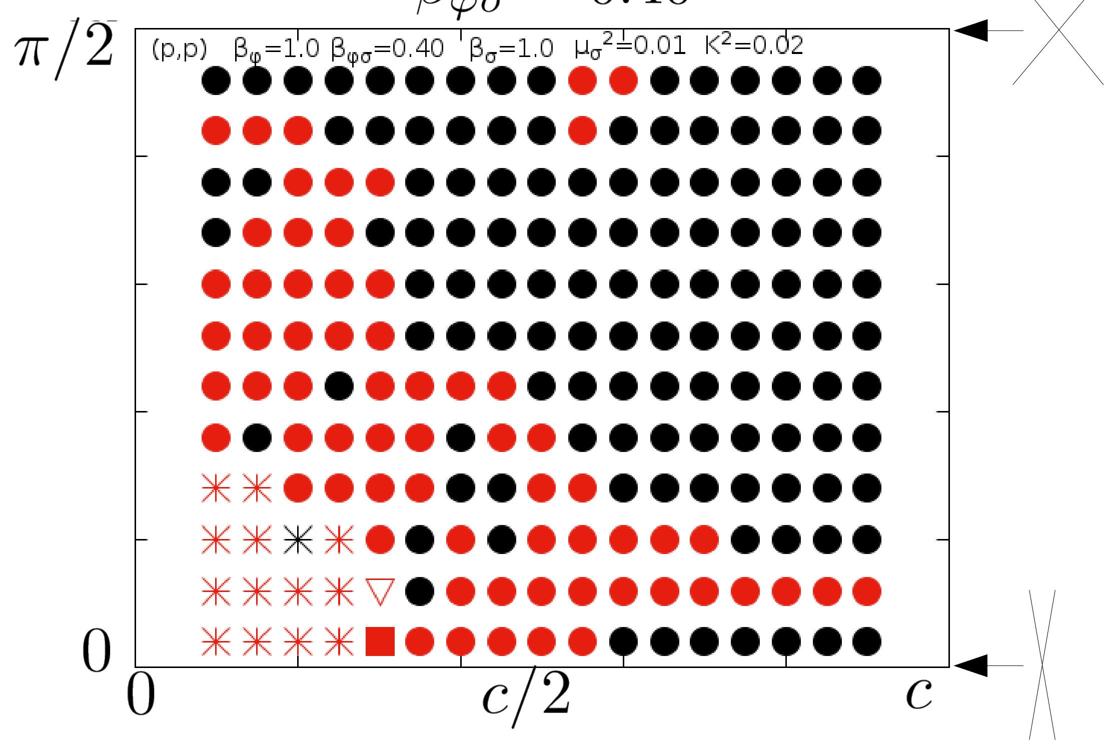
$\beta_{\varphi\sigma} = 0.49$ (fiducial)

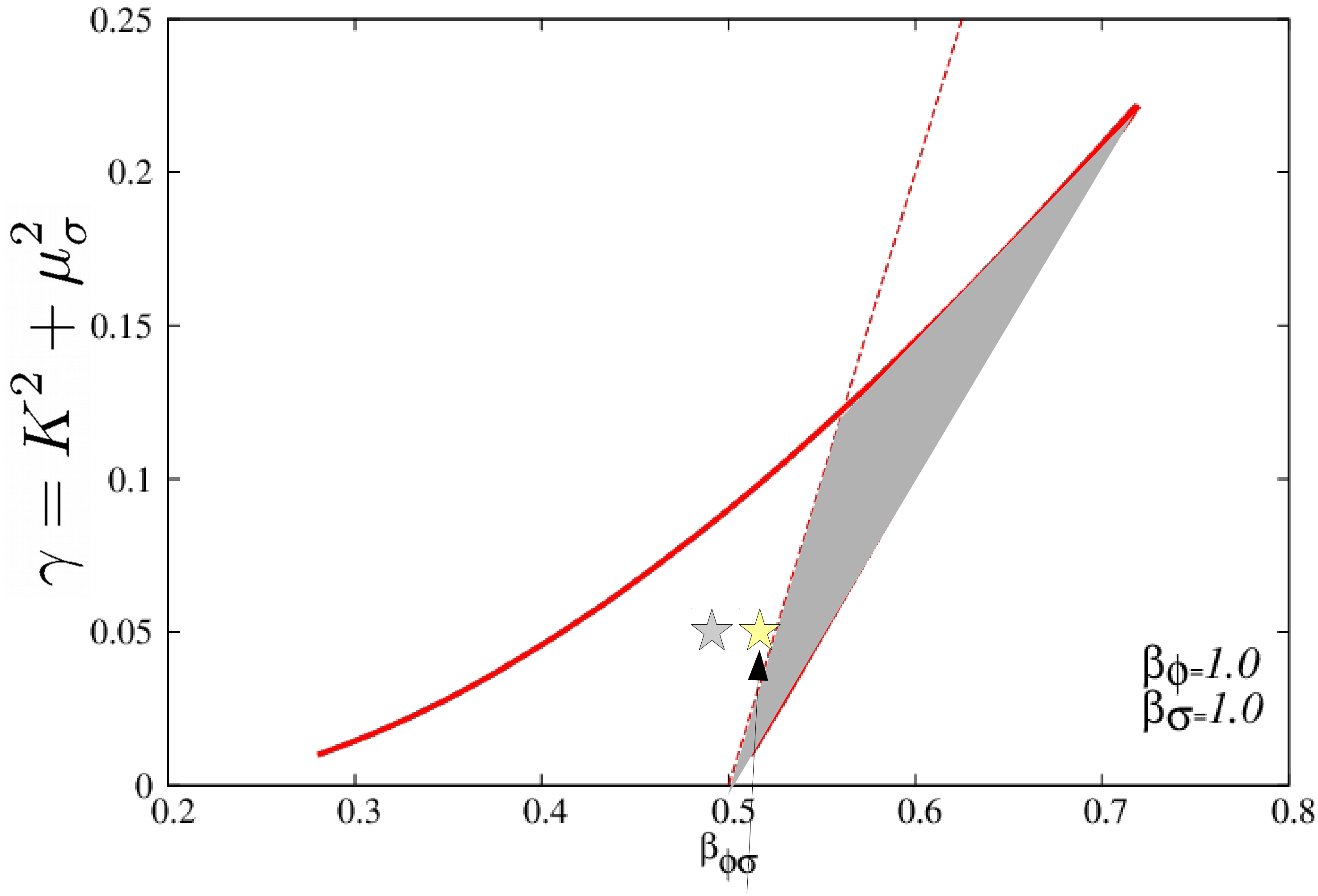


$\beta_{\varphi\sigma} = 0.40$



- The small $\beta_{\varphi\sigma}$ leads to less possibilities for forming bound states.
- $\beta_{\varphi\sigma}$ is not responsible for the final current strength.

‘Phase diagram’ : $\beta_{\varphi\sigma}$ dependence



Stable superconducting string,
but there is an unstable region for $\gamma \lesssim 0.02$

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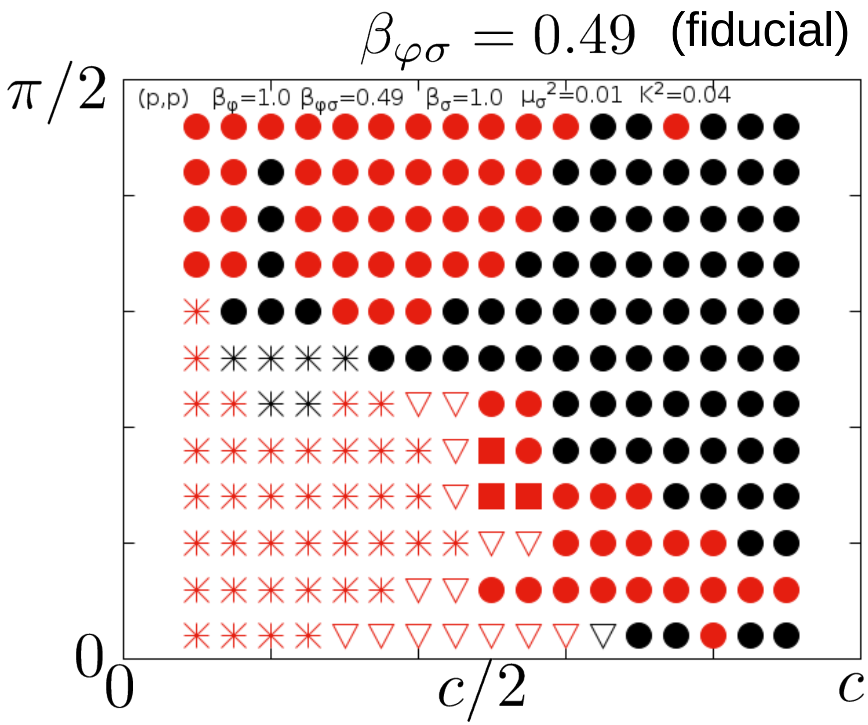

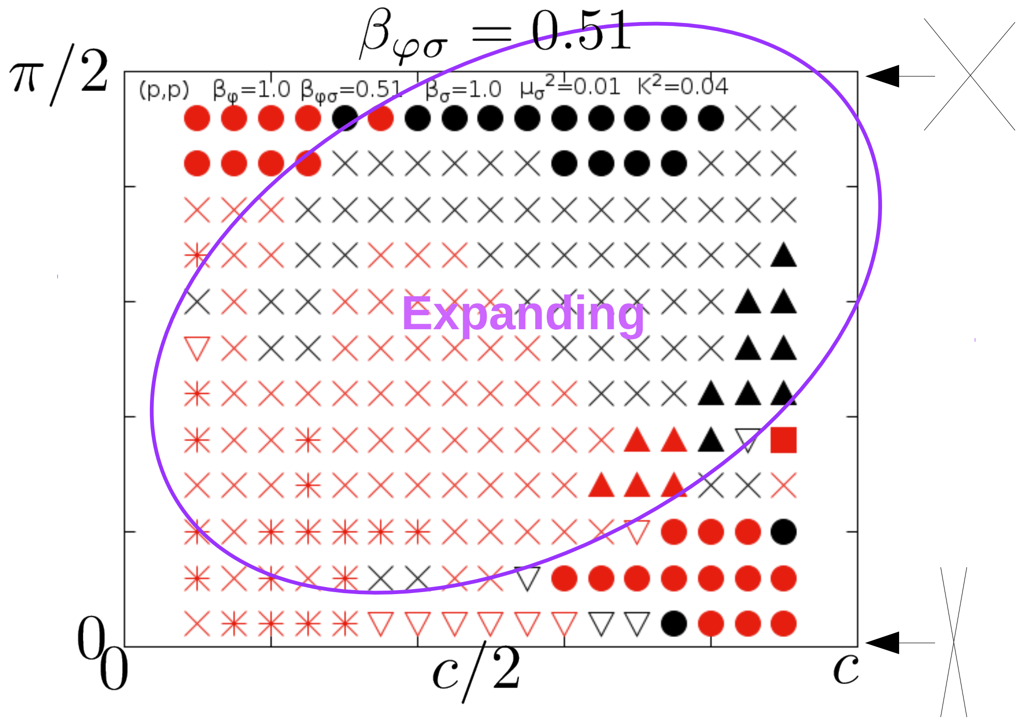

- The large $\beta_{\varphi\sigma}$ leads to the expanding bubble.
- Even so, high-speed collisions avoid the bubble nucleation.

Summary

Q. Can strings safely reconnect even if they couple to matter ?

Yes, but they have a rich diversity of their final states

⟷ Always successful for critical AH strings

- Stable pairs can form a bound state like Type-I AH strings.
- They can pass through each other by double-reconnection like Type-II AH strings.
- The final configuration depends on $\beta_{\varphi\sigma}$, not K^2.
- K^2 is responsible only for the final current strength.

If bound states form and double-reconnection takes place frequently, the network is prevented from loop production (efficient energy release process) ?

work in progress….