HEALTHY IMPERFECT DARK MATTER FROM EFFECTIVE THEORY OF MIMETIC COSMOLOGICAL PERTURBATIONS

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Mimetic theory

Mimetic dark matter Chamseddine, Mukhanov (2013)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \lambda(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 1)\right]$$

λ : Lagrangian multiplier, $\mathcal{R}$: 4-dim. Ricci scalar

$\delta\lambda$: $g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = -1$ **"mimetic constraint"**

$\Rightarrow$ <u>Pressureless dust</u> in Einstein gravity $\Leftrightarrow$ DM ?

no nontrivial propagating mode in FR sp. .

Imperfect Dark Matter (IDM)

Chamseddine et al. (2014) Milzagholi, Vikman (2015)

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \lambda(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 1) + \frac{\alpha}{2}(\Box\phi)^2\right]$$

α : constant

- **imperfect fluid**, dusty behavior on FR sp.
- non-zero sound speed (very small)

⇒ IDM could improve the behavior of density perturbation on small scale (at linear level).

Missing satellites and core-cusp problem could be solved. Capela, Ramazanov (2015)

Imperfect Dark Matter (IDM)

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α : constant

- imperfect fluid, dusty behavior on FLRW spacetime
- non-zero sound speed (very small)

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Missing satellites and core-cusp problem could be solved. Capela, Ramazanov (2015)

Phenomenologically interesting !

Our work

theoretical structure in mimetic theory $\overset{\text{same}}{\Leftrightarrow}$ the one in the **projectable** Horava-Lifshitz gravity

(mimetic constraint = projectable condition)

Potential issues: gradient instabilities , caustic singularity
solar system constraint …

Our work:
- We find the reason for these instabilities.
- We consider the large class of scalar-tensor theory with mimetic constraint to construct a healthy IDM.

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- We find **the reason** for these instabilities.
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Theoretical structure

Chamseddine et al. (2014) Milzagholi & Vikman (2015)

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}\mathcal{R} - \lambda(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 1) - V(\phi) + \frac{\alpha}{2}(\Box\phi)^2$$

$\delta\lambda : \quad g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = -1$ “mimetic constraint”

$\Rightarrow$ it is convenient to use unitary gauge: $\phi(t, \boldsymbol{x}) = t$.

- **ADM decomposition**

$$ds^2 = -N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$n_\mu = -N\delta^0_\mu,\, K_{ij} = \frac{E_{ij}}{N} = \frac{1}{2N}(\dot{\gamma}_{ij} - D_i N_j - D_j N_i)$$

$$\mathcal{R} = R + K_{ij}K^{ij} - K^2 + (\text{total derivative})$$

$$\Box\phi = -\frac{K}{N} + \frac{\Xi}{N}\,,\quad \Xi = \frac{\dot{N}}{N^2} - \frac{N^i\partial_i N}{N^2}$$

Theoretical structure

$$\frac{\mathcal{L}}{\sqrt{\gamma}} = \frac{N}{2}(R + K_{ij}K^{ij} - K^2) + \lambda\left(\frac{1}{N} - N\right) - NV(t) + \frac{\alpha}{2N}(K - \Xi)^2$$

$$\Xi = \frac{\dot{N}}{N^2} - \frac{N^i\partial_i N}{N^2}$$

$$\delta\lambda \, : \, N(t, \boldsymbol{x}) = 1$$

$$\delta N \, : \, \frac{1}{2}(R + K_{ij}K^{ij} - K^2) - 2\lambda - V - \frac{3\alpha}{2}E^2 + \frac{\alpha}{2}[\dot{E} - D_i(N^i E)] = 0$$

is the equation to determine the value of λ .

$\Rightarrow$ cannot determine the dynamics of spacetime.

$$\delta N_i \, : \, D_j \pi^{ij} = 0, \; \pi^{ij} := E^{ij} - (1-\alpha)\gamma^{ij}E$$

$$\delta\gamma_{ij} \, : \, \frac{1}{\sqrt{\gamma}}\frac{d}{dt}(\sqrt{\gamma}\pi^{kl})\gamma_{ik}\gamma_{jl} + \cdots$$

can only determine the dynamics of spacetime.

Theoretical structure

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Mimetic theory is equivalent to the theory with N=1 !

$$\delta N : \frac{1}{2}(R + K_{ij}K^{ij} - K^2) - 2\lambda - V - \frac{3\alpha}{2}E^2 + \frac{\alpha}{2}[\dot{E} - D_i(N^i E)] = 0$$

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Theoretical structure

$$\Leftrightarrow \qquad \frac{\mathcal{L}}{\sqrt{\gamma}} = \frac{1}{2}(R + E_{ij}E^{ij} - E^2) - V(t) + \frac{\alpha}{2}E^2$$

The theory with N=1 gives the same dynamics!

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can determine
the dynamics of spacetime.

Cosmological solution

Background metric: $N=1,\ N_i=0,\ \gamma_{ij}=a^2(t)\delta_{ij}$

Evolution equation: $3H^2+2\dot{H}=\dfrac{2}{2-3\alpha}V(t)$, $H:=\dfrac{\dot{a}}{a}$

Given $V=V(t)$, one can integrate evolution eq.
to obtain $H=H(t)$.

integration constant

For $V=\Lambda=\mathrm{const.}$, $\dfrac{3(2-3\alpha)}{2}H^2=\boxed{\dfrac{C}{a^3}}+\Lambda$

earlier work: Mukohyama (2010)

Cosmological perturbations

Perturbed metric $N_i = \partial_i \chi, \;\; \gamma_{ij} = a^2 e^{2\zeta} \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$

Tensor part $$S_h^{(2)} = \frac{1}{8} \int d^4x \, a^3 \left[\dot{h}_{ij}^2 - \frac{(\partial_k h_{ij})^2}{a^2} \right]$$

Scalar part $$S_\zeta^{(2)} = \int d^4x a^3 \left[\left(\frac{3\alpha - 2}{\alpha} \right) \dot{\zeta}^2 - (-1) \frac{(\partial \zeta)^2}{a^2} \right]$$

$\frac{3\alpha - 2}{\alpha} > 0$: no ghost (for proper α)

$-1 < 0$: gradient instabilities

Unstable !

Ramazanov et al.(2016)
Ijjas et al.(2016)

Curing mimetic theory

The origin of gradient instabilities $+\dfrac{(\partial\zeta)^2}{a^2} \subset \sqrt{\gamma}R$

In usual theory, this wrong sign is flipped when one removes the perturbation of the lapse function by using the constraint equations.

$\Rightarrow$ **This flip cannot happen in mimetic theory because the lapse function is fixed to unity.**

Instead of imposing extra symmetry or field as the case of HL gravity, we introduce new couplings between extrinsic and intrinsic curvature.

$\Rightarrow$ general theory with mimetic constraint without instabilities

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$\Rightarrow$ **general theory with mimetic constraint without instabilities**

Building block formulation

Gleyzes et al. (2013), Tsujikawa (2014)

Essentially building block ($N=1,\ \phi=t$)

$$S=\int d^4x\sqrt{\gamma}L(E,\mathcal{S},R,\mathcal{Y},\mathcal{Z};t)$$

$$E=K/N$$

$\mathcal{S}:=E_{ij}E^{ij}$, $\mathcal{Y}:=R_{ij}E^{ij}$

$$\mathcal{Z}:=R_{ij}R^{ij}$$

Expansion around flat FR sp. ($E_i^j=H\delta_i^j+\delta E_i^j$)

$$L=L_0+\mathcal{F}\delta E+(L_R+HL_{\mathcal{Y}})R+L_{\mathcal{S}}\delta E_{ij}\delta E^i+\frac{\mathcal{A}}{2}\delta E^2$$
$$+\left(\mathcal{C}-\frac{L_{\mathcal{Y}}}{2}\right)R\delta E+L_{\mathcal{Z}}R_{ij}R^{ij}+\frac{\mathcal{G}}{2}R^2+\cdots$$

$L_0:=L(3H,3H^2,0,0,0;t)$, $L_E:=\frac{\partial L}{\partial E},\ L_{EE}:=\frac{\partial^2 L}{\partial E^2},\ \cdots$

$$\mathcal{F},\ \mathcal{A},\ \mathcal{G},\ \mathcal{C}\ \supset\ H,L_E,L_R,\cdots,L_{EE},L_{ER},\cdots$$

Tensor perturbation

Perturbed metric

$$N_i = \partial_i \chi, \;\; \gamma_{ij} = a^2 e^{2\zeta} \left(\delta_{ij} + h_{ij} + \frac{1}{2} h_{ik} h_{kj} \right)$$

Tensor sector

$$S_h^{(2)} = \frac{1}{4} \int d^4 x a^3 \left[L_{\mathcal{S}} \dot{h}_{ij}^2 - \frac{\mathcal{E}}{a^2} (\partial_k h_{ij})^2 + \frac{L_{\mathcal{Z}}}{a^4} (\partial^2 h_{ij})^2 \right]$$

Stability conditions

$$L_{\mathcal{S}} > 0, \;\; \mathcal{E} := L_R + \frac{1}{2a^3} \frac{d}{dt} (a^3 L_{\mathcal{Y}}) \geq 0, L_{\mathcal{Z}} \leq 0$$

Scalar perturbation

Expanding action to second order

$$\frac{\mathcal{L}^{(2)}}{a^3} = 2\left\{\mathcal{E} - \frac{3}{a}\frac{d}{dt}(a\mathcal{C})\right\}\frac{(\partial\zeta)^2}{a^2} + \left(\frac{9}{2}\mathcal{A} + 3L_{\mathcal{S}}\right)\dot{\zeta}^2 + 2(3L_{\mathcal{Z}} + 4\mathcal{G})\left(\frac{\partial^2\zeta}{a^2}\right)^2$$
$$+\frac{1}{2}(\mathcal{A} + 2L_{\mathcal{S}})\left(\frac{\partial^2\chi}{a^2}\right)^2 - (3\mathcal{A} + 2L_{\mathcal{S}})\dot{\zeta}\frac{\partial^2\chi}{a^2} + 4\mathcal{C}\frac{\partial^2\chi\partial^2\zeta}{a^4}$$

constraint equation ($\delta\chi$): $\dfrac{\partial^2\chi}{a^2} = \left(\dfrac{3\mathcal{A} + 2L_{\mathcal{S}}}{\mathcal{A} + 2L_{\mathcal{S}}}\right)\dot{\zeta} - \dfrac{4\mathcal{C}}{\mathcal{A} + 2L_{\mathcal{S}}}\dfrac{\partial^2\zeta}{a^2}$

(assumption: $\mathcal{A} + 2L_{\mathcal{S}} \neq 0$)

spacial case: $\mathcal{A} + 2L_{\mathcal{S}} = 0 \;\Rightarrow\; (3\mathcal{A} + 2L_{\mathcal{S}})\dot{\zeta} = 4\mathcal{C}\dfrac{\partial^2\zeta}{a^2}$

One always have $\zeta = \text{const.}$

(mimetic Horndeski, original mimetic gravity)

$\mathcal{A} \supset L_{EE}$

Scalar perturbation

Quadratic action

$$S_\zeta^{(2)} = \int d^4x a^3 \left[q_1 \dot{\zeta}^2 - q_2 \frac{(\partial\zeta)^2}{a^2} - q_3 \frac{(\partial^2\zeta)^2}{a^4} \right]$$

Stability conditions

One need $\mathcal{C} \neq 0$

$\mathcal{C} \supset L_{ER},\ L_{\mathcal{Y}}$

$$q_1 = 2\left(\frac{3\mathcal{A}+2L_{\mathcal{S}}}{\mathcal{A}+2L_{\mathcal{S}}}\right)L_{\mathcal{S}} > 0$$

$$q_2 = -2\mathcal{E} + \frac{8}{a}\frac{d}{dt}\left(\frac{a\mathcal{C}L_{\mathcal{S}}}{\mathcal{A}+2L_{\mathcal{S}}}\right) \geq 0$$

$\mathcal{E} \geq 0$ from the stability of **tensor part**

$$q_3 = -2(3L_{\mathcal{Z}} + 4\mathcal{G}) + \frac{8\mathcal{C}^2}{\mathcal{A}+2L_{\mathcal{S}}} \geq 0$$

$\mathcal{G} \supset L_{RR}$

Healthy example

Simple extension of IDM

$$L = \frac{1}{2}(R + \mathcal{S} - E^2) + \frac{\alpha}{2}E^2 + \beta(t)RE + \frac{\beta^2}{2\alpha}R^2$$

Background evolution (mimetic matter dominance)

$$\delta a : \ \ 3H^2 + 2\dot{H} = 0$$

integration : $3H^2 = \dfrac{C}{a^3} \quad \Leftrightarrow \quad Ht = 2/3$ (=MD era)

same equations as the case of IDM

Stability conditions

Simple extension of IDM

$$L = \frac{1}{2}(R + \mathcal{S} - E^2) + \frac{\alpha}{2}E^2 + \beta(t)RE + \frac{\beta^2}{2\alpha}R^2$$

- **tensor sector** $L_{\mathcal{S}} = 1/2$, $\mathcal{E} = 1/2 + 3\beta H$, $L_{\mathcal{Z}} = 0$
- **scalar sector** $\mathcal{A} + 2L_{\mathcal{S}} = \alpha$ ⇒ dynamical

  $\mathcal{C} = \beta$

Stability conditions

Simple extension of IDM

$$L = \frac{1}{2}(R + \mathcal{S} - E^2) + \frac{\alpha}{2}E^2 + \beta(t)RE + \frac{\beta^2}{2\alpha}R^2$$

special combination

- **tensor sector** $\quad L_{\mathcal{S}} = 1/2\ ,\ \mathcal{E} = 1/2 + 3\beta H\ ,\ L_{\mathcal{Z}} = 0$
- **scalar sector** $\quad \mathcal{A} + 2L_{\mathcal{S}} = \alpha \quad \Rightarrow$ dynamical

$\mathcal{C} = \beta$

$$\Rightarrow \quad q_1 = \frac{3\alpha - 2}{\alpha}\ ,\ q_2 = \frac{4\dot{\beta}}{\alpha} - (1 + 2q_1\beta H)\ ,\ q_3 = 0$$

Healthy example

To be more specific

$$\alpha = -\epsilon \;\; , \beta = -\xi\epsilon t \qquad 0 < \epsilon \ll 1 \, , \xi = o(1)$$

- **tensor sector**

$$L_{\mathcal{S}} = 1/2 \, , \mathcal{E} = 1/2 - 2\xi\epsilon > 0 \quad \Rightarrow \quad c_h = \sqrt{\frac{\mathcal{E}}{L_{\mathcal{S}}}} = 1 - o(\epsilon)$$

Gravitational Cherenkov radiation:
(the origin of high energy cosmic ray)

$$1 - c_h < 2 \times 10^{-15}$$

Moore et al. (2001)

- **scalar sector**

$$q_1 = \frac{2 + 3\epsilon}{\epsilon} > 0 \, , q_2 = \frac{20}{3}\xi - 1 + 4\xi\epsilon > 0 \quad \Rightarrow \quad c_s = \sqrt{\frac{q_2}{q_1}} = o(\epsilon^{1/2}) \ll 1$$

- **covariantization**

$$\mathcal{Q} = \mathcal{R} + 2\mathcal{R}_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi - [(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{\mathcal{R}}{2} - \lambda(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 1) - \frac{\alpha}{2}\left[\Box\phi - \frac{\beta(\phi)}{\alpha}\mathcal{Q}\right]^2$$

Summary

- **The cause of gradient instabilities**

  - no flip of the wrong sign in mimetic theory

- **Effective theory of cosmological perturbations in unitary gauge on a general theories with mimetic constraint**

  - stability conditions, new coupling $ER,\ R^2$

- **A healthy imperfect dark matter**

  - stable in every stage of universe and $c_h \sim 1,\ c_s \ll 1$

future works: Ostrogradski ghost, solar system test,
caustic singularity, generation,…