COSMO 17, Paris, France

Generic instabilities of non-singular cosmologies in Horndeski theory:
A no-go theorem

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Based on

TK, PRD94 (2016) 04351 [arXiv:1606.05831]

Introduction

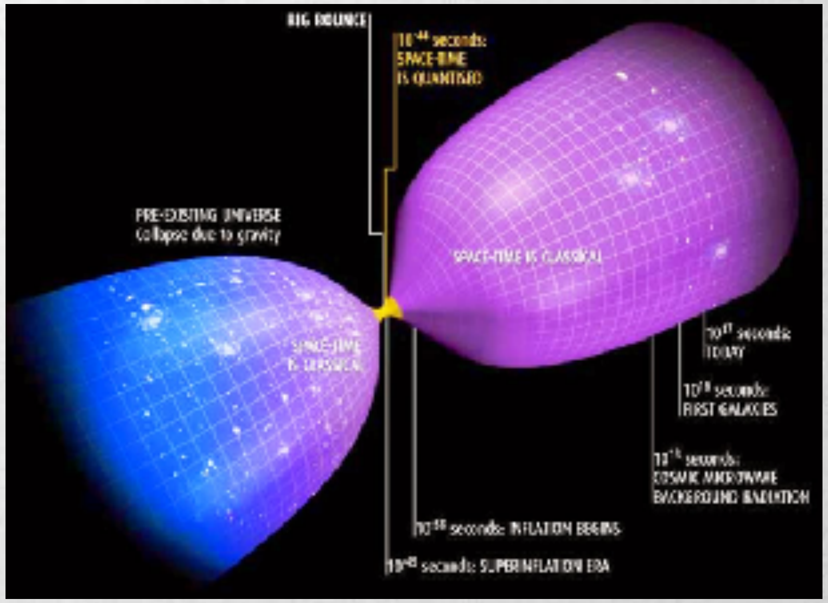
- Inflation is an almost perfect scenario, but
  - Initial singularity — Borde & Vilenkin gr-qc/9612036
  - Trans-Planckian problem — Martin & Brandenberger hep-th/0005209

- Non-singular cosmologies: bounce, Galilean Genesis, …
  - ~~Null energy condition~~ (NEC)

$$\dot{H} = -4\pi G\left(\rho + p\right) > 0$$

  - Stability?

Battefeld & Peter 1406.2790
Brandenberger & Peter 1603.05834



~~NEC~~ and stability

- Quadratic Lagrangian for curvature perturbation

$$\mathcal{L} = a^3 \left[\mathcal{G}_S(t)\dot{\zeta}^2 - a^{-2}\mathcal{F}_S(t)(\vec{\nabla}\zeta)^2 \right]$$

Ghost/gradient instabilities if $\mathcal{G}_S < 0 \,/\, \mathcal{F}_S < 0$

- $P(\phi, X)$ cosmologies are <u>unstable</u> if NEC is violated:

$X := -\frac{1}{2}(\partial\phi)^2$

$$\mathcal{F}_S = (8\pi G)^{-1}(-\dot{H})/H^2$$

- Galileon-type 2nd-order theories,

$$\mathcal{L} = \frac{R}{2\kappa} + G_2(\phi, X) - G_3(\phi, X)\Box\phi + \cdots$$

<u>admit stable NEC-violating solutions</u>, because $\mathcal{F}_S$ and $\mathcal{G}_S$ are not correlated with the sign of $\dot{H}$

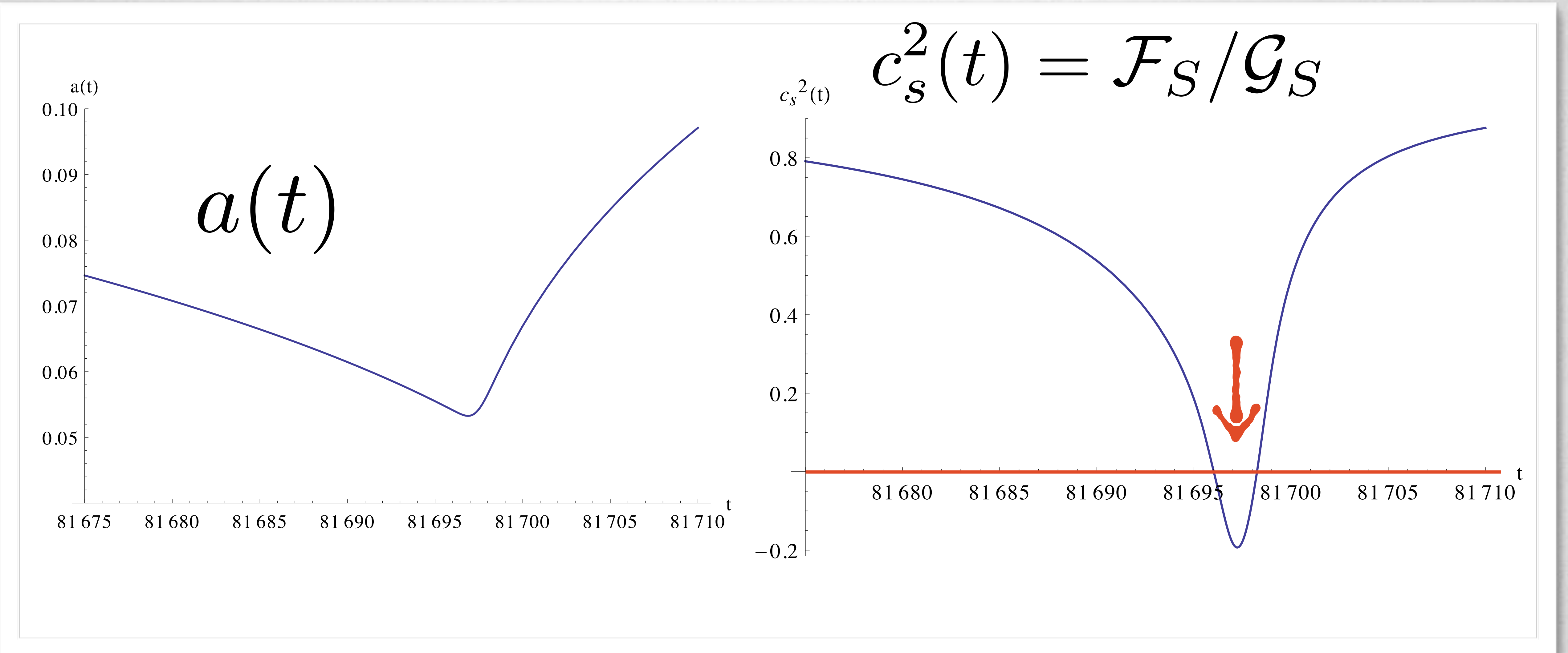
Stable non-singular cosmologies?

- Stable NEC-violating phases are possible
- But, known examples exhibit instability at some moment in the entire history
- Typically,
  - gradient instability at the bounce point
  - at the transition from NEC-violating phase to another
  - or at some moment after the bounce

Creminelli, *et al.* 1007.0027; Qui, *et al.* 1108.0593; Easson, *et al.* 1109.1047; Osipov & Rubakov 1303.1221; Cai, *et al.* 1206.2382; Koehn, *et al.* 1310.7577; Pirtskhalava, *et al.* 1410.0882; TK, Yamaguchi, Yokoyama 1504.05710; ……

Example 1 (Bounce)

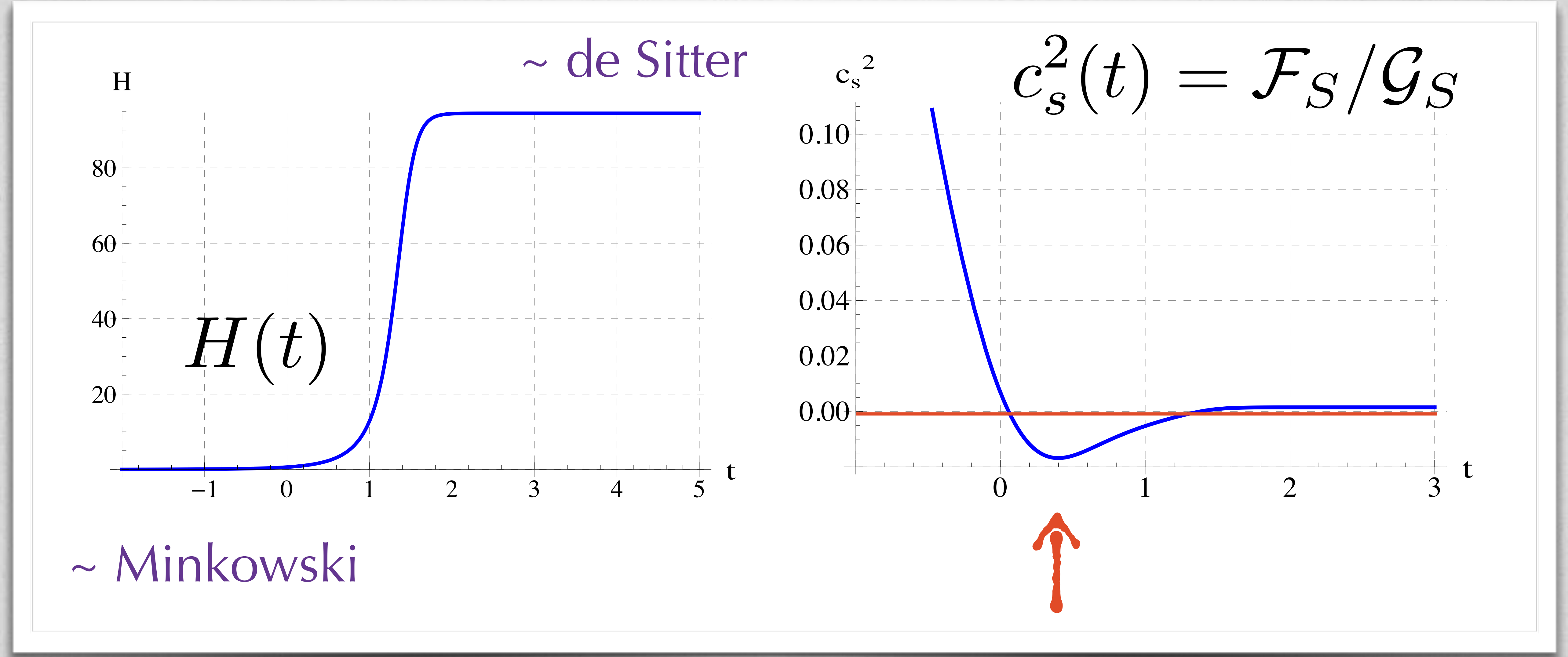
Koehn, Lehners, Ovrut 1310.7577



Gradient instability at the bounce

Example 2 (Galilean Genesis)

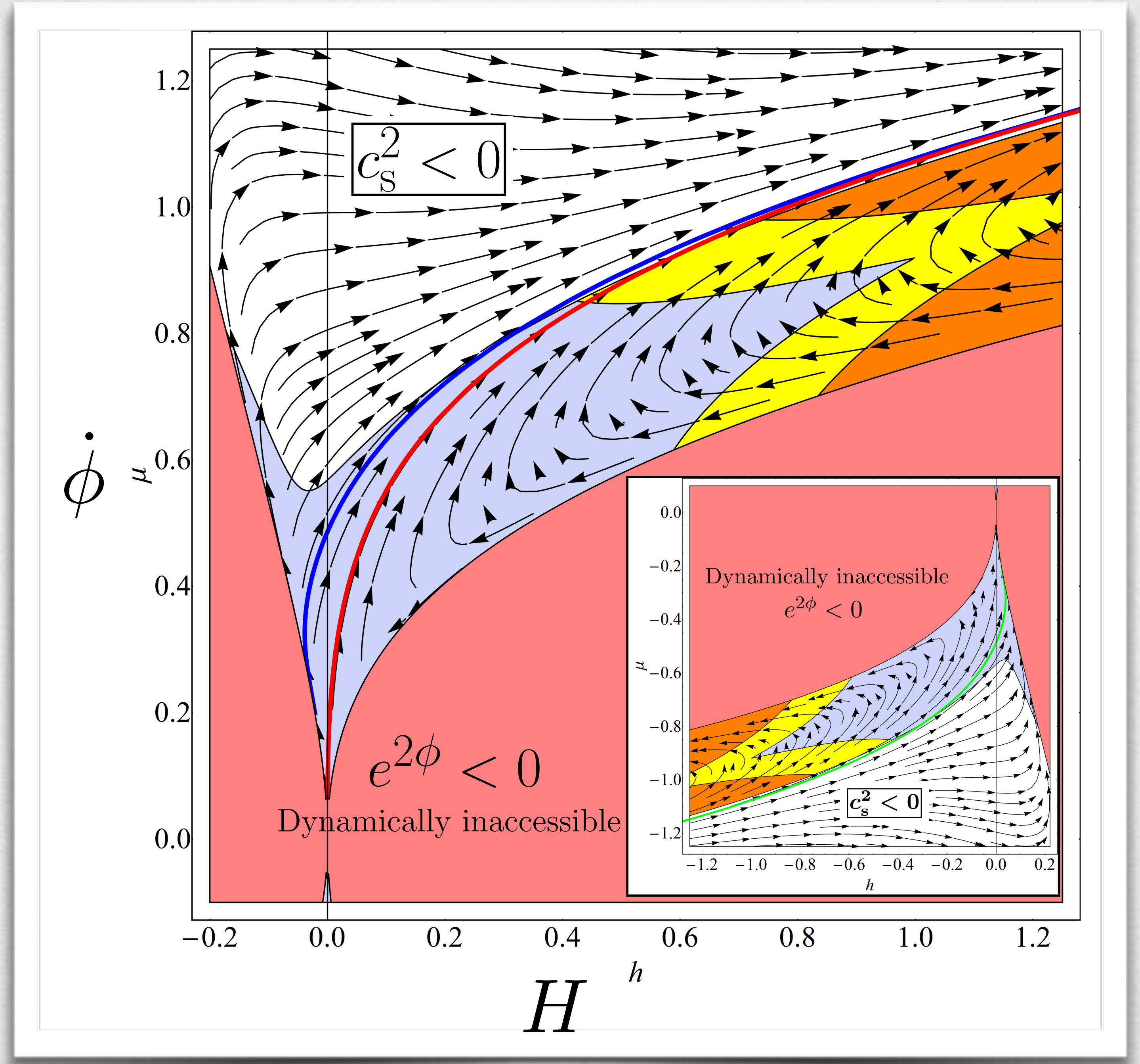
Pirtskhalava, *et al.* 1410.0882
TK, Yamaguchi, Yokoyama 1504.05710



Gradient instability at the transition from Galilean Genesis to inflation

Example 3 (Bounce)

Easson, Sawicki & Vikman 1109.1047



Stable bounce, but eventually $c_s^2 < 0$

Question to be addressed

- **Is the appearance of gradient instabilities generic or model-dependent nature?**

- A partial answer was given by Libanov, Mironov & Rubakov 1605.05992
  - All non-singular cosmological solutions in

$$\mathcal{L} = \frac{R}{2\kappa} + G_2(\phi, X) - G_3(\phi, X)\Box\phi$$

are plagued with gradient instabilities

- This talk: *The no-go theorem can be extended to the most general 2nd-order scalar-tensor theory (Horndeski)*

See Shingo Akama's talk for the case of <u>multiple</u> scalar fields

Horndeski theory

$$\begin{aligned}\mathcal{L} &= G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R \\ &\quad + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi, X)(\cdots)\end{aligned}$$

Horndeski (1974); Deffayet, *et al.* 1103.3260;
TK, Yamaguchi, Yokoyama 1105.5723

- The most general scalar-tensor theory with 2nd-order field equations
    - Einstein + canonical scalar, Brans-Dicke, *f*(*R*), k-essence, covariant Galileons, dilatonic Gauss-Bonnet, … as specific cases

Perturbations in Horndeski

TK, Yamaguchi, Yokoyama 1105.5723

- Tensor perturbations

$$\mathcal{L} = \frac{a^3}{8}\left[\mathcal{G}_T(t)\dot{h}_{ij}^2 - a^{-2}\mathcal{F}_T(t)(\vec{\nabla}h_{ij})^2\right]$$

Stability

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5,X} + G_{5,\phi}\right)\right] > 0$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4,X} - X\left(H\dot{\phi}G_{5,X} - G_{5,\phi}\right)\right] > 0$$

- Curvature perturbation

$$\mathcal{L} = a^3\left[\mathcal{G}_S(t)\dot{\zeta}^2 - a^{-2}\mathcal{F}_S(t)(\vec{\nabla}\zeta)^2\right]$$

Stability

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a\mathcal{G}_T^2}{\Theta}\right) - \mathcal{F}_T > 0, \ \mathcal{G}_S := \cdots > 0$$

with $\Theta := -\dot{\phi}XG_{3,X} + 2HG_4 - 8HXG_{4,X}\cdots$

Proof of no-go

TK 1606.05831

- Consider a non-singular, spatially flat FLRW solution,

$$a(t) > 0, \quad H, \dot{H}, \cdots < \infty \quad (-\infty < t < \infty)$$

- Stability

(Scalar stability) (Tensor stability)

$$\mathcal{F}_S = \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\underbrace{\left(\frac{a\mathcal{G}_T^2}{\Theta}\right)}_{=:\,\xi} - \mathcal{F}_T > 0 \quad \Rightarrow \quad \boxed{\frac{\mathrm{d}\xi}{\mathrm{d}t} > a\mathcal{F}_T > 0}$$

$$(-\infty < t < \infty)$$

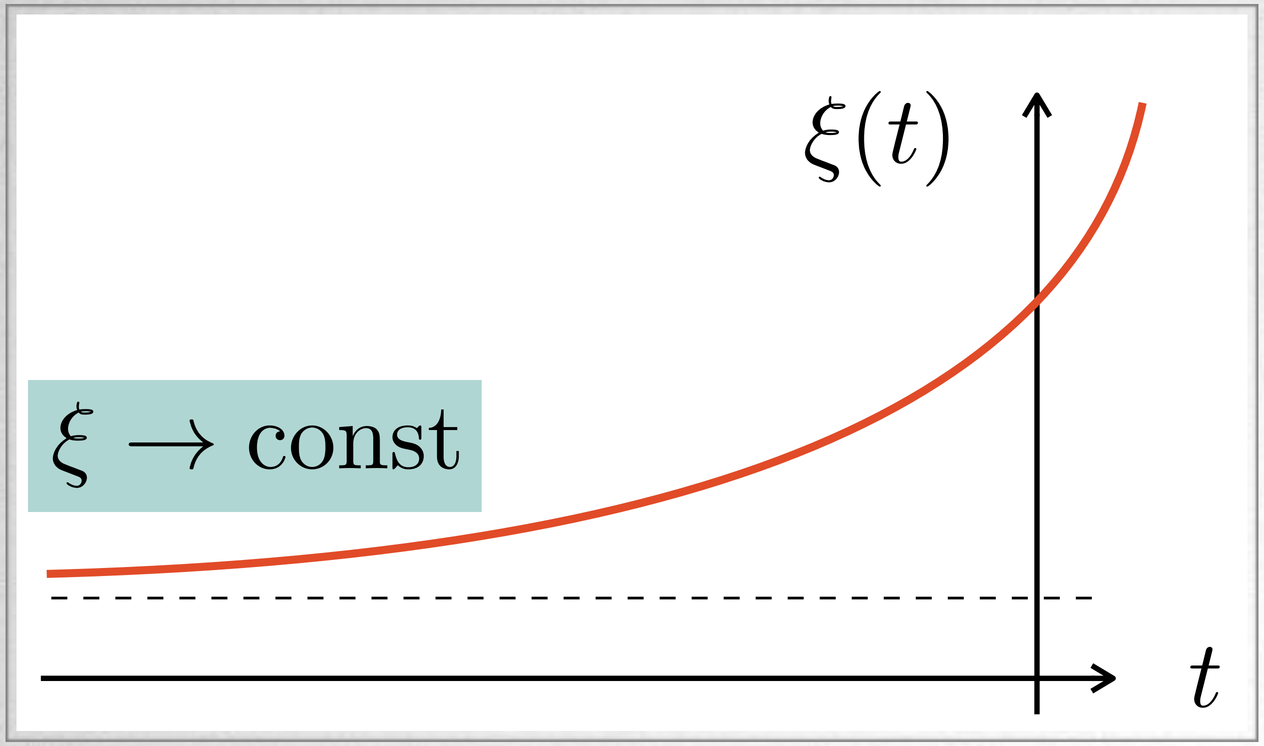
- $$\xi = \frac{a\mathcal{G}_T^2}{-\dot{\phi}XG_{3,X} + 2HG_4 + \cdots}$$ never vanishes

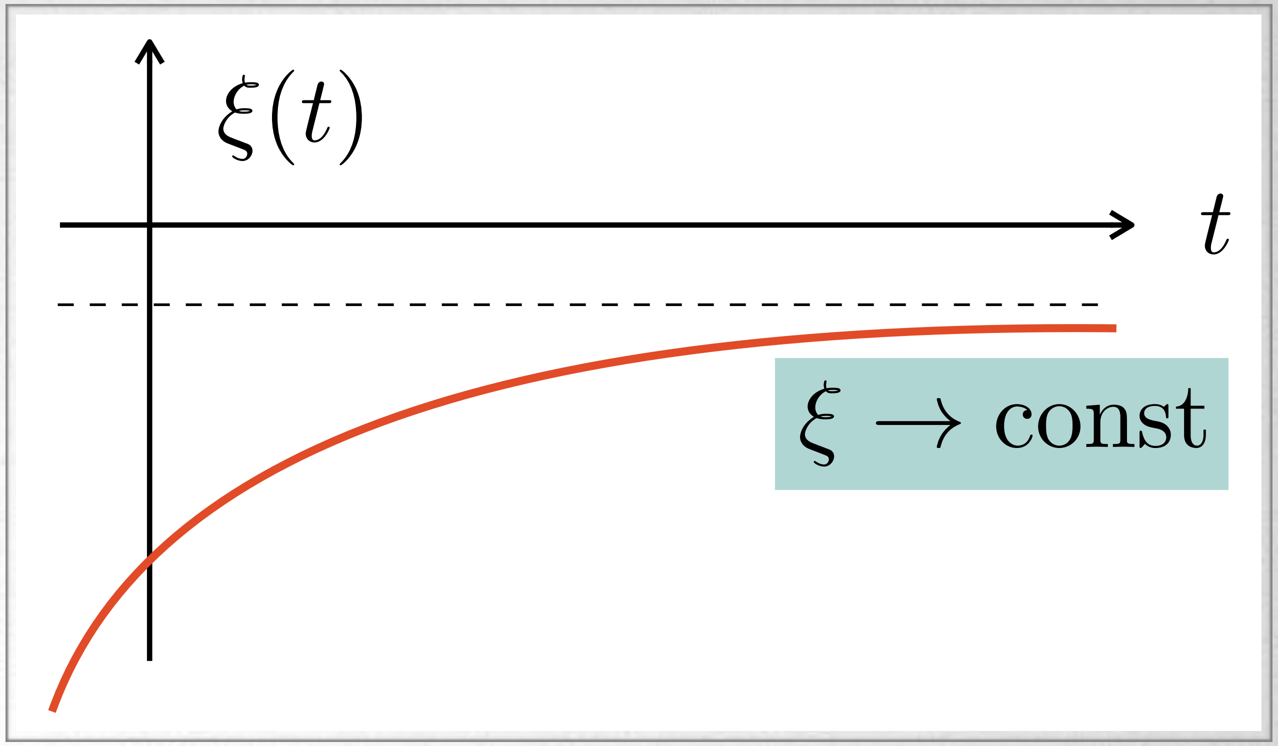
(> 0, Tensor stability)

in a stable non-singular universe (< ∞, non-singular)

- $\xi(t)$ satisfying these requirements must be …

$\xi(t)$: monotonically increasing function that never crosses zero





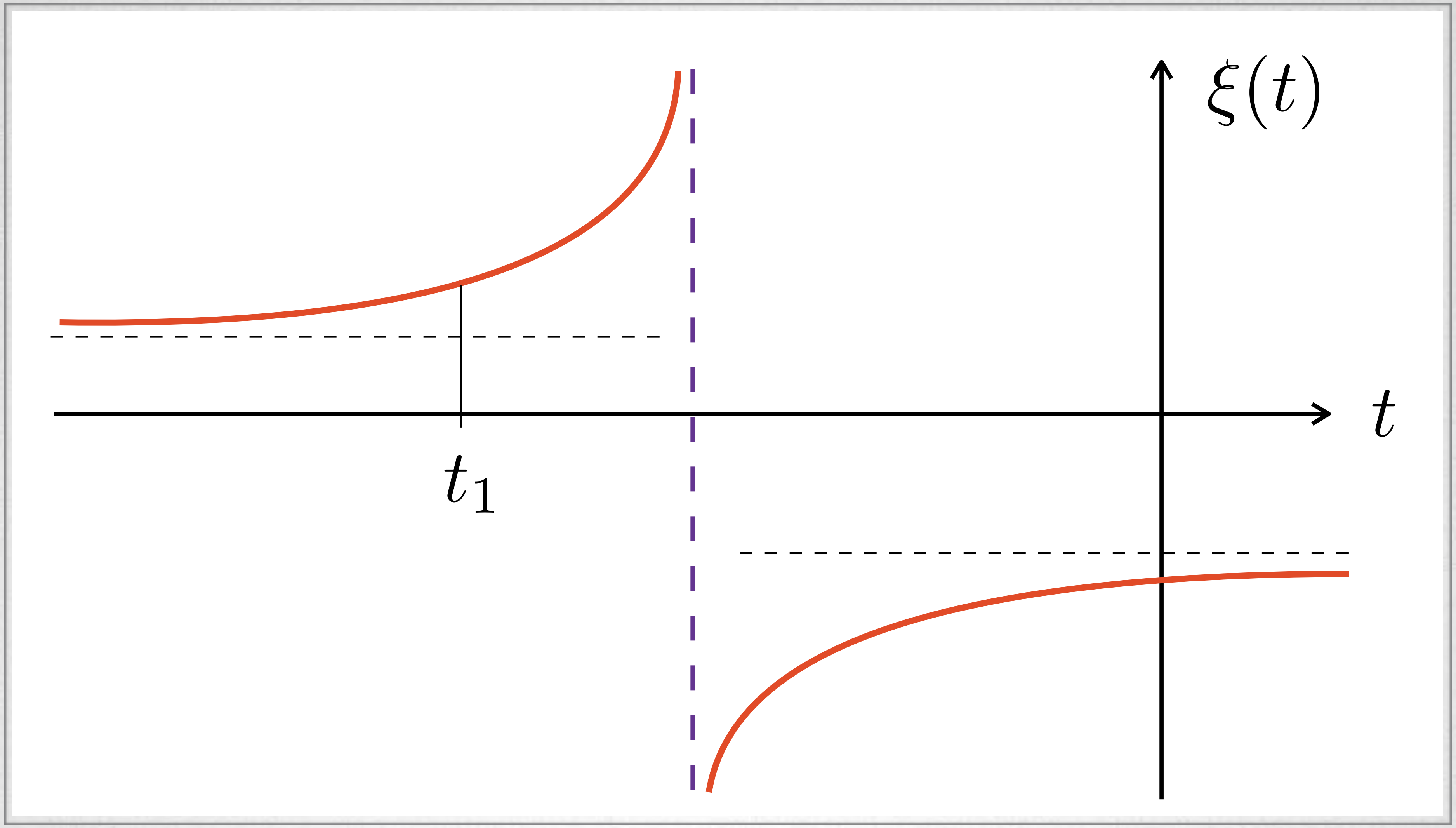
Stability

$$0 < a\mathcal{F}_T < \frac{\mathrm{d}\xi}{\mathrm{d}t} \quad \Rightarrow \quad 0 < \int_{-\infty}^{t} a\mathcal{F}_T \mathrm{d}t' < \underbrace{\xi(t) - \xi(-\infty)}_{< \infty}$$

Non-singular cosmological solutions are stable at any moment in the entire history only if

$$\int_{-\infty}^{t} a\mathcal{F}_T \mathrm{d}t' \quad \text{or} \quad \int_{t}^{\infty} a\mathcal{F}_T \mathrm{d}t'$$ is convergent

Discontinuity in ξ does not change the conclusion



$$\xi = \frac{a\mathcal{G}_T^2}{\boxed{-\dot{\phi}XG_{3,X} + 2HG_4 + \cdots}} \rightarrow = 0 \quad \text{here}$$

$$\int_{-\infty}^{t_1} a\mathcal{F}_T \mathrm{d}t' < \underbrace{\xi(t_1) - \xi(-\infty)}_{<\infty}$$

What does $\int_{-\infty} a\mathcal{F}_T \mathrm{d}t < \infty$ mean?

- *Disformal* transformation

$$a_{\mathrm{E}} = M_{\mathrm{Pl}}^{-1}\mathcal{F}_T^{1/4}\mathcal{G}_T^{1/4}a, \quad \mathrm{d}t_{\mathrm{E}} = M_{\mathrm{Pl}}^{-1}\mathcal{F}_T^{3/4}\mathcal{G}_T^{-1/4}\mathrm{d}t$$

- "Einstein frame" — Gravitons propagate along null geodesics

$$\mathcal{L}_{\mathrm{E}} = \frac{M_{\mathrm{Pl}}^2 a_{\mathrm{E}}^3}{8}\left[\dot{h}_{ij}^2 - a_{\mathrm{E}}^{-2}(\vec{\nabla}h_{ij})^2\right]$$

Creminelli, *et al.* 1407.8439;
Creminelli, *et al.* 1610.04207

- Past incompleteness in the Einstein frame

$$\int_{-\infty}^{t} a\mathcal{F}_T \mathrm{d}t = \underbrace{\int_{-\infty}^{t_{\mathrm{E}}} a_{\mathrm{E}}\mathrm{d}t_{\mathrm{E}}}_{\text{Affine parameter of a null geodesic}} < \infty$$

— Geodesically incomplete for the propagation of the gravitons

Summary

- All non-singular cosmological solutions are plagued with gradient instabilities at some moment in the entire expansion history in the Horndeski theory (the most general 2nd-order scalar-tensor theory) if graviton geodesics are past and future complete

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- To evade no-go, one must go ***beyond* Horndeski**

  - A new operator $R^{(3)}\delta N$ (in EFT) that is present in **GLPV**

    Cai, *et al.* 1610.03400; 1701.04330; 1705.03401;
    Creminelli, *et al.* 1610.04207;
    Kolevatov, *et al.* 1705.06626

    Gleyzes, *et al.* 1404.6495

  - Higer spatial derivatives, $\zeta\partial^4\zeta$, …

    Pirtskhalava, *et al.* 1410.0882;
    TK, Yamaguchi, Yokoyama 1504.05710

    Gao 1406.0822