Reconstruction of the Scalar Field Potential in Inflation with a Gauss-Bonet term

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Outline

Introduction

- If we consider higher order (quantum) corrections to general relativity or fundametal theory (string theory), the Gauss-Bonnet term seems to be qunite natural.

$$R^2_{GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu} + R^2$$

- It seems to be necessary to consider inflation with the Gauss-Bonnet term.
- Inflation with a Gauss-Bonnet term was studied in many literature and their predictions were found to be consistent with the observations
- In this talk, we try to reconstruct the inflaton potential and the Gauss-Bonnet coupling by using observationally favored configuratoins of the observable quantities.

Planck 2015 results

- tilt of the curvature power spectrum

$$n_s = 0.9655 \pm 0.0062 (68\%\mathrm{CL}, \mathrm{PlanckTT} + \mathrm{lowP})$$

- running of the spectral index

$$\frac{d \ln n_s}{d \ln k} = -0.0084 \pm 0.0082 (68\%\mathrm{CL}, \mathrm{PlanckTT} + \mathrm{lowP})$$

- upper bound on r

$$r_{0.002} < 0.10 \;\; (95\%\mathrm{CL}, \mathrm{PlanckTT} + \mathrm{lowP})$$

implications for selected inflationary models

- power law potentials

$$V(\phi) = \lambda M_{pl}^4 \left(\frac{\phi}{M_{pl}}\right)^n$$

- predictions

$$n_s - 1 = -\frac{2(n+2)}{4N_* + n}$$

$$r = \frac{16n}{4N_* + n}$$

- R^2 inflation:
  potential in Einstein frame through the conformal transformation

$$V(\phi) = \Lambda^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{pl}}\right)^2$$

- slow-roll predicitions

$$n_s - 1 \approx -\frac{2}{N}, \quad r \approx \frac{12}{N^2}$$

Outline

Inflation with a Gauss-Bonnet term

- action with a Gauss-Bonnet term

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)R^2_{GB}\right]$$

- flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2)$$

- equations of motion

$$\begin{aligned} H^2 &= \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V + 12\dot{\xi}H^3\right] \\ \dot{H} &= -\frac{\kappa^2}{2}\left[\dot{\phi}^2 - 4\ddot{\xi}H^2 - 8\dot{\xi}H\dot{H} + 4\dot{\xi}H^3\right] \\ \ddot{\phi} &+ 3H\dot{\phi} + V' + 12\xi' H^2(\dot{H} + H^2) = 0 \end{aligned}$$

Solw-roll approximations

- usual slow-roll conditions

$$\dot{\phi}^2 \ll V, \quad \ddot{\phi} \ll 3H\dot{\phi}$$

additional slow-roll conditions

$$4\dot{\xi}H \ll 1, \quad \ddot{\xi} \ll \dot{\xi}H$$

- slow-roll equations of motion

$$H^2 \simeq \frac{\kappa^2}{V}$$

$$\dot{H} \simeq -\frac{\kappa^2}{2}(\dot{\phi}^2 + 4\dot{\xi}H^3)$$

$$3H\dot{\phi} + V_\phi + 12\xi' H^4 \simeq 0$$

slow-roll approximations

- slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{2\kappa^2}\frac{V'}{V}Q$$

$$\eta = \frac{\ddot{H}}{H\dot{H}} = -\frac{1}{\kappa^2}\left(\frac{V''}{V'}Q + Q'\right)$$

$$\delta_1 = 4\kappa^2\dot{\xi}H = -\frac{4\kappa^2}{3}\xi' VQ$$

$$\delta_2 = \frac{\ddot{\xi}}{\dot{\xi}H} = -\frac{1}{\kappa^2}\left(\frac{\xi''}{\xi'}Q + \frac{V'}{2V}Q + Q'\right)$$

where

$$Q = \frac{V'}{V} + \frac{4}{3}\kappa^4\xi' V$$

- amount of inflationary expansions

$$N = \int_t^{t_e} H dt \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi$$

Observable quantities

$$\begin{aligned} n_s - 1 &= -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1} \\ n_t &= -2\epsilon \\ r &= 8(2\epsilon - \delta_1) \end{aligned}$$

$V(\phi) \sim \phi^n,\ \xi(\phi) \sim \phi^{-n}$

(Z. Guo, D. Schwarz, 2010)

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^{-n}$$

$$\Longrightarrow V(\phi)\xi(\phi) = \text{const.}$$

$$n_s - 1 = -\frac{2(n+2)}{4N+n},$$

$$r = \frac{16n(1-\alpha)}{4N-n}$$

where $\alpha = \frac{4}{3}V_0\xi_0$.

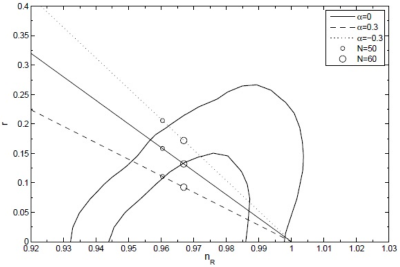


Figure: $n = 2$

$V(\phi) \sim \phi^n, \ \xi(\phi) \sim e^{-\lambda\phi}$

(P. Jiang, J. HU, Z. Guo, 2013)

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0 e^{-\lambda\phi}$$

$$\begin{aligned} n_s - 1 = \phi^{-2}[&-n(n+2) \\ &+ \alpha\lambda e^{-\lambda\phi}\phi^{n+1}(2\lambda\phi - n)] \\ r = 8\phi^{-2}(&n - \alpha\lambda e^{-\lambda\phi}\phi^{n+1}) \end{aligned}$$

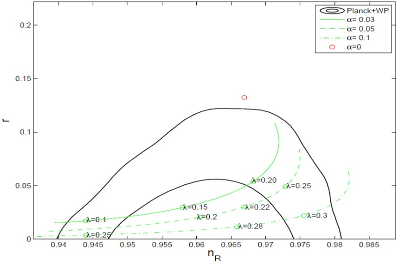


Figure: $n = 2$

$V(\phi), \xi(\phi) \sim \phi^n$

(SK, W. Lee, B. Lee, G. Tumurtushaa, 2014)

$$V(\phi) = V_0\phi^n, \quad \xi(\phi) = \xi_0\phi^n$$

$$n_s - 1 = -\frac{n+2}{2N} + \frac{n(3n+2)(2nN)^n\alpha}{2(1+n)N}$$
$$r = \frac{4n}{N} + \frac{4n(2n+1)(2nN)^n\alpha}{(1+n)N}$$

where $\alpha = \frac{4}{3}V_0\xi_0$

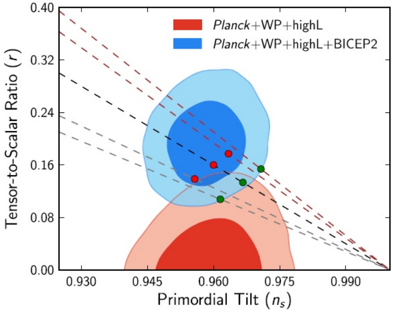


Figure: $n = 2$

Outline

Strategy

- Write the observable quantities in terms of N

$$N \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi, \quad Q = \frac{V'}{V} + \frac{4}{3}\kappa^4 \xi' V$$

$$\begin{aligned} n_s - 1 &\approx -2\epsilon - \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2 - \epsilon)}{2\epsilon - \delta_1}, \\ r &\approx 8(2\epsilon - \delta_1), \\ n_t &\approx -2\epsilon \end{aligned}$$

Strategy

- Write the observable quantities in terms of N

$$N \simeq \int_{\phi_e}^{\phi} \frac{\kappa^2}{Q} d\phi, \quad Q = \frac{V'}{V} + \frac{4}{3}\kappa^4 \xi' V$$

$$n_s(N) - 1 \approx \left[\ln \frac{Q^{(N)}}{V}\right]_N,$$

$$r(N) \approx 8Q^{(N)},$$

$$n_t(N) \approx -\frac{V_N}{V},$$

where $Q^{(N)} = \frac{V_N}{V} + \frac{4}{3}\kappa^4 \xi_N V$

Strategy

- Solving in terms of $V(N)$ and $\xi(N)$,

$$V(N) = \frac{1}{8c_1} r(N) e^{-\int [n_s(N)-1] dN},$$
$$\xi(N) = \frac{3}{4\kappa^4}\left[\frac{1}{V(N)} + \int \frac{r(N)}{8V(N)} dN + c_2\right]$$

- Finally, we obtain by replacing N with ϕ

$$V(\phi), \quad \xi(\phi)$$

Inverse Problem

General expressions from Planck 2015

$$n_s - 1 = -\frac{\beta}{N+\alpha},$$
$$r = \frac{q}{N^p + \gamma N + \alpha}$$

Inverse Problem, $\gamma = 1$

$\gamma = 1, \beta = p = 2, q = 8$

(T. Chiba, 2015)

$$V(\phi) = c_1 \tanh^2\left(\frac{1}{2}\kappa(\phi - C)\right),$$

$$\xi(\phi) = \frac{3}{4\kappa^4}(c_1 - c_2)$$

Since $\xi(\phi) \sim$ const., the Gauss-Bonnet term does not contribute to the background dynamics.

Inverse Problem, $\gamma = 0$

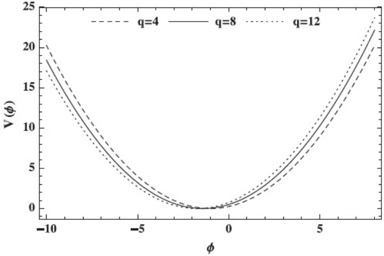
$p = 1$

$$V(\phi) = \frac{8}{8c_1}\mathcal{F}(\phi),$$

$$\xi(\phi) = \frac{3}{4\kappa^4}\left[\frac{q + 8(1-\beta)}{q(1-\beta)}\frac{1}{\mathcal{F}(\phi)}c_1 + c_2\right]$$

$$\mathcal{F}(\phi) \equiv \left(\alpha + \frac{2}{q}\kappa^2\phi^2 + \sqrt{\frac{8\alpha}{q}}\kappa\phi\right)^{\beta-1}$$

Inverse Problem, $\gamma = 0$



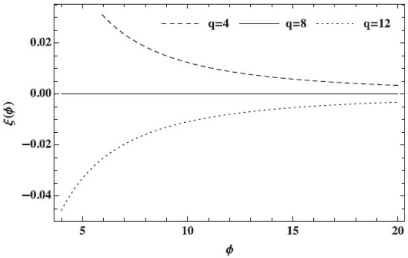


Figure: $\alpha = 1/2, \ \beta = 2, \ c_1 = 1, \ c_2 = 0$

- $V(\phi)\xi(\phi) \sim$ const
- reproduce (Z. Guo and M. Schwarz, 2010) model

Inverse Problem, $\gamma = 0$

$p = 2$

$$V(\phi) = \frac{q}{8c_1\alpha}\operatorname{sech}^2\tilde{\phi}\mathcal{F}(\tilde{\phi})^{\beta}$$

$$\xi(\phi) = \frac{3}{4\kappa^4}\left[\frac{q\mathcal{F}(\tilde{\phi}) + 8(1-\beta)\alpha\cosh^2\tilde{\phi}}{q(1-\beta)\mathcal{F}^{\beta}(\tilde{\phi})}c_1 + c_2\right]$$

where $\tilde{\phi} = \sqrt{\frac{8}{q}}\kappa\phi$ and $\mathcal{F}(\tilde{\phi}) = \alpha + \sqrt{\alpha}\sinh\tilde{\phi}$

Inverse Problem, $\gamma = 0$

$p = 2, \ \alpha \ll 1, \ c_2 = 0$

$$V(\phi) \sim \tanh^2 \tilde{\phi}$$

$$\xi(\phi) \sim -\frac{3c_1}{4\sqrt{\alpha}\kappa^4} \operatorname{csch} \tilde{\phi}$$

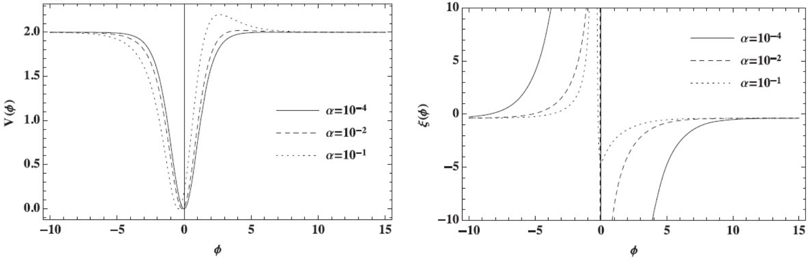
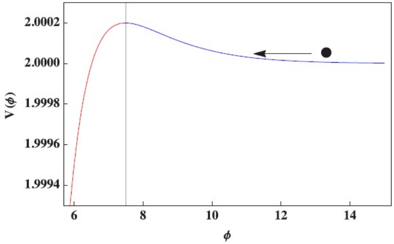


Figure: $q = 16, \ \beta = 2, \ c_1 = 1, \ c_2 = 0$

Outline

Blue tilted tensor spectrum for $\gamma = 0,\ p = 2$



- For $\gamma = 0,\ p = 2$, $\epsilon = \frac{1}{2\kappa^2}\frac{V'}{V}Q < 0$ during slow-roll expansion leads to $n_t \simeq -2\epsilon > 0$.
- This implies that null energy condtion is violated, $\rho + P < 0$.
- From $Q > 0$, $\xi' > -\frac{3}{4\kappa^4}\frac{V'}{V^2} > 0$ *i.e.* $\xi(\phi)$ has positive slope.

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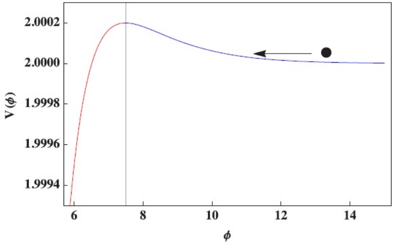

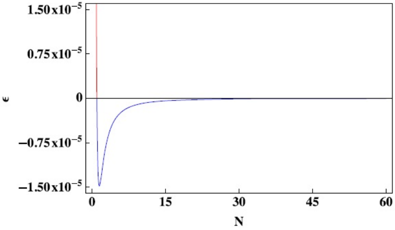

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stability for the blue tilted spectrum

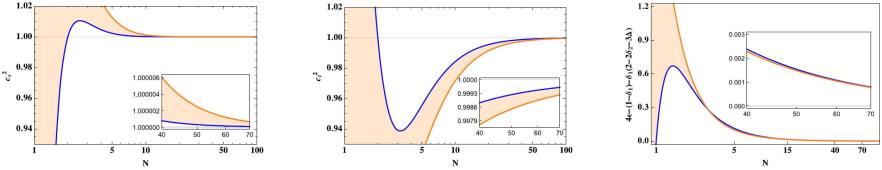
(Calcagni, de Carlos, De Felice (2006)l De Felice, Hindmarsh, Trodden (2006); Hikmawan, Soda, Suroso, Zen (2016)

- positive and non-superluminal propagation speed

$$0 \leqslant c_s^2,\ c_t^2 \leqslant 1$$

- ghost-free conditions

$$4\epsilon(1-\delta_1) - \delta_1(2 - 2\delta_2 - 3\Delta) > 0, \quad 1 - \delta_1 > 0$$



Outline

Summary

- We have considered inflationary models with a Gauss-Bonnet term to reconstruct the scalar-field potentials and the Gauss-Bonnet coupling from the observable quantities.
- Assuming a specific ansatz for $n_s(N)$ and $r(N)$ which are in good agreement with the observational data, we obatin the analytic results for $V(N)$ and $\xi(N)$.
- Especaillay, for $\gamma = 0$ and $p = 2$, the reconstructed $V(\phi)$ and $\xi(\phi)$ provide the blue tilted tensor spectrum, $n_t > 0$, by violating null energy conditions. We have checked the ghost-free and stability conditions.

Appendix: linear perturbations

- linearized metric

$$ds^2 = a^2(\tau)[-d\tau^2 + \{(1-2\mathcal{R})\delta_{ij} + h_{ij}\}dx^i dx^j]$$

- introducing canonical variables

$$v_{\mathcal{R}} = z_{\mathcal{R}}\mathcal{R}, \quad v_t = z_t h$$

- Sasaki-Mukhanov equations

$$v_A'' + \left(c_A^2 k^2 - \frac{z_A''}{z_A}\right) v_A = 0$$

where

$$c_s^2 = 1 - \frac{(4\epsilon + \delta_1(1-4\epsilon-\delta_2)^2_\Delta}{4\epsilon - 2\delta_1 - 2\delta_1(2\epsilon-\delta_2) + 3\delta_1\Delta}, \quad c_t^2 = 1 + \frac{\delta_1(1-\delta_2)}{1-\delta_1}$$

$$z_s = \sqrt{\frac{a^2}{\kappa^2}\frac{2\epsilon - \delta_1(1+2\epsilon-\delta_2) + 3\delta_1\Delta/2}{(1-\Delta/2)^2}}, \quad z_t = \sqrt{\frac{a^2}{\kappa^2}(1-\delta_1)}$$

$$\Delta = \frac{\delta_1}{1-\delta_1}$$

Appendix: power spectrum

- positive frequency mode solutions

$$v_A = \frac{\sqrt{\pi|\tau|}}{2} e^{i(\nu_A+1/2)\frac{\pi}{2}} H^{(1)}_{\nu_A}(c_A k|\tau|)$$

- power spectrum

$$\begin{aligned}
\mathcal{P}_A &= \frac{k^3}{2\pi^2}\frac{1}{z_A^2}\langle|v_A|^2\rangle \\
\mathcal{P}_s &\simeq \frac{\csc^2 \nu_s \pi}{\pi \mathcal{D}_s^2 \Gamma^2(1-\nu_s)} \frac{1}{c_s^3|\tau|^2 a^2}\left(\frac{c_s k|\tau|}{2}\right)^{3-2\nu_s} \\
\mathcal{P}_t &\simeq 8 \frac{\csc^2 \nu_t \pi}{\pi \mathcal{D}_t^2 \Gamma^2(1-\nu_t)} \frac{1}{c_t^3|\tau|^2 a^2}\left(\frac{c_t k|\tau|}{2}\right)^{3-2\nu_t}
\end{aligned}$$

where

$$\nu_s \simeq \frac{3}{2} + \epsilon + \frac{2\epsilon(2\epsilon+\eta) - \delta_1(\delta_2-\epsilon)}{4\epsilon - 2\delta_1}, \quad \nu_t \simeq \frac{3}{2} + \epsilon$$