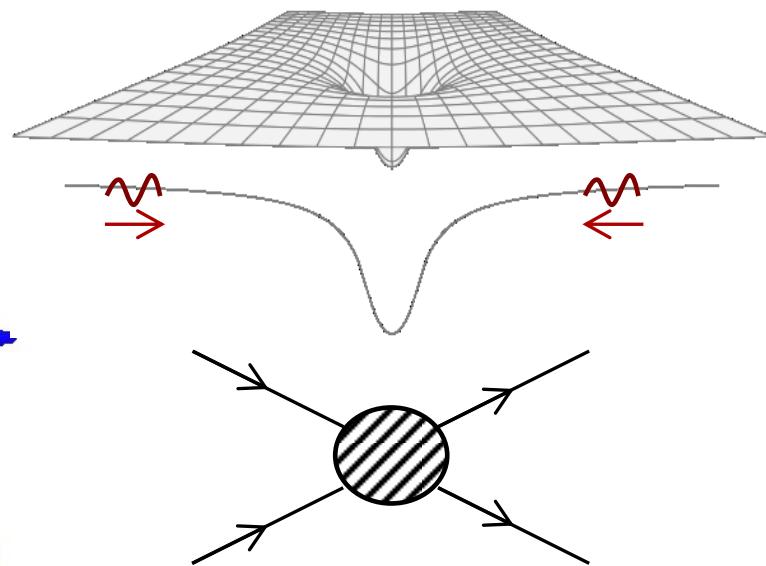
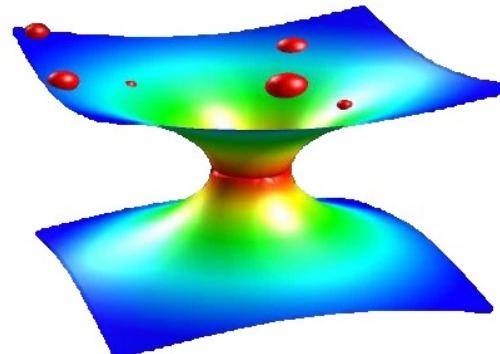


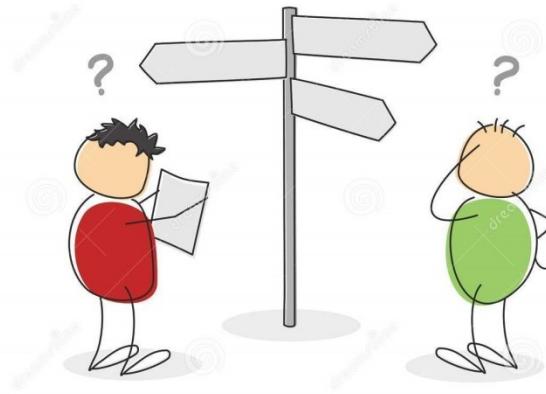
Unitary NEC Violation in Scalar Field Cosmologies

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London**



- Null Energy Condition Can we violate it...
- Unitarity Bounds ... without also violating unitarity?
- P(X) Bounce The answer is 'no'...
- Higher Derivatives ... unless we have some higher derivatives.



Barrow, PRD 48 (1993)
Nonsingular scalar-tensor cosmologies

Khoury et al., PRD 65 (2002), [hep-th/0108187]
From big crunch to big bang

N. Arkani-Hamed et al., JHEP 05(2004)074, [hep-th/0312099]
Ghost condensation and a consistent infrared modification of gravity

Koehn et al., PRD 93 (2016), [1512.03807]
Nonsingular bouncing cosmology: Consistency of the effective description

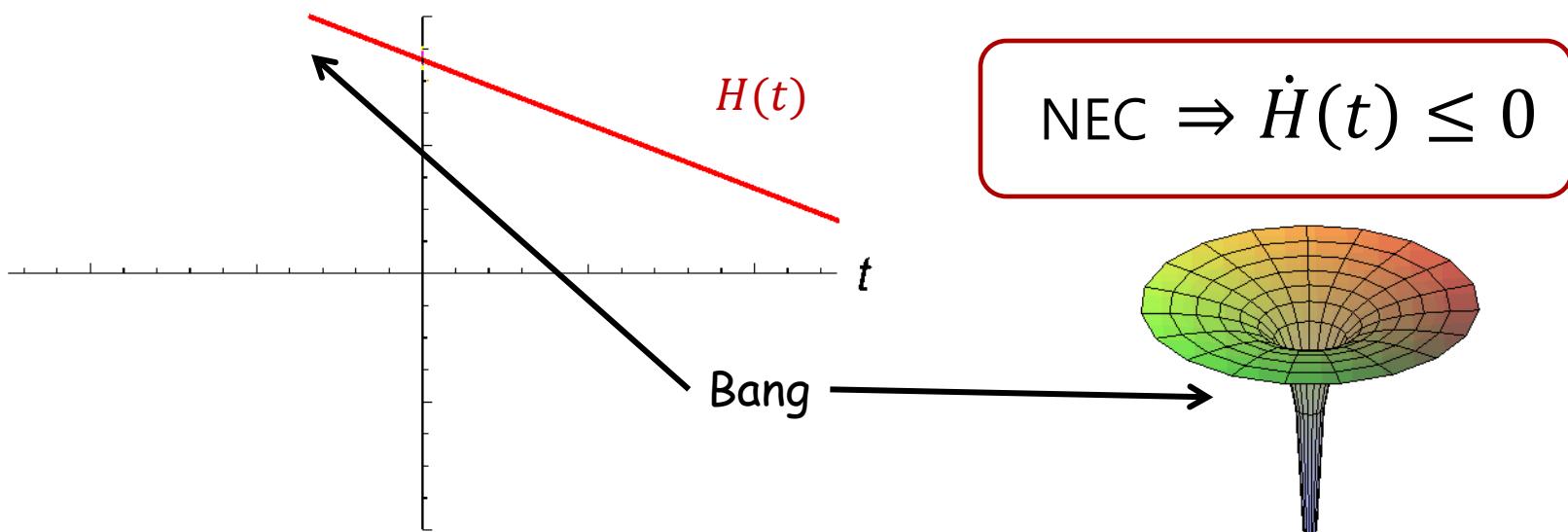
Ijjas and Steinhardt, PRL 117 (2016) 12, [1606.08880]
Classically stable nonsingular cosmological bounces

Bounces and NEC

FLRW:

$$3M_P^2 H^2 = \rho$$

$$M_P^2 \dot{H} = -\frac{1}{2} \underbrace{(\rho + P)}_{\geq 0} \text{NEC}$$

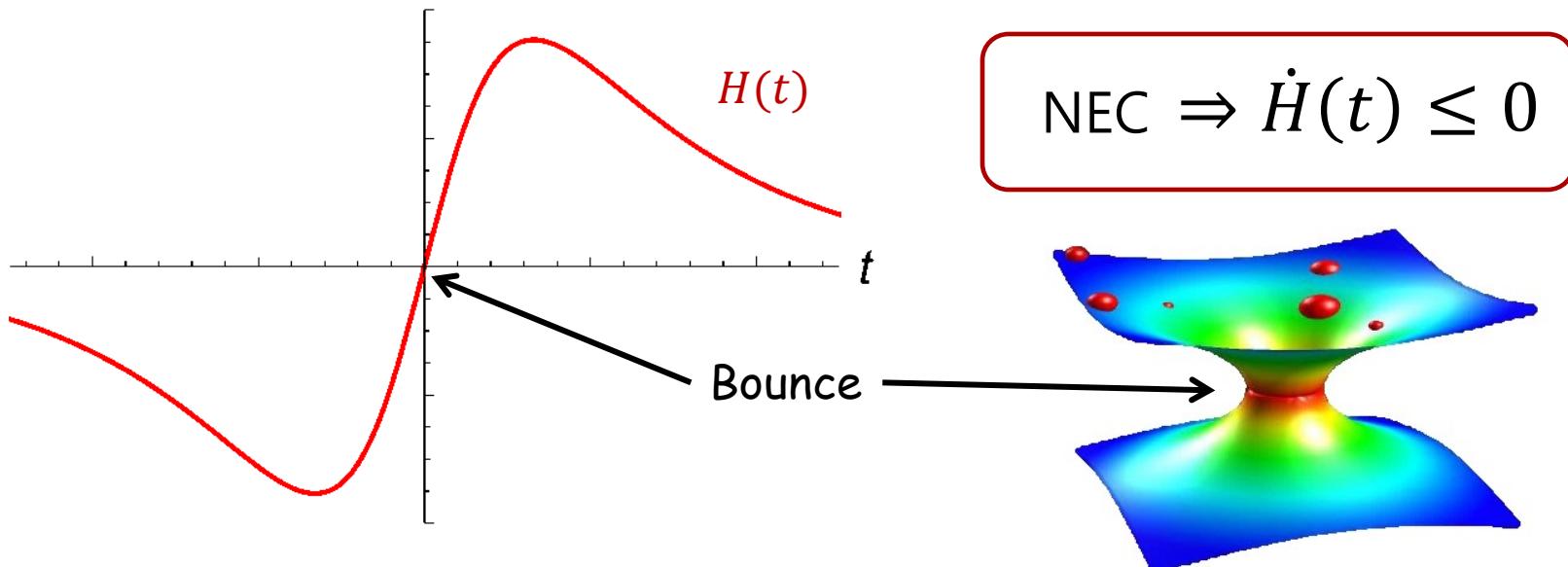


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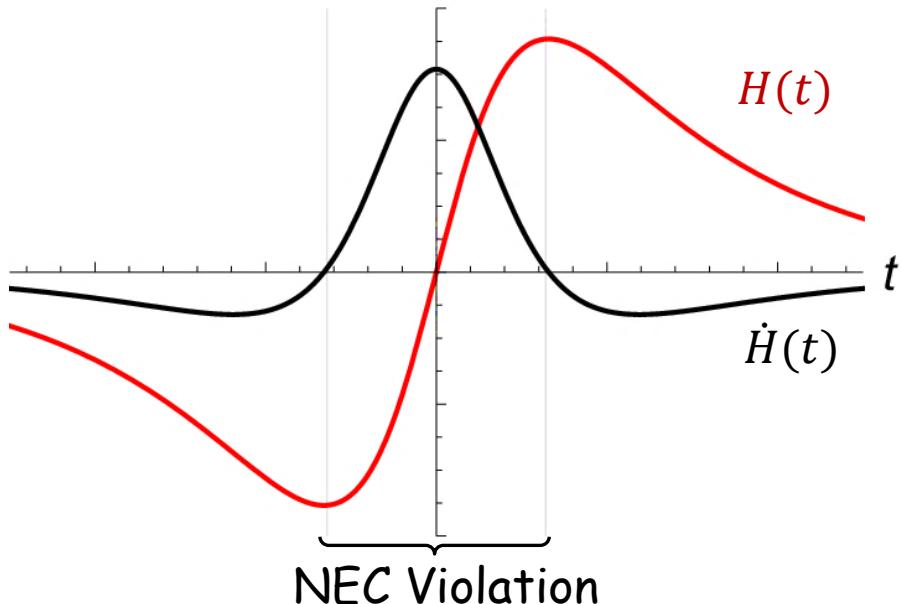


Bounces and NEC

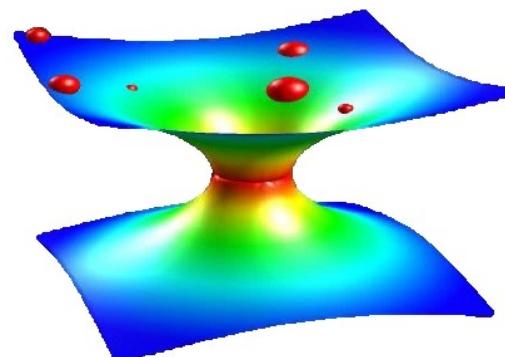
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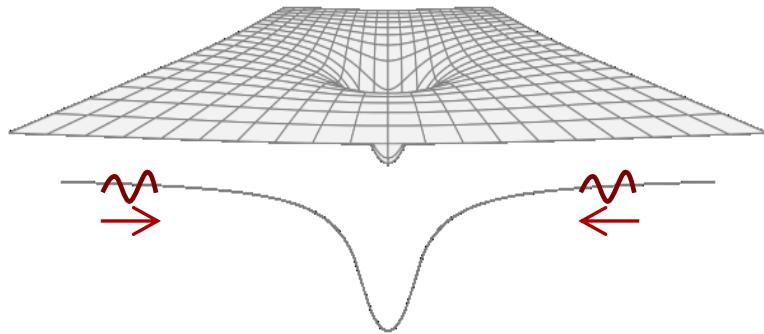


$$\text{NEC} \Rightarrow \dot{H}(t) \leq 0$$



Fluctuations

$$S[g_{\mu\nu}, \Phi] = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R(g_{\mu\nu}) + \mathcal{L}(g_{\mu\nu}, \Phi) \right)$$



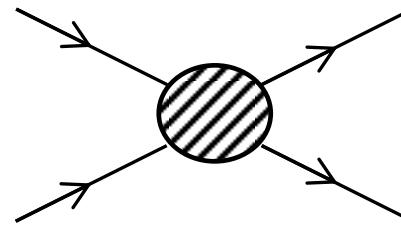
$$\begin{aligned} g_{\mu\nu} &= g_{\mu\nu}^{FLRW} + h_{\mu\nu} \\ \Phi &= \phi(t) + \varphi \end{aligned}$$

$$\mathcal{L}_{eff}[\varphi] = \frac{1}{2}\varphi(\partial_t^2 - c_s^2 \partial_i^2)\varphi + \mathcal{L}_{int}[\varphi] + \mathcal{O}\left(\frac{h_{\mu\nu}\varphi}{M_P}\right)$$

Depend on background \dot{H}

Unitarity

$$|A_{2 \rightarrow 2}| \leq 1$$



e.g. if relativistic,

$$\frac{\partial_i^4 \varphi^4}{\Lambda^4} \Rightarrow A_{2 \rightarrow 2} \sim \frac{\mathbf{k}^4}{\Lambda^4} \sim \frac{E^4}{\Lambda^4} \Rightarrow E \approx \Lambda \text{ breakdown}$$

for us, these depend on background \dot{H}

Strong Coupling Scale

$$\mathcal{L}_{eff}[\varphi] = \underbrace{\frac{1}{2}\varphi(\partial_t^2 - c_s^2 \partial_i^2)\varphi}_{\text{1}} + \mathcal{L}_{int}[\varphi]$$

1

$$dt \rightarrow d\tilde{t} = dt/c_s$$

$$\varphi \rightarrow \tilde{\varphi} = \varphi/\sqrt{c_s}$$

$$\frac{\partial_i^4 \varphi^4}{\Lambda^4}$$

$$\frac{\partial_t^M \partial_i^{2L} \varphi^N}{\Lambda^P}$$

$$\frac{1}{2}\tilde{\varphi}(\partial_{\tilde{t}}^2 - \partial_i^2)\tilde{\varphi}$$

$$\frac{\partial_i^4 \tilde{\varphi}^4}{c_s^3 \Lambda^4}$$

$$\frac{\partial_{\tilde{t}}^M \partial_i^{2L} \tilde{\varphi}^N}{\tilde{\mu}^P}$$

2 Breakdown scale

$$\tilde{E} \approx c_s^{3/4} \Lambda$$

3 $d\tilde{t} \rightarrow dt = c_s d\tilde{t}$

$$E \approx c_s^{7/4} \Lambda$$

$$\mu^{2P} = c_s^{3N+4L-6} \Lambda^{2P}$$

$$E \approx \Lambda \operatorname{Min}_{N,M,L} \left[c_s^{\frac{3N+4L-6}{2N+2M+4L-8}} \right] \text{ breakdown}$$

Bounds

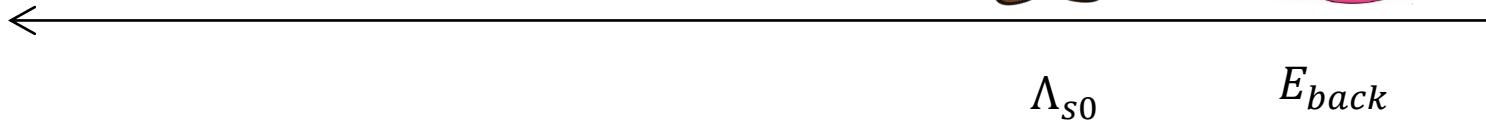
$$\mathcal{L}(g_{\mu\nu}, \Phi)$$

$$g_{\mu\nu} = g_{\mu\nu}^{FLRW} + h_{\mu\nu}$$

$$\Phi = \phi(t) + \varphi$$

Background $E_{back} = \text{Max}[H, \dot{H}, \dot{\phi}/\phi]$

Breakdown $\Lambda_{s0} \approx \Lambda \text{Min}_{N,M,L} \left[c_s^{\frac{3N+4L-6}{2N+2M+4L-8}} \right]$



Bounds

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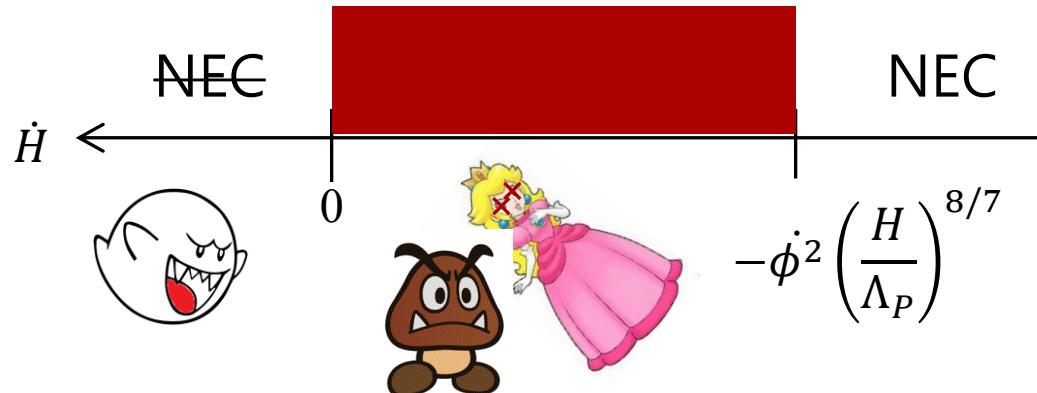
P(X) Theories

$$\mathcal{L}(g_{\mu\nu}, \Phi, \partial\Phi, \dots) = P(X), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi$$

$$\dot{H} = -\frac{1}{2}\dot{\phi}^2 c_s^2$$

$$\text{NEC} \Rightarrow c_s \rightarrow 0$$

Breakdown $\Lambda_{s0} \approx \Lambda_P c_s^{7/4} \rightarrow 0$ ($\mathcal{L} \sim P_{,XX}(\partial_i \varphi)^4 \sim \frac{(\partial_i \varphi)^4}{\Lambda_P^4}$)



Higher Derivative Corrections

$$\begin{aligned}\mathcal{L}[\Phi] &= -\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2\Lambda_c^2}(\partial^2\Phi)^2 + \dots \\ &= \frac{1}{2}\varphi(\partial_t^2 - c_s^2\partial_i^2)\varphi + \frac{1}{2\Lambda_c^2}(\ddot{\varphi} - \partial_i^2\varphi)^2 + \dots\end{aligned}$$

Relativistic ($c_s = 1$) Propagator

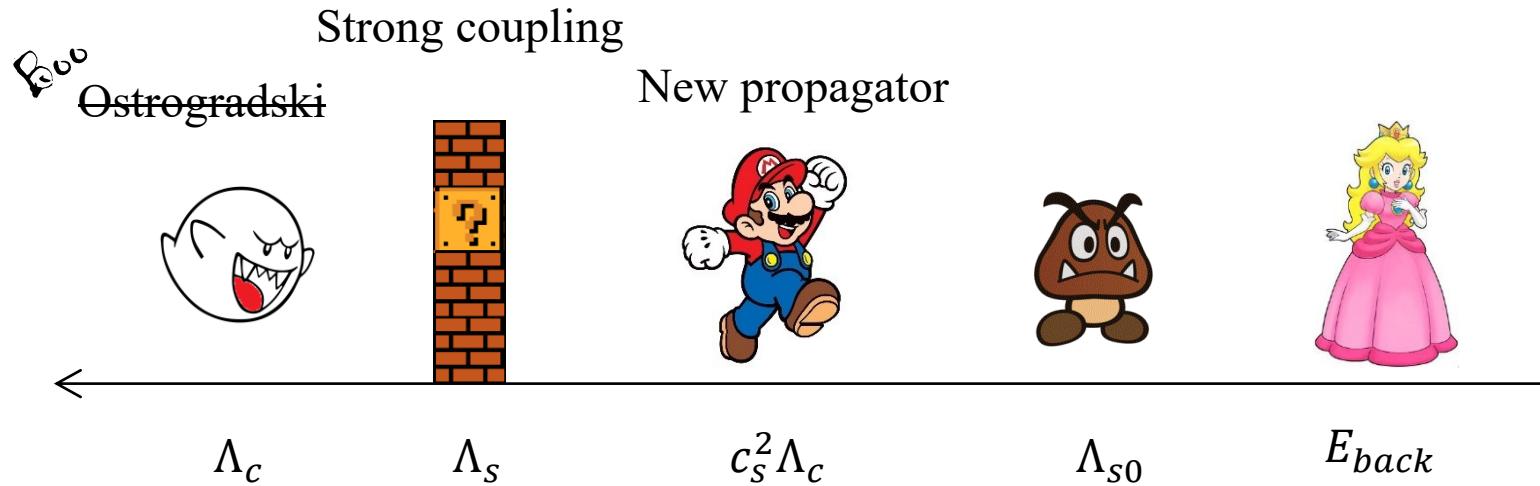
$$\begin{aligned}&= \frac{1}{-\omega^2 + k^2 - \frac{1}{\Lambda_c^2}(-\omega^2 + k^2)^2} \\ &= \frac{1}{p^2(1 - \frac{p^2}{\Lambda_c^2})} = \frac{1}{p^2} \left(1 + \frac{p^2}{\Lambda_c^2} + \dots\right)\end{aligned}$$

Non-relativistic Propagator

$$\begin{aligned}&= \frac{1}{-\omega^2 + c_s^2 k^2 - \frac{1}{\Lambda_c^2}(-\omega^2 + k^2)^2} \\ &= \frac{1}{(\omega^2 - c_s^2 k^2 + (1 - c_s^2)^2 \frac{k^4}{\Lambda_c^2} + \dots)(1 - \frac{\omega^2}{\Lambda_c^2} + \dots)}\end{aligned}$$

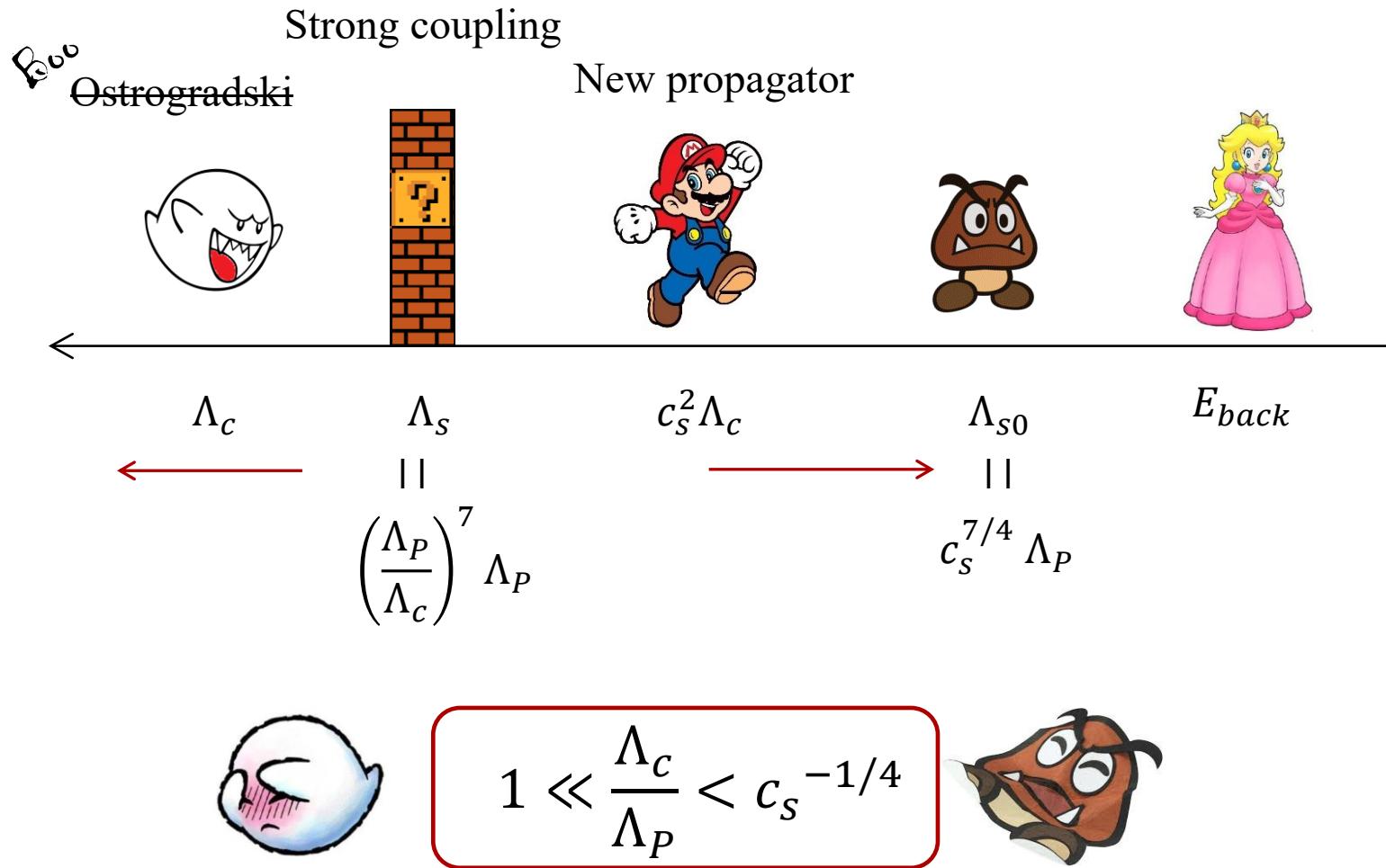
$$\omega^2 \ll \Lambda_c^2 \Rightarrow \text{Prop} = \begin{cases} \frac{1}{\omega^2 - c_s^2 k^2 + \dots}, & c_s^2 k^2 \ll c_s^4 \Lambda_c^2 \quad 0 \leq E \ll c_s^2 \Lambda_c \quad \Lambda_{s0} \\ \frac{1}{\omega^2 - \frac{k^4}{\Lambda_c^2} + \dots}, & c_s^2 k^2 \gg c_s^4 \Lambda_c^2 \quad c_s^2 \Lambda_c \ll E \ll \Lambda_c \quad \Lambda_s \end{cases}$$

Higher Derivative Corrections

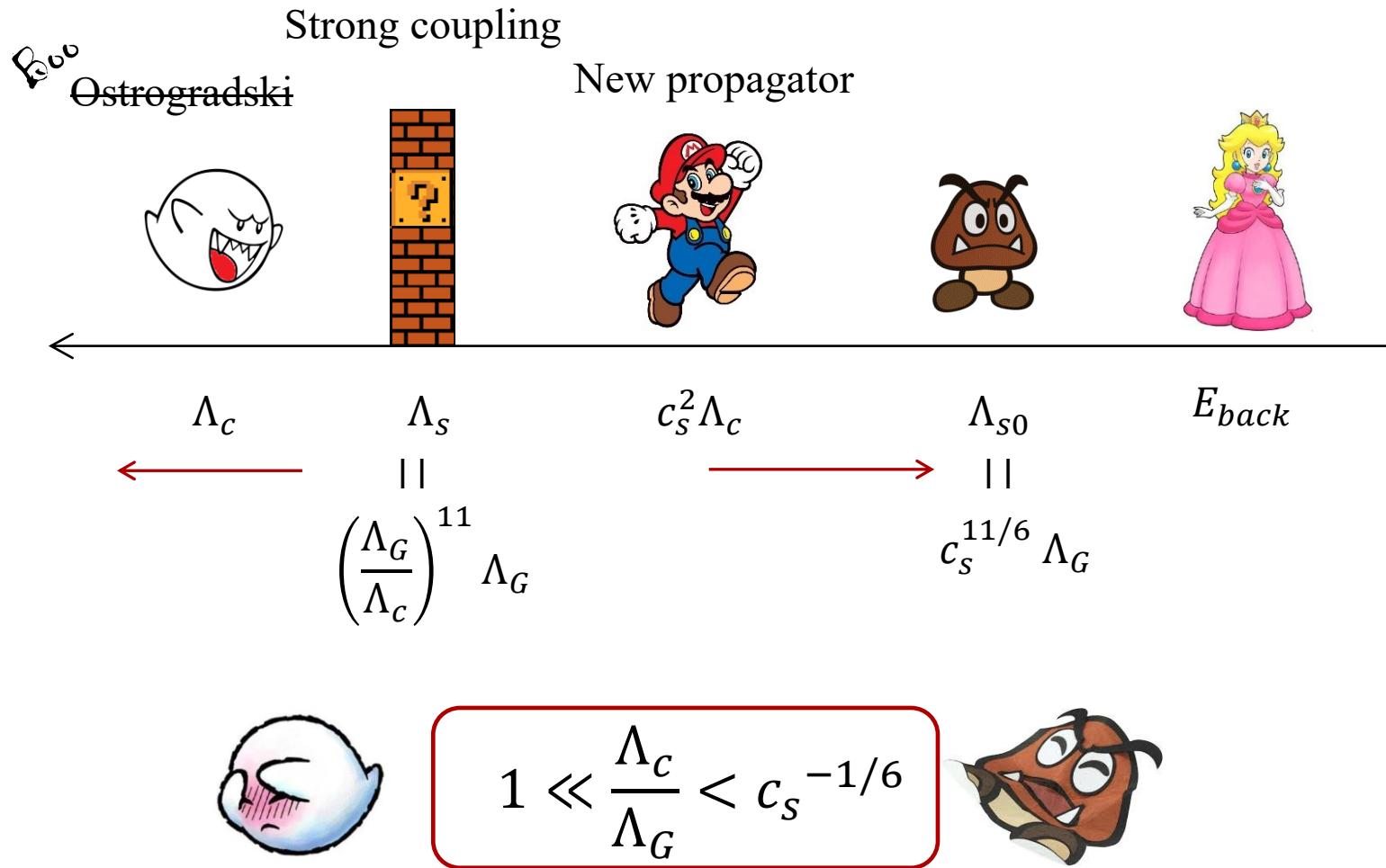


$$\omega^2 \ll \Lambda_c^2 \Rightarrow \text{Prop} = \begin{cases} \frac{1}{\omega^2 - c_s^2 k^2 + \dots}, & c_s^2 k^2 \ll c_s^4 \Lambda_c^2 \quad 0 \leq E \ll c_s^2 \Lambda_c \quad \Lambda_{s0} \\ \frac{1}{\omega^2 - \frac{k^4}{\Lambda_c^2} + \dots}, & c_s^2 k^2 \gg c_s^4 \Lambda_c^2 \quad c_s^2 \Lambda_c \ll E \ll \Lambda_c \quad \Lambda_s \end{cases}$$

Higher Derivative Corrections



Higher Derivative Corrections



1. Beware strong coupling issues



$$\mathcal{L}(g_{\mu\nu}, \Phi) = \frac{1}{2}\varphi(\partial_t^2 - c_s^2 \partial_i^2)\varphi + \frac{\mathcal{O}_P}{\Lambda^{P-4}} + \dots$$

$c_s^2 > 0$ is only half the battle!

Also need $\Lambda_s > E_{back}$ to trust theory (perturbatively)

2. Fix strong coupling issues

$$\mathcal{L}(g_{\mu\nu}, \Phi) + \frac{1}{2\Lambda_c^2} (\partial^2 \Phi)^2 \quad \text{with } \Lambda_c > \Lambda_s > \Lambda_{s0}$$

3. $P(X)$ and Galileon bounces

$$1 \ll \Lambda_c^4 P_{XX} < 1/c_s$$

$$1 \ll \Lambda_c^6 G_{3,X}^2 < 1/c_s$$