A new formalism for cosmological observables (without coordinates)

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Observation and tetrads

- *What do we measure?*
  Spectrum, flux, intensity, polarization, angular position, shape and size in the sky of light beams

- *Where does this information lie?*
  The momentum vector and polarization tensor of photons

  $\Rightarrow$ We measure the components of tensors at $\mathcal{O} \in \mathcal{M}$

  $\Rightarrow$ w.r.t. a basis of $T_{\mathcal{O}}\mathcal{M}$ privileged by the apparatus

- This basis e_a^μ is orthonormal, i.e. a tetrad

$$g_{\mu\nu}(\mathcal{O})\, e_a^\mu e_b^\nu = \eta_{ab} \qquad \text{and} \qquad e_0^\mu \equiv u_{\mathcal{O}}^\mu$$

and we measure $k^a(\mathcal{O}) := e_\mu^a k^\mu(\mathcal{O}) \equiv \omega_{\mathcal{O}}\left(1, -n_{\mathcal{O}}^i\right)$

- In the metric formalism the only available frames are ∂_μ

$$g(\mathcal{O})\,(\partial_\mu, \partial_\nu) \equiv g_{\mu\nu}(\mathcal{O}) \neq \eta_{\mu\nu}$$

especially in the usual gauges (Newtonian and synchronous)

$\Rightarrow$ the components k^μ are *not* the measured ones

- We do have, however, $u^\mu_\mathcal{O}$ and $u^\mu_\mathcal{S}$

$\Rightarrow$ we can define *some* observables such as redshift

$$1 + z := \frac{(u_\mu k^\mu)_\mathcal{S}}{(u_\mu k^\mu)_\mathcal{O}}$$

- But we still lack a spatial orthonormal frame $e^\mu_{i=1,2,3}$

$\Rightarrow$ we cannot define angle-dependent observables

- Solution 1: choose coordinates such that $g_{\mu\nu}(\mathcal{O}) = \eta_{\mu\nu}$

  $\Rightarrow$ fixes the gauge freedom in the vicinity of $\mathcal{O}$

  $\Rightarrow$ no matching with standard gauges on the rest of $\mathcal{M}$

- Solution 2: invoke a tetrad e_a^μ at $\mathcal{O}$ only
  and build it out of $g_{\mu\nu}(\mathcal{O})$ and $u_\mathcal{O}^\mu$

  $\Rightarrow$ 16 components in e_a^μ versus 13 in $g_{\mu\nu}(\mathcal{O})$ and $u_\mathcal{O}^\mu$

  $\Rightarrow$ partial gauge-fixing – usual choices break diffs near $\mathcal{O}$

  $\Rightarrow$ again, no freedom to go to standard gauges in $\mathcal{M}$

Our solution: tetrad formulation of GR

- Replace the metric information by a tetrad *field* $e_a^\mu(x)$

$$g_{\mu\nu}(x) = \eta_{ab}\, e_\mu^a(x)\, e_\nu^b(x)$$

- Advantage 1:
  Unifies observer and sources into an "observer family", e.g.

$$u_{\mathcal{O}}^\mu \equiv e_0^\mu(\mathcal{O})\,, \qquad u_{\mathcal{S}}^\mu \equiv e_0^\mu(\mathcal{S})$$

  $\Rightarrow$ e_i^μ spans the rest-frame directions by construction

- Advantage 2:
  Work directly with $T^{a_1\ldots a_n}_{b_1\ldots b_m}(x)$ on $\mathcal{M}$ (diff scalars)
  $\Rightarrow$ $T^{a_1\ldots a_n}_{b_1\ldots b_m}(\mathcal{O})$ are the observed quantities
  $\Rightarrow$ explicitly coordinate-independent formalism

– Advantage 3:
New gauge symmetry: local Lorentz transformations (LLT)

$$\tilde{e}^a_\mu(x) = \Lambda^a{}_b(x)\, e^b_\mu(x) \qquad \Rightarrow \qquad \tilde{g}_{\mu\nu}(x) = g_{\mu\nu}(x)$$

- 16 tetrad components $\rightarrow$ 10 metric components
- LLT = "observer family transformations"

  k^a measured by e^μ_a $\rightarrow$ $\tilde{k}^a$ measured by $\tilde{e}^\mu_a$

  $\Rightarrow$ local SO(1,3) allows to reach all possible observer families
- Conceptual transparency

  Symmetry $\Rightarrow$ observer-*independent* physics
  Relativity $\Rightarrow$ observer-*dependent* measurements

  $\Rightarrow$ Coordinates are no longer related to these concepts

- Advantage 4:
  Covariance under both gauge transformations
  $\Rightarrow$ Freedom to choose diff gauges (e.g. Newtonian)
  $\Rightarrow$ LLT gauge amounts to observer dynamics (e.g. free-fall)

- Advantage 5:
  Unifies gravitational and observer information:
  Gravity = internal norm of the tetrad (metric, LLT-scalar)
  Observer = orientation of the tetrad (LLT-vector)

- Advantage 6:
  Symmetry-based definition of observables $C(\mathcal{S}, \mathcal{O})$
  - Invariant under coordinate transformations
  - Can only depend on $\Lambda(\mathcal{O})$ and $\Lambda(\mathcal{S})$ under LLTs

Example: the angular diameter distance

- Deviation of the light-like geodesic in the tetrad basis

$$d^a := e^a_\mu(\gamma)\,\delta_{\rm dev}\gamma^\mu \qquad d^i_\perp := d^i - n^i n^j d^j$$

- Deviation of the observed position in the sky

$$m^i_\mathcal{O} := \delta_{\rm dev}\, n^i_\mathcal{O} \sim \dot{d}^i_\mathcal{O} \qquad m^i_\mathcal{O} n^i_\mathcal{O} \equiv 0$$

- Physical area of the source (visible face) in its rest-frame

$$\mathrm{d}A_\mathcal{S} := \varepsilon^{ijk}\, n^i_\mathcal{S} d^j_\mathcal{S} \tilde{d}^k_\mathcal{S}$$

- Observed solid angle

$$\mathrm{d}\Omega_\mathcal{O} := \varepsilon^{ijk}\, n^i_\mathcal{O} m^j_\mathcal{O} \tilde{m}^k_\mathcal{O}$$

Observation implies

$$d^i_{\mathcal{O}} = 0$$

$\Rightarrow$ geodesic deviation solution takes the form

$$d^i_{\perp,\mathcal{S}} = \mathcal{J}^{ij}_{\mathcal{S},\mathcal{O}} m^j_{\mathcal{O}}$$

where $\mathcal{J}$ is the lensing Jacobi map obeying

$$n^i_{\mathcal{S}} \mathcal{J}^{ij}_{\mathcal{S},\mathcal{O}} \equiv 0 \qquad \mathcal{J}^{ij}_{\mathcal{S},\mathcal{O}} n^j_{\mathcal{O}} \equiv 0$$

We then have

$$\mathrm{d}A_{\mathcal{S}} \equiv D^2_{\mathcal{S},\mathcal{O}}\, \mathrm{d}\Omega_{\mathcal{O}}$$

$$D^2_{\mathcal{S},\mathcal{O}} = \mathrm{det}_{\perp} \mathcal{J} \equiv \frac{1}{2} \left(n^i_{\mathcal{S}} \varepsilon^{ijk}\right) \left(n^k_{\mathcal{O}} \varepsilon^{klm}\right) \mathcal{J}^{jl} \mathcal{J}^{km}$$

No coordinates involved.

Thank you!