Thermalized Axion Inflation

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Outline

Inflation

- Inflation is the best candidate for initial conditions of our Universe
- Can explain:
  - Observed smallness of spatial curvature Ω_k (“flatness problem").
  - Makes Universe causally connected on large scales (“horizon problem").
  - In addition: provides the seeds for observed density fluctuations.

Inflation

- In FLRW metric, expansion described by $a(t)$

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

- Einstein equations for homogeneous/isotropic flat space ($H \equiv \frac{\dot{a}}{a}$)

$$\begin{aligned} H^2 &= \frac{\rho}{3M^2} \\ \dot{\rho} &= -3H(\rho + p) \end{aligned}$$

- $\rho \sim const \Rightarrow H \sim const$
- $\Rightarrow a(t) = e^{Ht}$

Standard slow-roll Inflation

- We need $p \sim -\rho$ dominating energy density
- Simple approach: a new scalar field

$$\rho = V(\phi) + \frac{\dot{\phi}^2}{2} \qquad p = -V(\phi) + \frac{\dot{\phi}^2}{2} \tag{1}$$

$(\dot{\phi}^2/2 \ll V)$

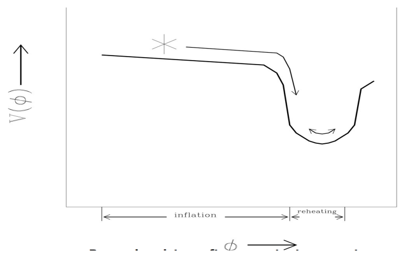
Hubble friction dominates:

$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0 \tag{2}$$

$(\ddot{\phi} \ll 3H\dot{\phi})$

- $\Rightarrow$ Slowly rolls for $\left(\frac{a_I}{a_F}\right) = e^N \gtrsim e^{60}$

Slow-roll



- "Slow-roll": vacuum state
- Then fast roll and decay: creation of particles, thermalization ("Reheating")
- It also fluctuates ⇒ Density fluctuations

Slow-roll Inflation: simple but...

- Which V?
- Why unusually flat V?
- Unknown couplings and Reheating process
- Very few measured parameters (A, n_s, upper limits on r, f_{NL}...)
- Interesting to explore non-minimal features during inflation:
  - New predictions?
  - New observables?
  - New dynamics? (not based on flat potentials)

Non-standard features of inflation

- We explore non-standard features of inflation
- We analyze:
  - Axion models
  - Dissipation & Temperature, *already during inflation*
  - Strong Friction during inflation (?)

Inflation with Axial coupling

- ϕ coupled to U(1) gauge fields, “axion-like":

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\phi}{4f_\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

- $F_{\mu\nu}\tilde{F}^{\mu\nu}$ odd under CP (and so T) $\implies$ Instability $\propto \dot{\phi}$
- Photons are massless at constant ϕ (ϕ coupled derivatively) $\implies$ Efficient production
- Derivative coupling $\implies$ No corrections to $V(\phi)$

Inflation with Axial coupling

- In a time dependent ϕ and in FLRW[2] (conformal time $a d\tau = dt$, $\pm$ positive (negative) helicity) :

$$A''_{\pm} + \left(k^2 \mp \frac{k\phi'}{f} \right) A_{\pm} = 0 \,,$$

- $\phi' = a\dot{\phi} \neq 0$

- One helicity is unstable: gauge fields becomes quickly large

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[2] *e.g.* I. Tkachev, Pisma Astron.Zh. 12 (1986).

Constant $\dot\phi$ and de Sitter

- Assume: $\dot\phi = const$ in de Sitter and $a(t) = -\frac{1}{H\tau}$ (Sorbo & Anber '09)

$$A''_\pm + \left(k^2 \mp \frac{2k\xi}{\tau}\right) A_\pm = 0\,, \qquad \xi \equiv \frac{\dot\phi}{2fH}$$

- Impose vacuum fluctuations $A_k = \frac{e^{ik\tau}}{\sqrt{2k}}$ at $\tau \to -\infty$ (past)

  (*Almost, up to a $\ln(\tau)$ phase.)

- Solution at $\tau \to 0^-$ (future):

$$A_+ \approx \frac{1}{\sqrt{2k}} \left(\frac{k|\tau|}{2\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k|\tau|}}$$

Consequences

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F}\rangle}{4} = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\, k\, \frac{d\left[|A_+|^2 - |A_-|^2\right]}{d\tau} \approx \frac{H^4}{\xi^4} e^{2\pi\xi}$$

$$\rho_\gamma = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\, k\, \frac{d\left[|A_+|^2 - |A_-|^2\right]}{d\tau} \approx \frac{H^4}{\xi^3} e^{2\pi\xi}$$

- New features: [3]
  - Fields are **not** in the vacuum:
    - Contribution to 2-point function $\langle \delta\phi\delta\phi\rangle_{loop}$
    - Contribution to 3-point function $\langle \delta\phi\delta\phi\delta\phi\rangle_{loop}$
    - Contribution to tensors
  - At large ξ: Backreaction on ϕ dynamics (friction)

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[3] Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016). Ferreira & Sloth, JHEP 1412 (2014) 139. Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst. Phys.), Phys.Lett. B718 (2013),....

Consequences

Generic conclusions:[4] :

- Contribution to 3-point function $\langle \delta\phi\delta\phi\delta\phi \rangle_{loop}$ can be very large at $\xi \gtrsim 2.5 - 3$ (good & bad)
- Constraints from 3-point function seems to forbid other effects to be visible:
  - Large r
  - Large Backreaction (friction) on ϕ dynamics
- Moreover loop expansion [5] seems to break down at $\xi \gtrsim 3.5 - 4.5$

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[4] Barnaby & Peloso PRL 106 (2011), Barnaby et al. PRD85 (2012), Namba et al. JCAP 1601 (2016). Anber & Sorbo PRD85 (2012) 123537. Lin & Ng (Taiwan, Inst. Phys.), Phys.Lett. B718 (2013),....

[5] Ferreira & Sloth, JHEP 1412 (2014) 139

Thermalized Axion Inflation

- Include scattering of gauge bosons [6]:
  - Very large occupation number $N_\gamma \rightarrow$ scatterings enhanced $\gamma\gamma \leftrightarrow \gamma\gamma$, $\gamma\gamma \leftrightarrow \phi\phi$, $\gamma\phi \leftrightarrow \gamma\phi$
  - $\Longrightarrow$ Thermalization during Inflation, with $T > H$
  - Very efficient if Standard Model gauge fields: $\gamma\gamma \leftrightarrow \ell^+\ell^-$

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[6] *"Thermalized Axion Inflation"* Ricardo Z. Ferreira, A.N. 1706.00373.

Thermalized Axion Inflation

- New phenomenology:
  - Moves power of gauge fields to UV $k/a \sim T$, more inside the horizon
  - Expect: new dependence of $\langle \delta\phi\delta\phi \rangle_{loop}$, $\langle \delta\phi\delta\phi\delta\phi \rangle_{loop}$ on ξ (allow for larger ξ)
- Questions:
  - Is large friction allowed?
  - Perhaps: oscillations in spectra?

Particle production and thermalization

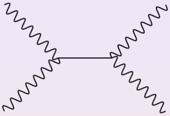
- Instability $\Rightarrow$ particle production of modes: $\frac{k}{a} \lesssim 2\xi H$.
- Instability starts subhorizon (if $\xi > 1$) where particle interpretation meaningful.
- Define Particle number per mode k as

$$\frac{\rho_\gamma(k)}{2k} = \frac{A'^2 + k^2 A^2}{2k} \equiv \frac{1}{2} + N_\gamma(k) \quad \Rightarrow$$

$$\begin{cases} N_\gamma(k) \simeq \quad 0, & k/a \gg H \\ N_\gamma(k) \simeq \frac{e^{2\pi\xi}}{8\pi\xi}, & k/a \ll H \end{cases}$$

Scatterings

Scatterings are enhanced by powers of N_γ



$$\frac{dN_\gamma(k)}{d\tau} = S(k)$$

$$S = \frac{1}{\omega(k)} \int \prod_{i=2}^{4} \left(\frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right) |M_n|^2 (2\pi)^4 \delta^{(4)} \left(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu \right) \cdot$$

$$\cdot N_\gamma(k) N_\gamma(p_2) \left[1 + N_\gamma(p_3)\right] \left[1 + N_\gamma(p_4)\right]$$

Outline

1. Introduction
2. Inflation with Axial coupling
3. Thermalization
    - Axion-mediated scatterings
    - SM scatterings
    - Perturbations

Scatterings

- Scattering rates $\propto N_\gamma^3 \implies$ For large N_γ:

$$t_{\text{scatterings}} \ll H^{-1} \quad \Rightarrow \quad \text{thermalization}$$

- $S \approx 10^{-4} \frac{\omega^5}{f^4} N_+^3$.
- Compare: $N_+ H \ll S$

$$N_+ \gg \sqrt{\beta_S \frac{Hf^4}{\omega^5}} \quad \xRightarrow[\omega\approx H]{} \quad \xi \gtrsim 0.45 \ln\left(\frac{f}{H}\right) + 2.7\,,$$

(Using $N_+ \approx 10^{-4} e^{4.5\xi}$)

- Expectation: thermal bath of photons with temperature

$$T \approx \rho_{\gamma,\text{initial}}^{1/4} \approx 0.1 H e^{\pi\xi/2}$$

Boltzmann-like equations

- We rewrite the eom as a Boltzmann-like eq. $(g \equiv A'_+(k,\tau)/A_+(k,\tau))$:

$$\frac{dN_{\gamma_+}(k)}{d\tau} = -\frac{4k\xi}{\tau}\,\frac{\mathrm{Re}\,[g]}{|g|^2+k^2}\left(N_{\gamma_+}(k)+\frac{1}{2}\right)+S$$

(approximation: g computed without S)

- Full system $(\gamma_+, \gamma_-, \phi)$

$$\begin{cases} N'_+ = -\frac{4k\xi}{\tau}\frac{\mathrm{Re}[g]}{|g|^2+k^2}\left(N_+ + \frac{1}{2}\right) + S^{++} + S^{+\phi} + D^{+\phi} + S^{+-}\,, \\ N'_u = -S^{+\phi} - D^{+\phi}\,, \\ N'_- = -S^{+-}\,, \end{cases}$$

(u$\equiv a\,\delta\phi$)

Numerical results

- Discretize: $\mathcal{O}(10)$ modes of comoving momentum: $k \in [1, \mathcal{O}(10)]H$.
- Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$
- Distribution of particles approaches Bose-Einstein distribution at ξ, f in agreement with estimations.

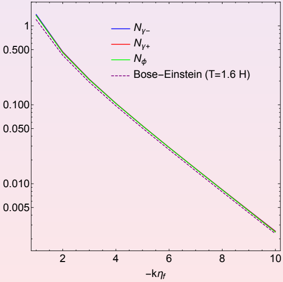
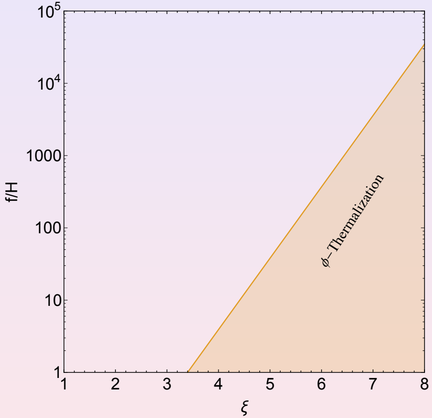


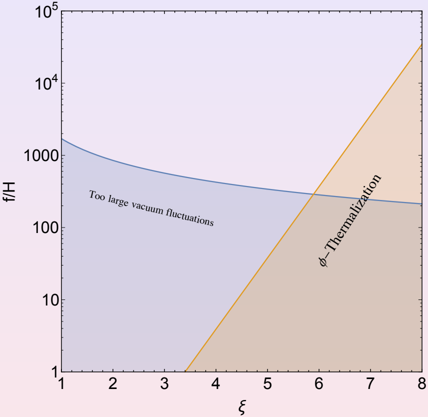
Figure: $\xi = 2$, $f/H = 0.1$

Thermalization



$$\Gamma_s \gg H \qquad \Longrightarrow \qquad \xi \gtrsim 0.45 \ln\left(\frac{f}{H}\right) + 2.7\,,$$

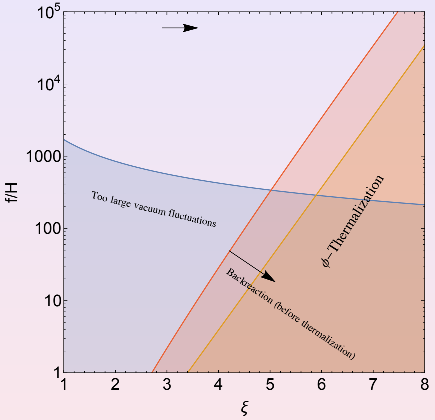
Thermalization



Vacuum fluctuations:

$$P_\zeta^{\mathrm{vac}} = \frac{H_*^4}{4\pi^2\dot{\phi}_*^2} = \frac{H^2}{16\pi^2 f^2\xi^2} \leq 2.2\times 10^{-9} \qquad \Longrightarrow \qquad \frac{f}{H} \gtrsim \frac{2\times 10^3}{\xi}\,.$$

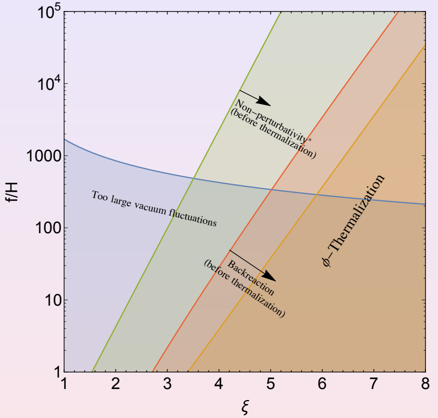
Thermalization



Backreaction on ϕ (before reaching thermalization):

$$\frac{\langle F\tilde{F}\rangle}{f} \gtrsim V'(\phi) \simeq 3H\dot{\phi} \qquad \Longrightarrow \qquad f/H \lesssim 4\times 10^{-3} e^{\pi\xi}/\xi^{5/2}.$$

Thermalization



* Requiring perturbativity on loop expansion for cosmological correlators, in absence of thermalization (Ferreira *et al.*,JCAP 1604 (2016)):

$$\frac{H^2}{f^2}\frac{e^{2\pi\xi}}{16\pi^2 I} < 1,$$

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Standard Model couplings

- If gauge fields belong to SM: many other interactions with known couplings ($\gamma\gamma \leftrightarrow e^+e^-$, ...)
- More predictive, only depends on ξ. Interactions not suppressed by powers of $1/f$.
- More realistic, the inflaton has anyway to couple efficiently to the SM to reheat the universe.
- Using $\sigma_{\gamma\gamma\leftrightarrow\ell^+\ell^-} \approx \frac{\alpha_{EM}^2}{\omega^2} \quad \Longrightarrow$

  Requirement for thermalization ($\Gamma_s \gg H$) :

$$N_\gamma H \ll \frac{\alpha_{EM}^2}{\omega^2} \cdot H^3 N_\gamma^2 \quad \Longrightarrow \quad \xi \gtrsim 2.9$$

SM Thermalization

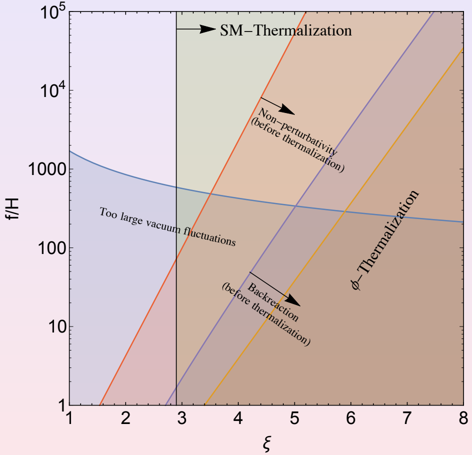

Figure: Summing over all $U_Y(1)$ charged particles in SM.

Enter a new regime...

- Thermal gauge field masses appear: $m_T \propto gT$

$$A''_\pm + \omega_T^2(k)A_\pm = 0, \qquad \omega_T(k) = \left(k^2 \pm \frac{2k\xi}{\tau} + \frac{m_T^2}{H^2\tau^2}\right).$$

- When $m_T \geq \xi H$ completely shields the instability band $(\omega^2 > 0)$
- Expect $T_{eq} \approx \frac{\xi H}{g}$ (or maybe oscillations?)
- New regime: linear in ξ, not exponential: $T_{eq} \ll (e^{2\pi\xi}H^4)^{1/4}$!

At equilibrium temperature

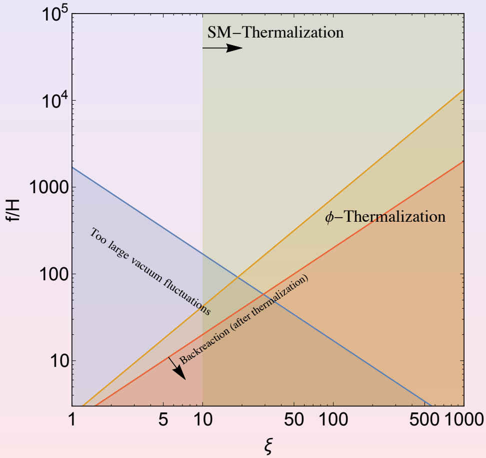

Figure: Using $T_{eq} \approx \frac{\xi H}{g}$, with $g = 0.5$.

Outline

Thermal Spectrum of ζ

- If $\zeta \equiv \frac{H}{\dot{\phi}}\delta\phi$ thermalizes $\Longrightarrow$ **not** in vacuum at Horizon Crossing
- $|u_k|^2 = \left|\frac{1}{\sqrt{2k}}\right|^2 = \frac{1}{k}\cdot\frac{1}{2} \qquad \Rightarrow \qquad \frac{1}{k}\left(\frac{1}{2} + N(k_*)\right)$
- $N_k = \frac{1}{e^{\frac{k/a}{T}}-1}. \qquad$ At $\frac{k}{a} = H \Longrightarrow N_k \approx \frac{T_*}{H_*}$
- $P_\zeta = P_\zeta^{vac}\cdot\frac{2T_*}{H_*}$

Thermal Spectrum of ζ

- $P_\zeta = P_\zeta^{vac} \cdot \frac{2T_*}{H_*} \quad \Longrightarrow$

$$n_s - 1 \equiv \frac{d \ln P_\zeta^{\text{therm}}}{d \ln k} \quad = \quad -6\epsilon_H + 2\eta + \frac{d \ln (T_*/H_*)}{d \ln k} \,,$$

- If $T = T_{eq} = \frac{\xi}{g} H, \quad \Longrightarrow$

$$\begin{cases} P_\zeta^{\text{therm}} = \frac{\xi}{\bar{g}} \frac{H_*^4}{2\pi^2 \dot{\phi}_*^2} = \frac{H^2}{8\pi^2 f^2 \bar{g} \xi} \,, \\ n_s - 1 \equiv = -6\epsilon_H + 2\eta + \frac{\dot{\xi}}{H\xi} = -4\epsilon_H + \eta \,. \end{cases}$$

New regime at T_{eq}

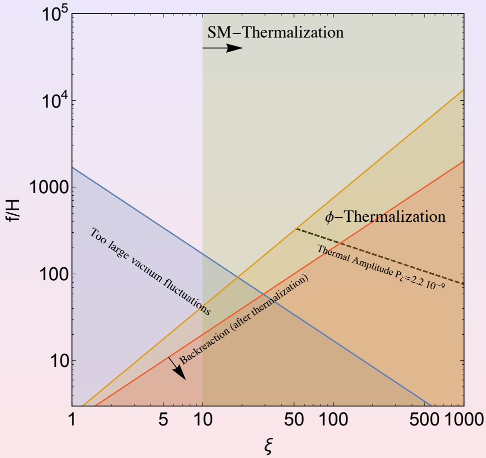

Figure: $P_\zeta = P_\zeta^{vac} \cdot \frac{2T_*}{H_*} = P_\zeta^{vac} \cdot \frac{2\xi}{g}$, $(g = 0.5.)$

Phenomenology in the thermal regime

- Loop effects on ζ drastically modified!
- Thermalization shifts gauge fields from horizon size to UV. At horizon crossing N_γ is reduced.
- We expect (work in progress) much smaller $\langle\zeta\zeta\zeta\rangle_{\rm loop}$
  - $f_{NL} \simeq \frac{\langle\zeta^3\rangle}{\langle\zeta^2\rangle^2} \propto P_\zeta^{\rm vac}\mathcal{O}\left(\frac{T^4}{H^4}\right) \propto c\xi^4$ (c small number)
  - Instead of non-thermal case: $f_{NL} \propto e^{4\pi\xi}$!
  - Constraints on ξ become weaker and (maybe) allow for the backreacting regime? (Work in progress)

Phenomenology of tensor modes

- Assuming tensor modes to be in the vacuum:

$$r \equiv \frac{P_T}{P_\zeta^{\text{therm}}} = 16\,\epsilon \frac{H_*}{2T_*} \,.$$

- Suppressed by $\frac{H_*}{2T_*} = \frac{g}{2\xi}$
- At least $\mathcal{O}(10^{-2})$ suppression.

Summary

- Thermalized Axion Inflation:
- Axion Inflation can be hot
- P_ζ can be thermal
- Reheating is automatic and fixed (when $\rho_\gamma > V(\phi)$)
- Work in progress: Inclusion of Thermal masses, Non-gaussianity, Backreaction
- Does the spectrum have small oscillations?