Interacting neutrinos in cosmology: exact description and constraints

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arXiv: 1706.02123 [astro-ph.CO]

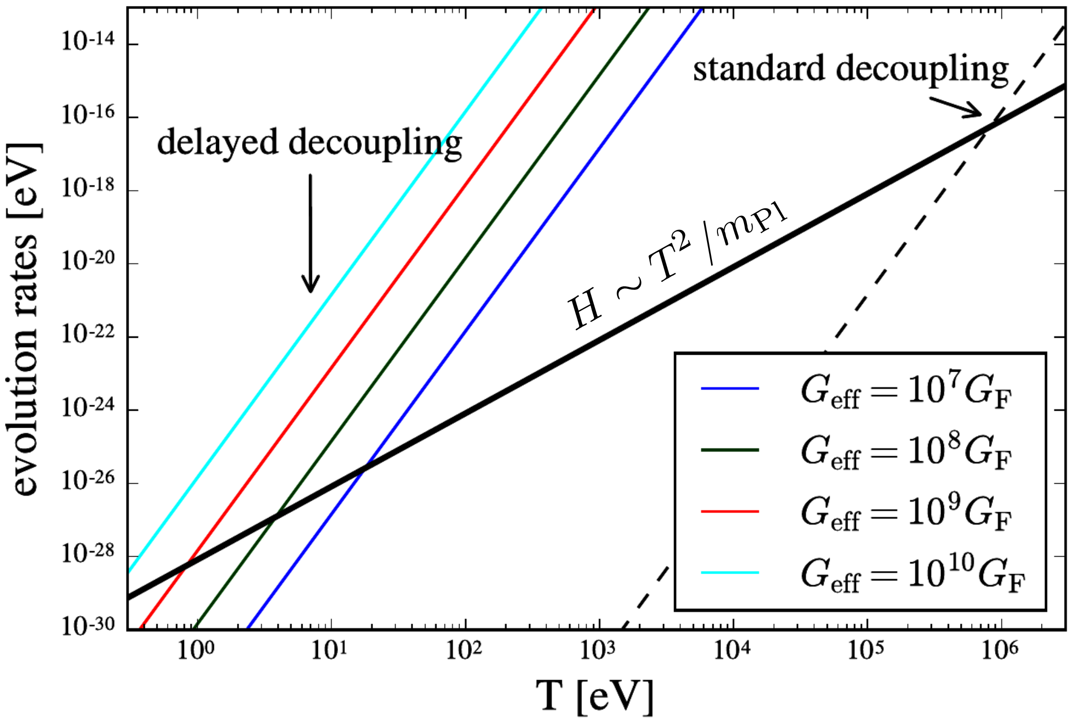
J. Cosmol. Astropart. Phys. 1504, 016 (2015), arXiv:1409.1577 [astro-ph.CO]

massless neutrinos ↯ observation of neutrino oscillations

→ Models of neutrino mass generation, “Majoron models“

$$\mathcal{L}_{\text{int}} = \mathfrak{g}_{ij}\bar{\nu}_i\nu_j\phi + \mathfrak{h}_{ij}\bar{\nu}_i\gamma_5\nu_j\phi$$

→ **non-standard neutrino interactions** $\Gamma_{\text{new}} \sim G_{\text{eff}}^2 T^5$ (massive scalar limit)



→ **cosmological signature?**

Impact on the CMB described by **Boltzmann hierarchy for interacting neutrinos**

… What's that...???

→ Cosmic perturbation theory

Small fluctuations from inflation are the seeds for the structures observed today

1.) Perturbed Einstein equation: $\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$ *Lifshitz, 1946*

2.) Perturbed Boltzmann equations: *Peebles & Yu 1970*

Perturbed phase-space density: $f(\mathbf{k},\mathbf{q},\eta) = \bar{f}(q)\,(1+\Psi(\mathbf{k},\mathbf{q},\eta))$

$$\boxed{\dot{\Psi}_i(\mathbf{k},\mathbf{q},\eta) + \mathrm{i}\frac{|\mathbf{q}||\mathbf{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\,\Psi_i(\mathbf{k},\mathbf{q},\eta) + \frac{\partial \ln \bar{f}_i(|\mathbf{q}|,\eta)}{\partial \ln |\mathbf{q}|}\left[\dot{\tilde{\eta}} - (\hat{k}\cdot\hat{q})^2\,\frac{\dot{h}+6\dot{\tilde{\eta}}}{2}\right] = \left(\frac{\partial f_i}{\partial \eta}\right)^{(1)}_{\mathrm{coll}}}$$

Apply on all relevant particle species:

	interacting	non-interacting
relativistic	photons	**neutrinos?**
non-relativistic	baryons	CDM

Decompose phase-space perturbation into Legendre polynomials:

$$\Psi(|\mathbf{k}|, |\mathbf{q}|, \hat{k}\cdot\hat{q}) = \sum_{\ell=0}(-i)^{\ell}(2\ell+1)\Psi_{\ell}(|\mathbf{k}|, |\mathbf{q}|)P_{\ell}(\hat{k}\cdot\hat{q})$$

→ Taking moments: $\int_{-1}^{1} \mathrm{d}(\hat{k}\cdot\hat{q})P_{\ell}(\hat{k}\cdot\hat{q})\,[\text{Boltzmann eq.}]$

→ Neutrino Boltzmann hierarchy: *Stewart 1970*

$$\dot{\delta}_{\nu} = -\frac{4}{3}\theta - \frac{2}{3}\dot{h},$$

$$\dot{\theta} = k^2\left(\frac{1}{4}\delta - \sigma\right),$$

$$\dot{F}_2 = 2\dot{\sigma} = \frac{8}{15}\theta - \frac{3}{5}kF_3 + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$

$$\dot{F}_{\ell\geq 3} = \frac{k}{2\ell+1}\left[lF_{\ell-1} - (\ell+1)F_{\ell+1}\right]$$

→ analogously for all other particle species

How to include neutrino interactions?

1.) Relaxation time approximation:

$$\dot{\mathcal{F}}_{\nu 2} = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} + \alpha_2 \dot{\tau}_\nu \mathcal{F}_{\nu 2} \ ,$$

$$\dot{\mathcal{F}}_{\nu \ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] + \alpha_\ell \dot{\tau}_\nu \mathcal{F}_{\nu\ell} \ , \quad \ell \geq 3 \ ,$$

→ motivated from the photon hierachy

F. Cyr-Racine, K. Sigurdson, arXiv:1306.1536

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2.) Parameterisation used to fit cosmological data:

$$\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu - \frac{2}{3}\dot{h} + \frac{\dot{a}}{a}(1-3c_{\rm eff}^2)\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right),$$

$$\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \frac{k^2}{4}(1-3c_{\rm eff}^2)\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right),$$

$$\dot{\mathcal{F}}_{\nu 2} = 2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} - (1-3c_{\rm vis}^2)\left(\frac{8}{15}\theta_\nu + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}}\right),$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right], \quad \ell \geq 3$$

$$\left(c_{\rm eff}^2, c_{\rm vis}^2\right) = \left(\frac{1}{3}, \frac{1}{3}\right)$$

→ standard case

$$\left(c_{\rm eff}^2, c_{\rm vis}^2\right) = \left(\frac{1}{3}, 0\right)$$

→ tightly coupled limit

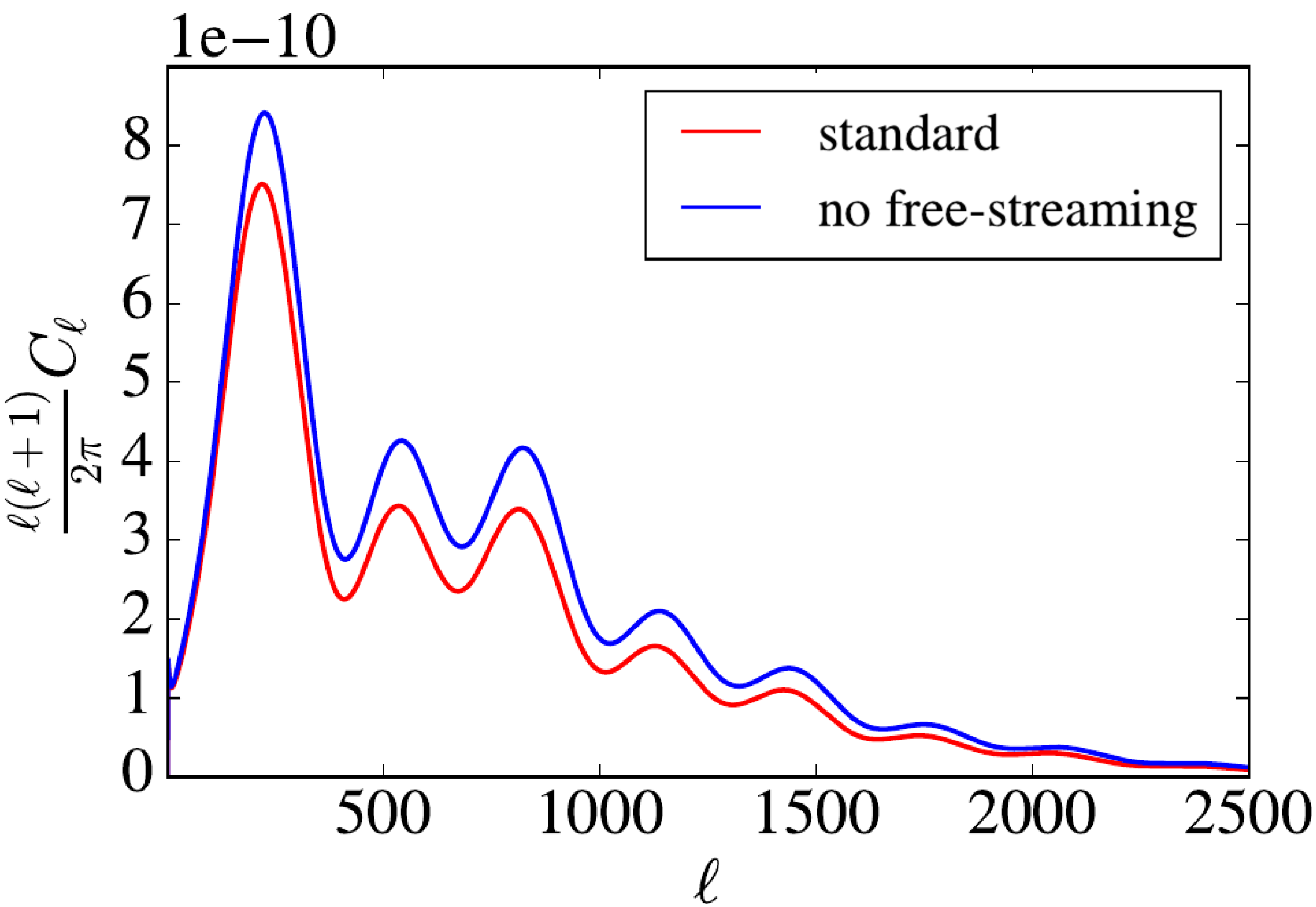
e.g. A. Melchiorri, arXiv:1109.2767, ...

General expected signal

suppression of free-streaming

→ enhancement of neutrino monopole/energy density

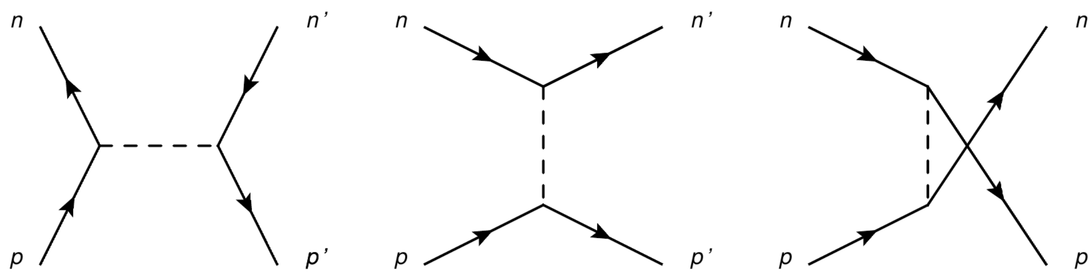
→ enhancement of temperature anisotropies



Exact description of interacting neutrinos needs calculation of the

collision integral.

$$\Rightarrow \dot{\Psi}_i(\mathbf{k},\mathbf{q},\tau) + \mathrm{i}\frac{|\mathbf{q}||\mathbf{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\,\Psi_i(\mathbf{k},\mathbf{q},\tau) + \frac{\partial\ln\bar{f}_i(|\mathbf{q}|)}{\partial\ln|\mathbf{q}|}\left[\dot{\eta} - (\hat{k}\cdot\hat{q})^2\,\frac{\dot{h}+6\dot{\eta}}{2}\right] = \boxed{\left(\frac{\partial f_i}{\partial\tau}\right)^{(1)}_{\mathrm{coll}}}$$



difference to photon case: Thomson scattering = low energy transfer

$$\left(\frac{\partial f_i}{\partial\tau}\right)^{(1)}_{ij\leftrightarrow kl}(\boldsymbol{k},\boldsymbol{q},\tau) = \frac{g_j g_k g_l}{2|\boldsymbol{q}|(2\pi)^5}\int\frac{\mathrm{d}^3\boldsymbol{q}'}{2|\boldsymbol{q}'|}\int\frac{\mathrm{d}^3\boldsymbol{l}}{2|\boldsymbol{l}|}\int\frac{\mathrm{d}^3\boldsymbol{l}'}{2|\boldsymbol{l}'|}\,\delta^4_{\mathrm{D}}(q+l-q'-l')$$
$$\times\,|\mathcal{M}_{ij\leftrightarrow kl}|^2\Big(\bar{f}_k(|\boldsymbol{q}'|)\,\bar{f}_k(|\boldsymbol{l}'|)\,\Psi_l(\boldsymbol{k},\boldsymbol{l}') + \bar{f}_l(|\boldsymbol{l}'|)\,\bar{f}_l(|\boldsymbol{q}'|)\,\Psi_k(\boldsymbol{k},\boldsymbol{q}')$$
$$-\,\bar{f}_i(|\boldsymbol{q}|)\,\bar{f}_i(|\boldsymbol{l}|)\,\Psi_j(\boldsymbol{k},\boldsymbol{l}) - \bar{f}_j(|\boldsymbol{l}|)\,\bar{f}_j(|\boldsymbol{q}|)\,\Psi_i(\boldsymbol{k},\boldsymbol{q})\Big)$$

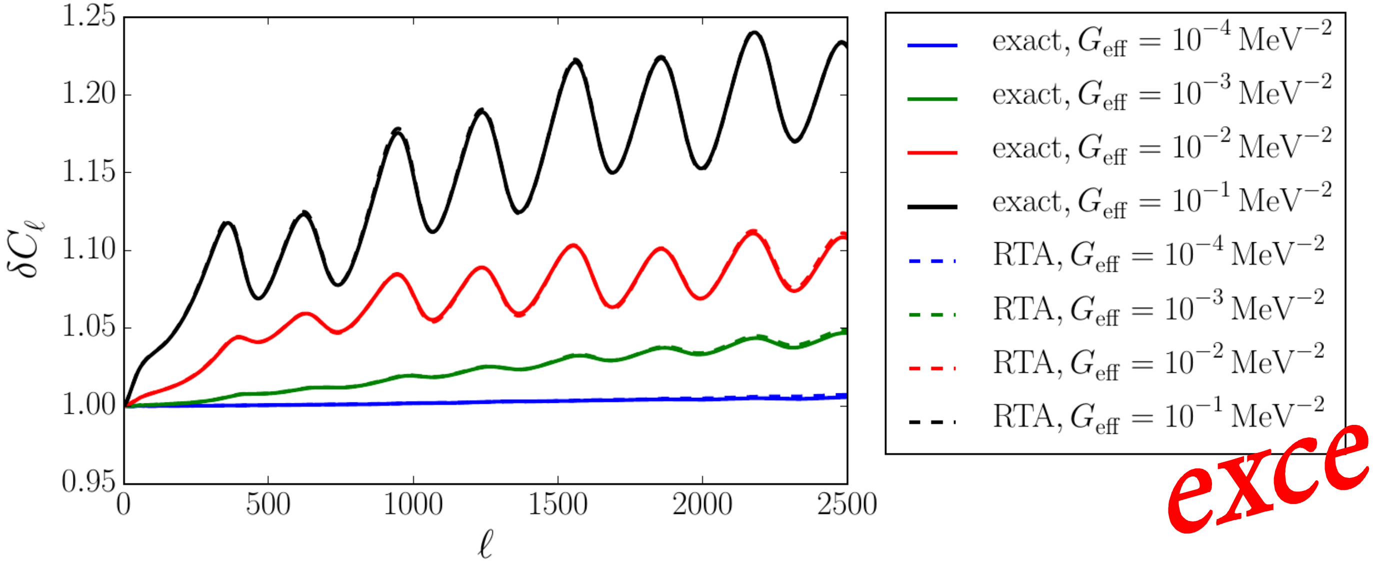
$$\begin{aligned}
\dot{\Psi}_0(q) =& - k\Psi_1(q) + \frac{1}{6}\frac{\partial \ln \bar{f}}{\partial \ln q}\dot{h} - \frac{40}{3}G^{\mathrm{m}} q\, T_{\nu,0}^4\, \Psi_0(q) \\
&\qquad + G^{\mathrm{m}} \int \mathrm{d}q'\, \frac{q'}{q\bar{f}(q)} \left[2K_0^{\mathrm{m}}(q,q') - \frac{20}{9} q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right] \bar{f}_\nu(q')\, \Psi_0(q')\,, \\
\dot{\Psi}_1(q) =& - \frac{2}{3}k\Psi_2(q) + \frac{1}{3}k\Psi_0(q) - \frac{40}{3}G^{\mathrm{m}} q\, T_{\nu,0}^4\, \Psi_1(q) \\
&\qquad + G^{\mathrm{m}} \int \mathrm{d}q'\, \frac{q'}{q\bar{f}(q)} \left[2K_1^{\mathrm{m}}(q,q') + \frac{10}{9} q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right] \bar{f}(q')\, \Psi_1(q')\,, \\
\dot{\Psi}_2(q) =& - \frac{3}{5}k\Psi_3(q) + \frac{2}{5}k\Psi_1(q) - \frac{\partial \ln \bar{f}}{\partial \ln q}\left(\frac{2}{5}\dot{\tilde{\eta}} + \frac{1}{15}\dot{h}\right) - \frac{40}{3}G^{\mathrm{m}} q\, T_{\nu,0}^4\, \Psi_2(q) \\
&\qquad + G^{\mathrm{m}} \int \mathrm{d}q'\, \frac{q'}{q\bar{f}(q)} \left[2K_2^{\mathrm{m}}(q,q') - \frac{2}{9} q^2\, {q'}^2 e^{-q/T_{\nu,0}}\right] \bar{f}(q')\, \Psi_2(q')\,, \\
\dot{\Psi}_{\ell>2}(q) =& \frac{k}{2\ell+1}\left[\ell\Psi_{\ell-1}(q) - (\ell+1)\Psi_{\ell+1}(q)\right] - \frac{40}{3}G^{\mathrm{m}} q\, T_{\nu,0}^4\, \Psi_\ell(q) \\
&\qquad + G^{\mathrm{m}} \int \mathrm{d}q'\, 2\frac{q'}{q\bar{f}(q)} K_\ell^{\mathrm{m}}(q,q')\, \bar{f}(q')\, \Psi_\ell(q')
\end{aligned}$$

- **momentum-dependence reflects non-negligible energy transfer**
- formally very different from other approaches

  → implement in Boltzmann code CLASS *(J. Lesgourgues, et al.)*

Comparison with other approaches

1.) Relaxation time approximation



excellent!

Comparison with other approaches

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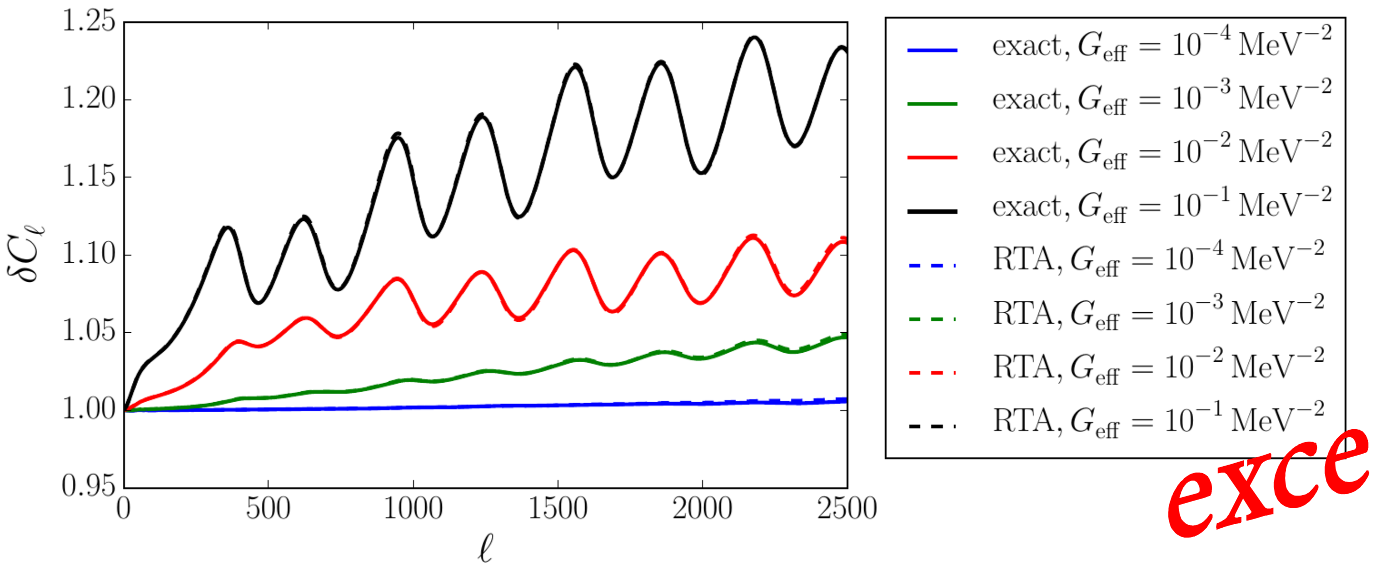
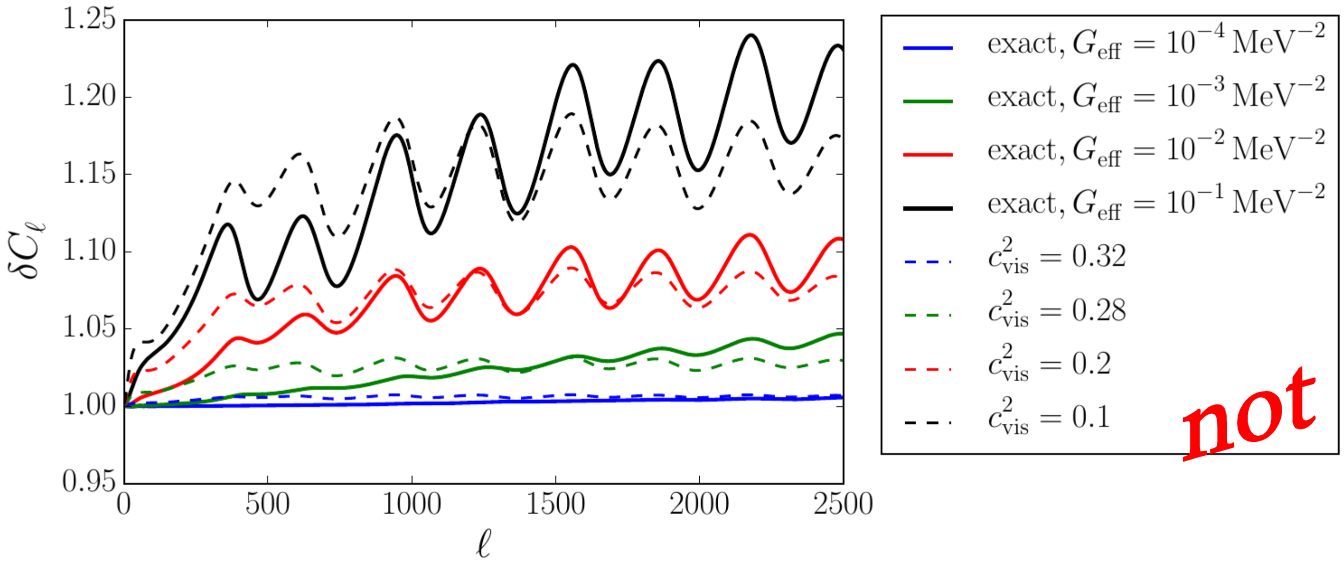


excellent!

2.) $(c_{\rm eff}^2, c_{\rm vis}^2)$

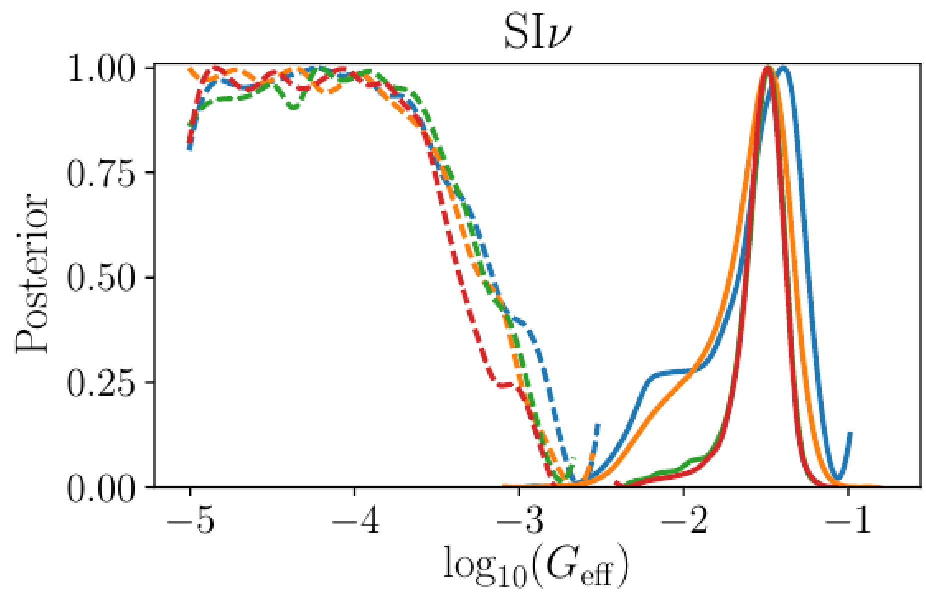


not really...

- Relaxation time approximation entirely sufficient.
- $(c_{\rm eff}^2, c_{\rm vis}^2)$ -parameterisation should not be used!

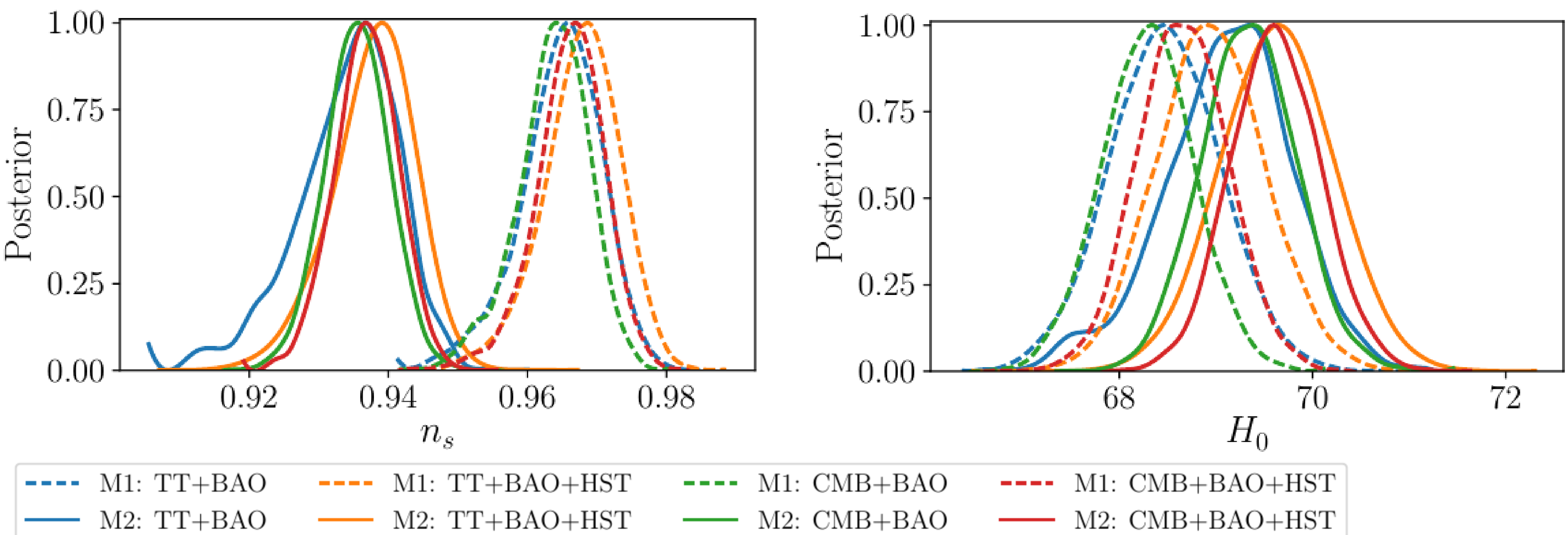
<u>MCMC results</u> (using the relaxation time approximation)

Compare with
arXiv: 1704.06657 (Lancaster, Cyr-Racine et al.) & arXiv:1306.1536 (Cyr-Racine, Sigurdson)



Interacting neutrino mode!

$$G_{\rm eff} = 3\times 10^9 G_{\rm F}$$



Summary:

Majoron models → non-standard neutrino interactions → impact on the CMB?

➔ Calculated the Boltzmann hierarchy for interacting neutrinos

➔ Implemented it in CLASS

Conclusions:

- Boltzmann hierarchy has **formally** a much richer structure than approximations by others
- … but **relaxation time approximation** is an **excellent** effective description
- $\left(c_{\mathrm{eff}}^2, c_{\mathrm{vis}}^2\right)$ -parameterisation does not describe neutrino interactions
- MCMC: there is an interacting neutrino mode!

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Conclusions:

- Boltzmann hierarchy has **formally** a much richer structure than approximations by others
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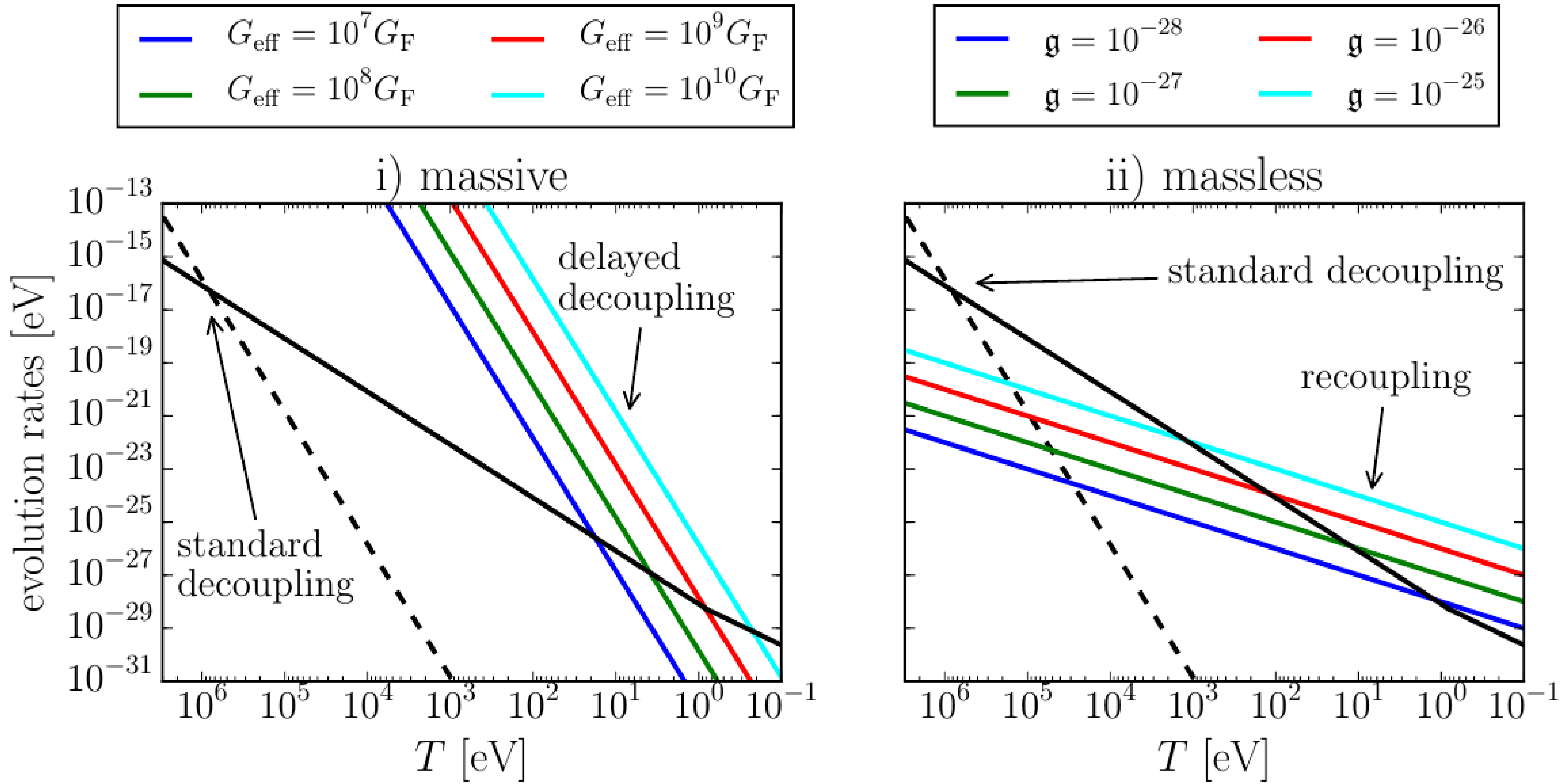
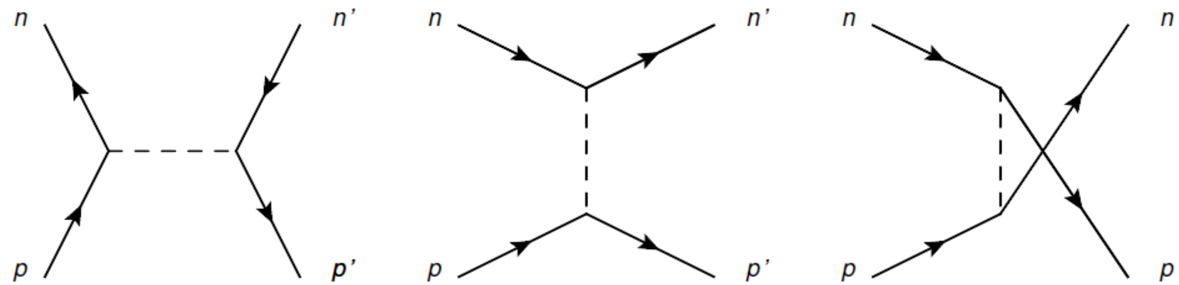
Thank you for your attention!



$$\Gamma_{\rm new} \sim G_{\rm eff}^2 T^5$$

$$\Gamma_{\rm new} \sim gT$$

$\nu\nu \leftrightarrow \nu\nu$:

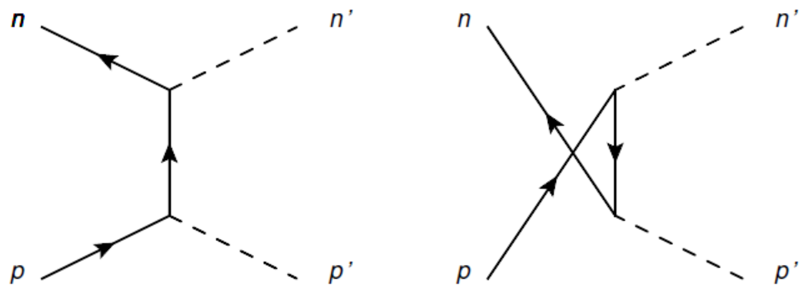


massive case:

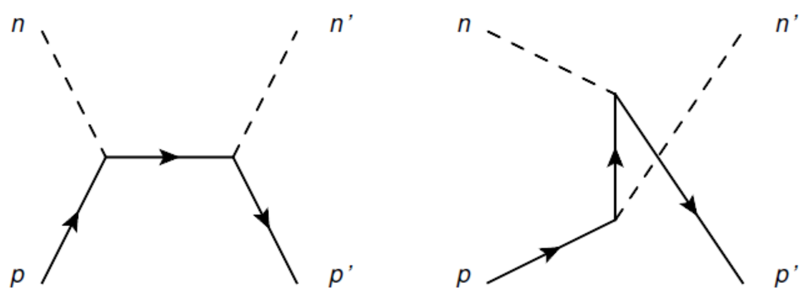
only neutrino

self-interactions

$\nu\nu \leftrightarrow \phi\phi$:



$\nu\phi \leftrightarrow \nu\phi$:



massless case:

need to include new hierarchy for scalar particle as well

recoupling → already at background level out of equilibrium

Ugly integral kernels...: $K_\ell^{\mathrm{m}}(|q|,|q'|) = \int_{-1}^{1} \mathrm{d}\cos\theta \, K^{\mathrm{m}}(|q|,|q'|,\cos\theta)\, P_\ell(\cos\theta)$

where $$K^{\mathrm{m}}(q,q',\cos\theta) \equiv \frac{1}{16P^5}{}^{-(Q_-+P)/(2T_{\nu,0})}\, T_{\nu,0}\left(Q_-^2 - P^2\right)^2 \times \left[P^2\left(3P^2 - 2PT_{\nu,0} - 4T_{\nu,0}^2\right) + Q_+^2\left(P^2 + 6PT_{\nu,0} + 12T_{\nu,0}^2\right)\right]$$

and $\mathrm{P} \equiv |q - q'|,\ Q_\pm \equiv q \pm q'$

Number, energy and momentum conservation

Number: $\int \mathrm{d}q\, q^2 \left(\frac{\partial f_\nu}{\partial \eta}\right)_{\mathrm{coll},\ell=0}(k,q) \overset{!}{=} 0$

Energy: $\int \mathrm{d}q\, q^3 \left(\frac{\partial f_\nu}{\partial \eta}\right)_{\mathrm{coll},\ell=0}(k,q) \overset{!}{=} 0$

Momentum: $\int \mathrm{d}q\, q^3 \left(\frac{\partial f_\nu}{\partial \eta}\right)_{\mathrm{coll},\ell=1}(k,q) \overset{!}{=} 0$

Numerical problems...

$$\dot{\Psi}_{\ell>2}(q) = \frac{k}{2\ell+1}\left[\ell\Psi_{\ell-1}(q) - (\ell+1)\Psi_{\ell+1}(q)\right]$$
$$-\frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, \Psi_\ell(q) + G^{\rm m}\int {\rm d}q'\, 2\frac{q'}{q\bar{f}(q)} K_\ell^{\rm m}(q,q')\, \bar{f}(q')\, \Psi_\ell(q')$$

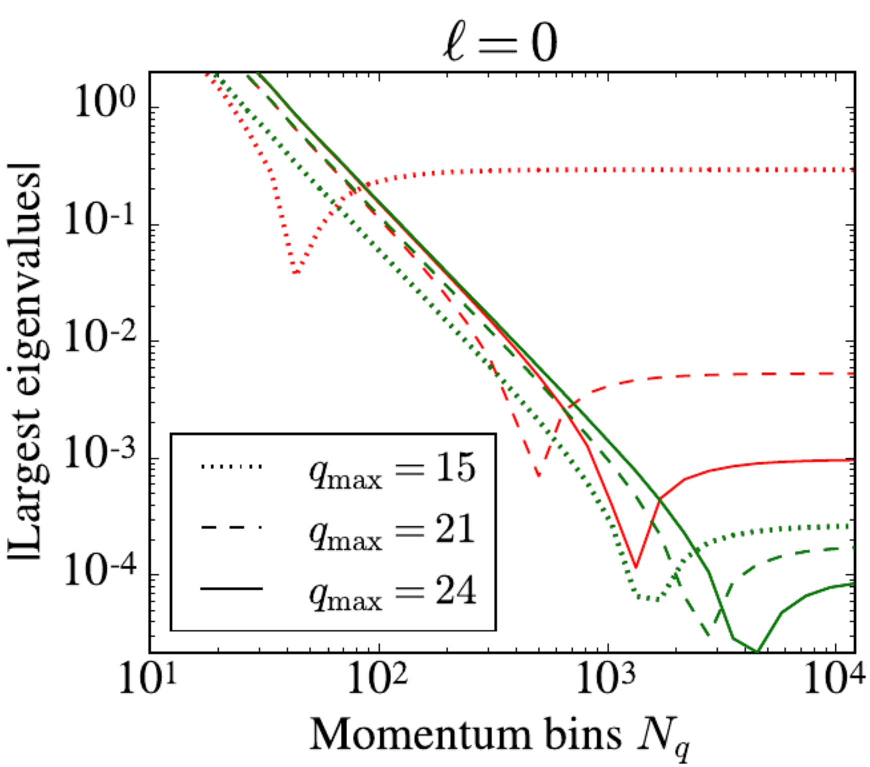
Discretized Boltzmann hierarchy

$$\dot{\Psi}_{\ell,i} = G_{\ell,i} + \sum_j M_{\ell,ij}\Psi_{\ell,j}$$

Homogenous solution: $\boxed{\boldsymbol{\Psi}_\ell^h = \sum_k c_k\, \mathbf{v}_k e^{\lambda_k \tau}}$

Exponential growth for positive eigenvalues!

Finite momentum-grid size → (small) positive eigenvalues...



Solution:

1) Calculate eigenvalues

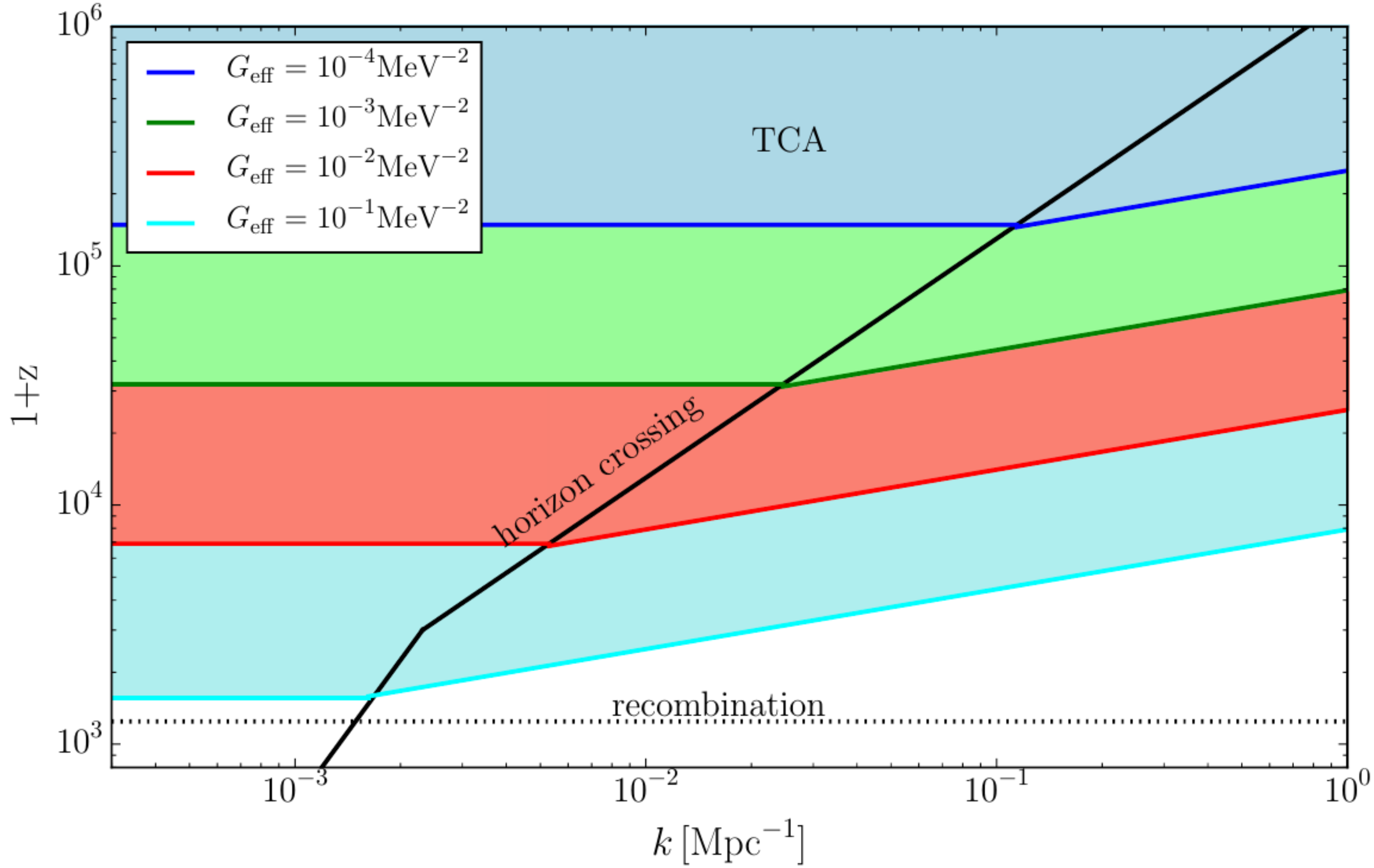
2) Set positive eigenvalues to zero

3) Obtain corrected scattering matrix

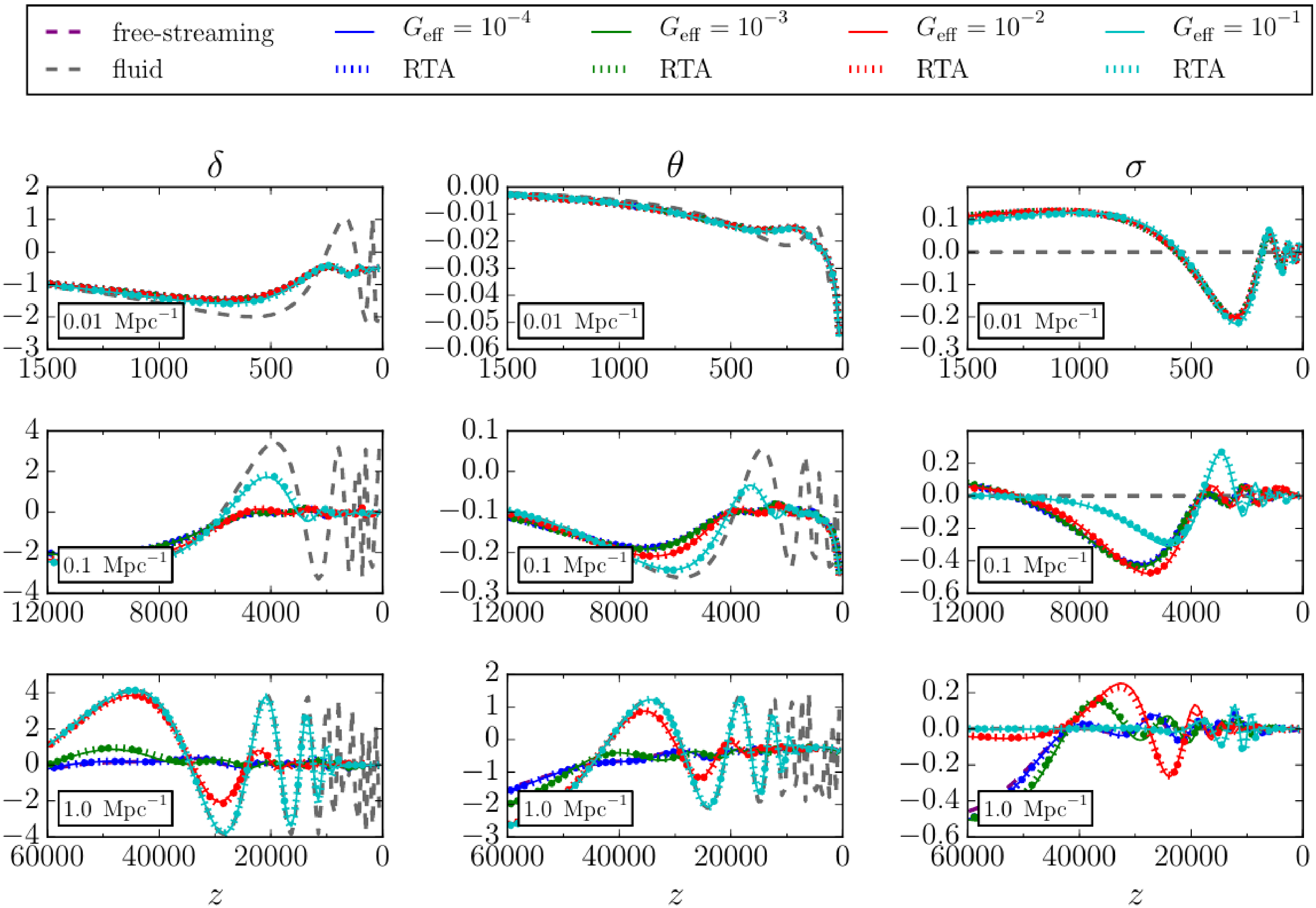
4) Run code only for sufficiently large $q_{\max}$

Tightly coupled limit:

$$\dot{\delta}_\nu = -\tfrac{4}{3}\theta_\nu - \tfrac{2}{3}\dot{h}$$
$$\dot{\theta}_\nu = \tfrac{1}{4}k^2\delta_\nu$$



$$T_{\rm dec} \simeq 7.66\times10^{-2}\left(\frac{{\rm MeV}^{-2}}{G_{\rm eff}}\right)^{2/3}\,{\rm eV} = 0.2\left(\frac{2.03\times10^{10}\,G_{\rm F}}{G_{\rm eff}}\right)^{2/3}\,{\rm eV},\quad {\rm (RD)}$$


free-streaming
fluid
$G_{\rm eff} = 10^{-4}$
RTA
$G_{\rm eff} = 10^{-3}$
RTA
$G_{\rm eff} = 10^{-2}$
RTA
$G_{\rm eff} = 10^{-1}$
RTA
δ
θ
σ
0.01 Mpc^{-1}
0.1 Mpc^{-1}
1.0 Mpc^{-1}
z