Axion excursions of the landscape

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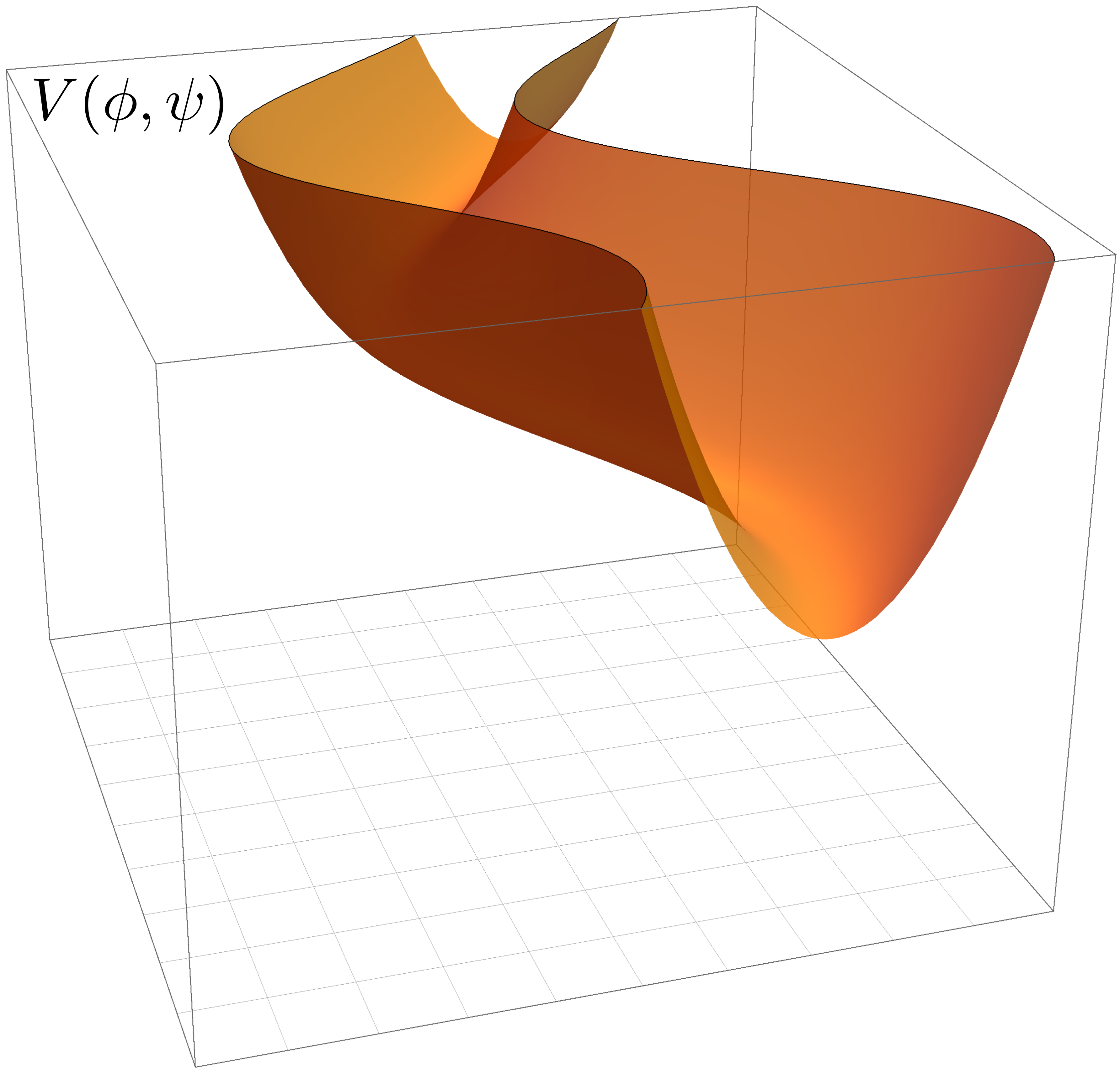
Motivation and idea



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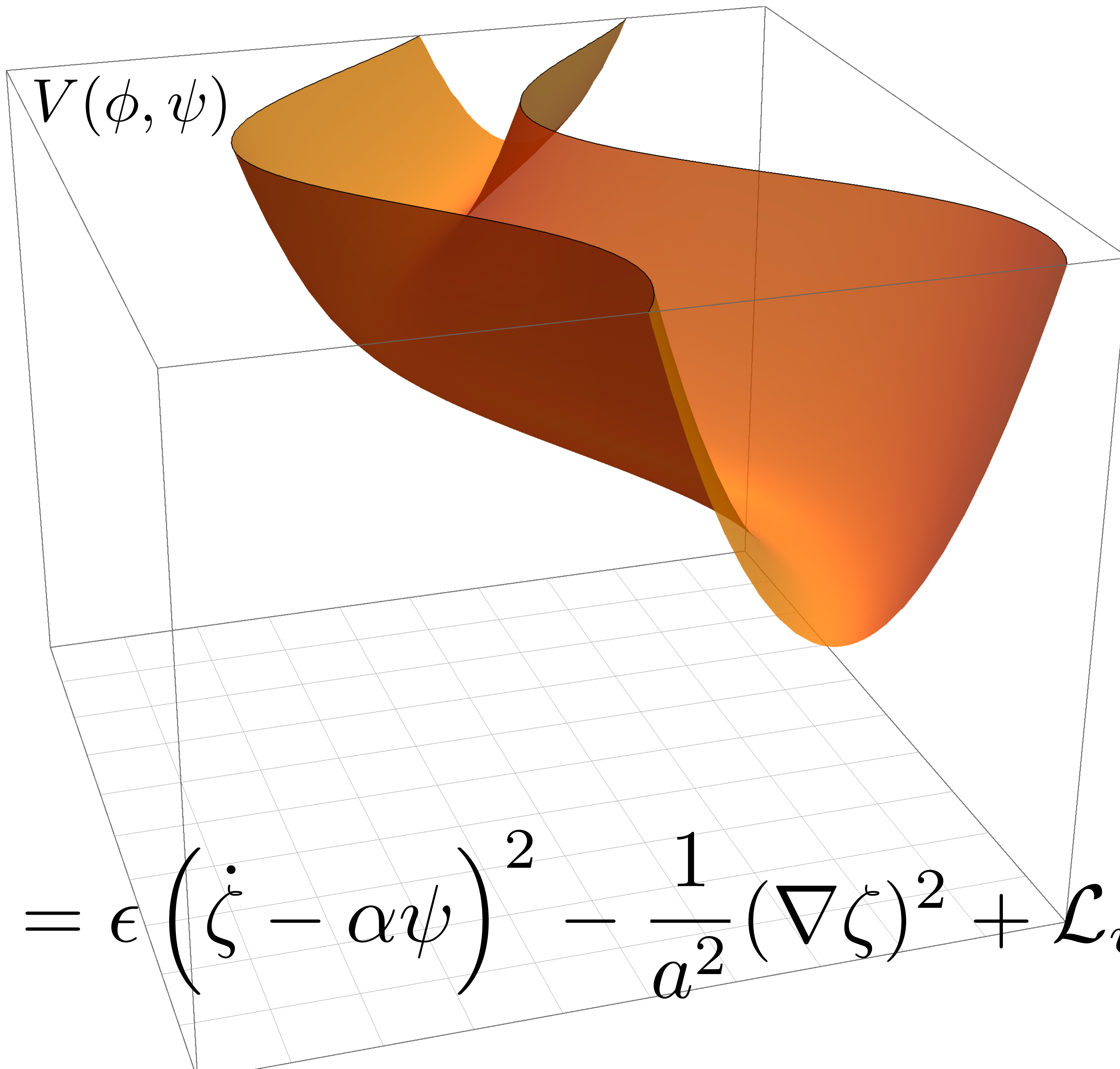
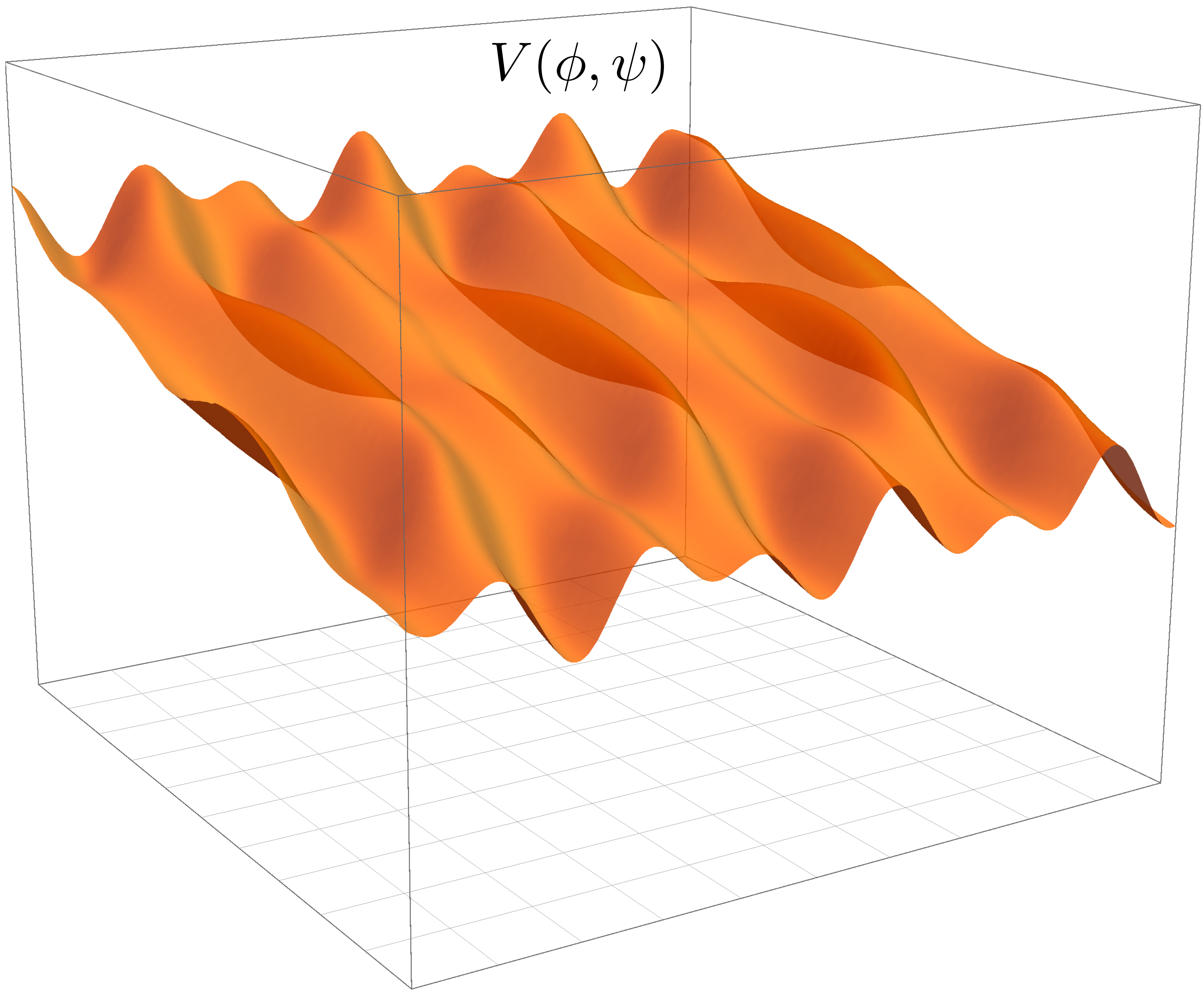


$$\mathcal{L} = \epsilon\left(\dot{\zeta} - \alpha\psi\right)^2 - \frac{1}{a^2}(\nabla\zeta)^2 + \mathcal{L}_\psi$$

Motivation and idea



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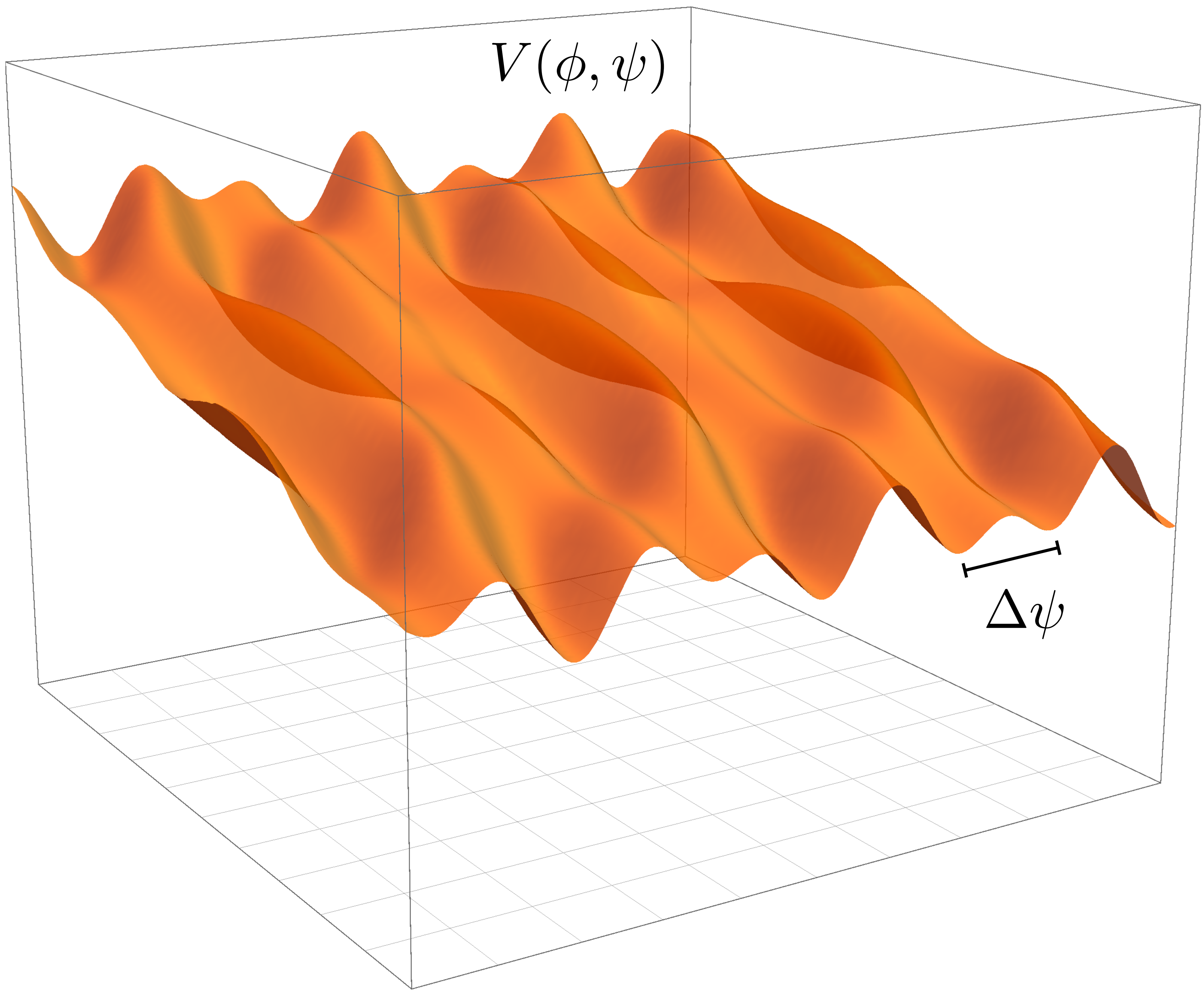

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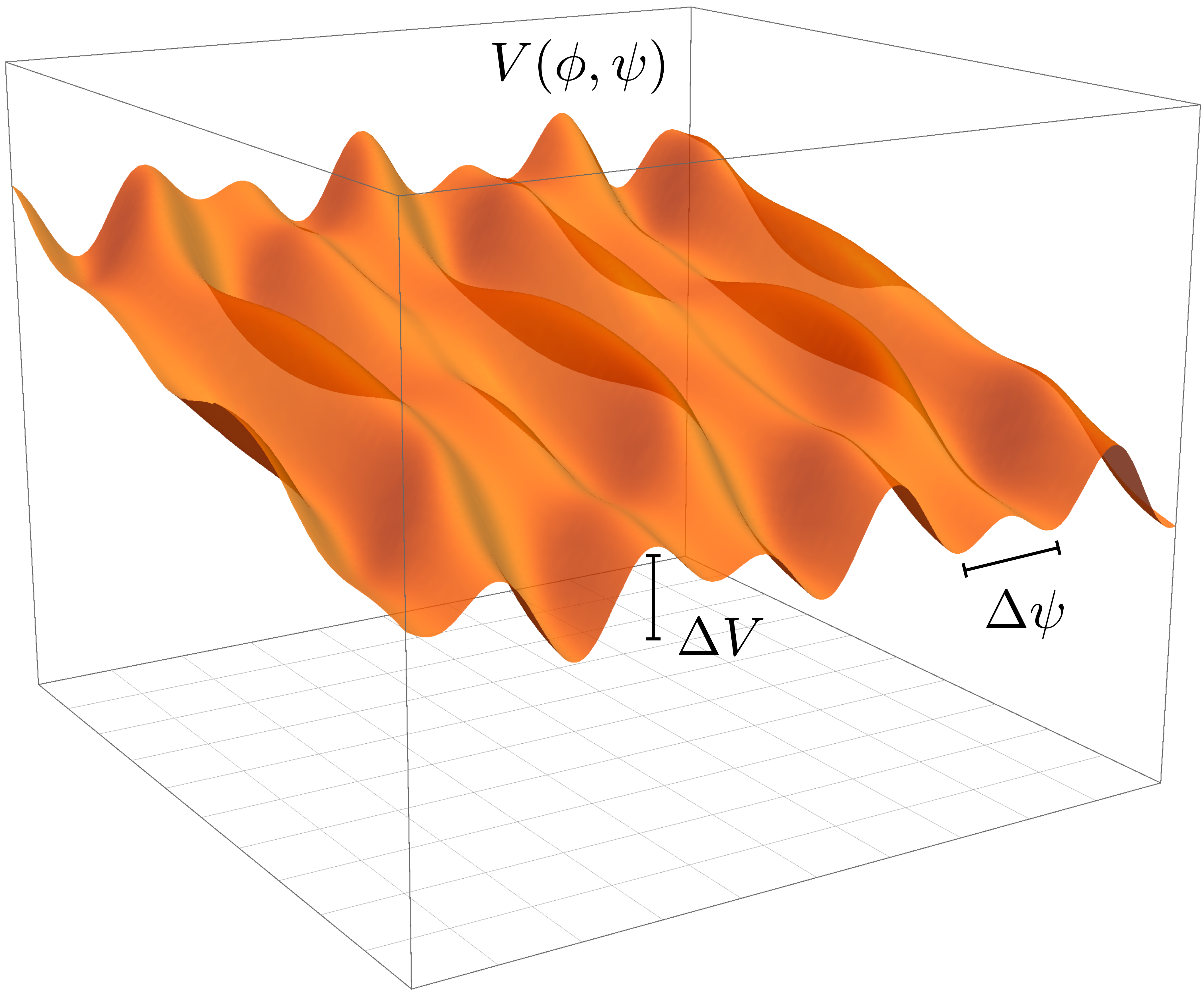

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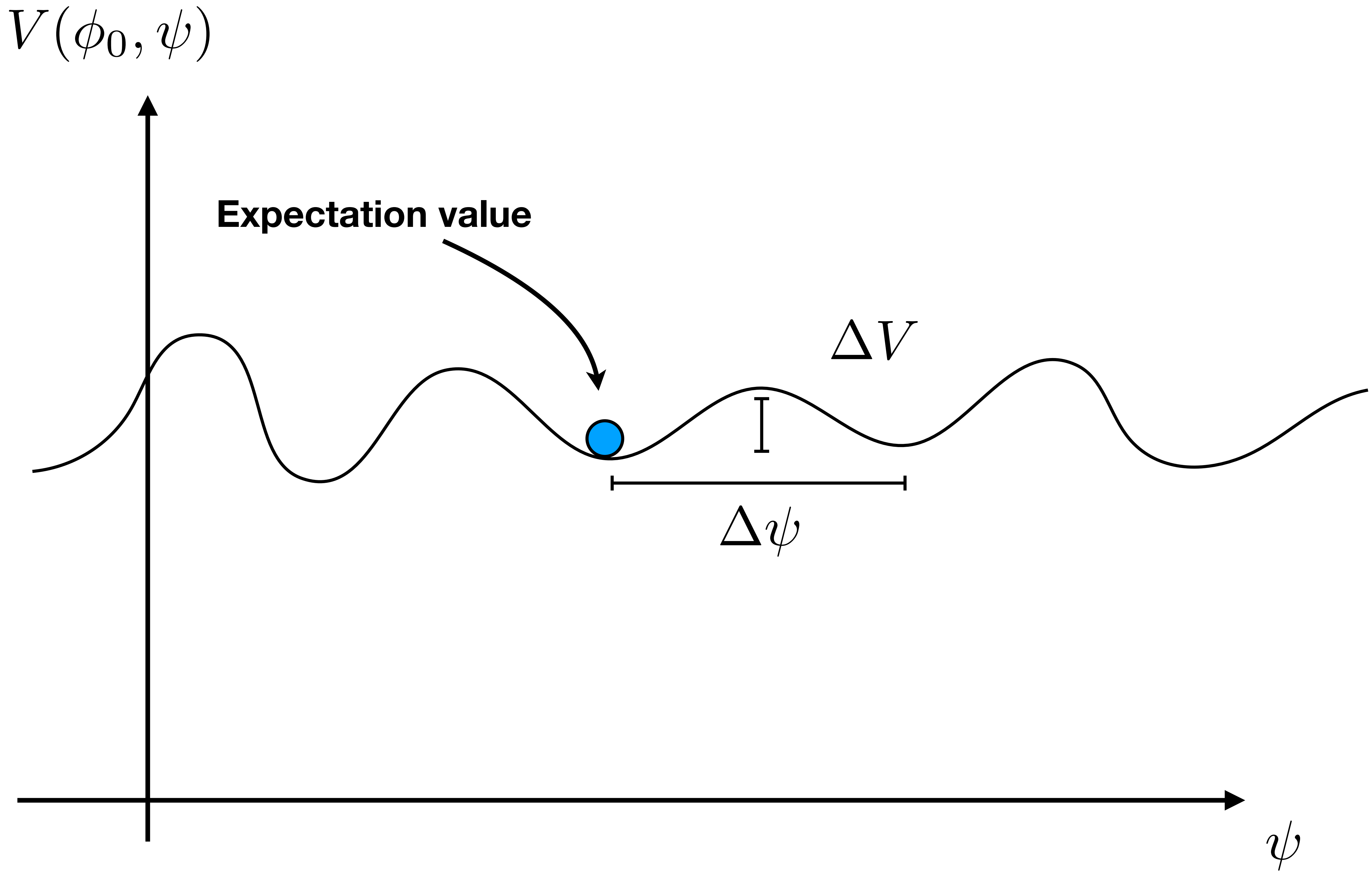

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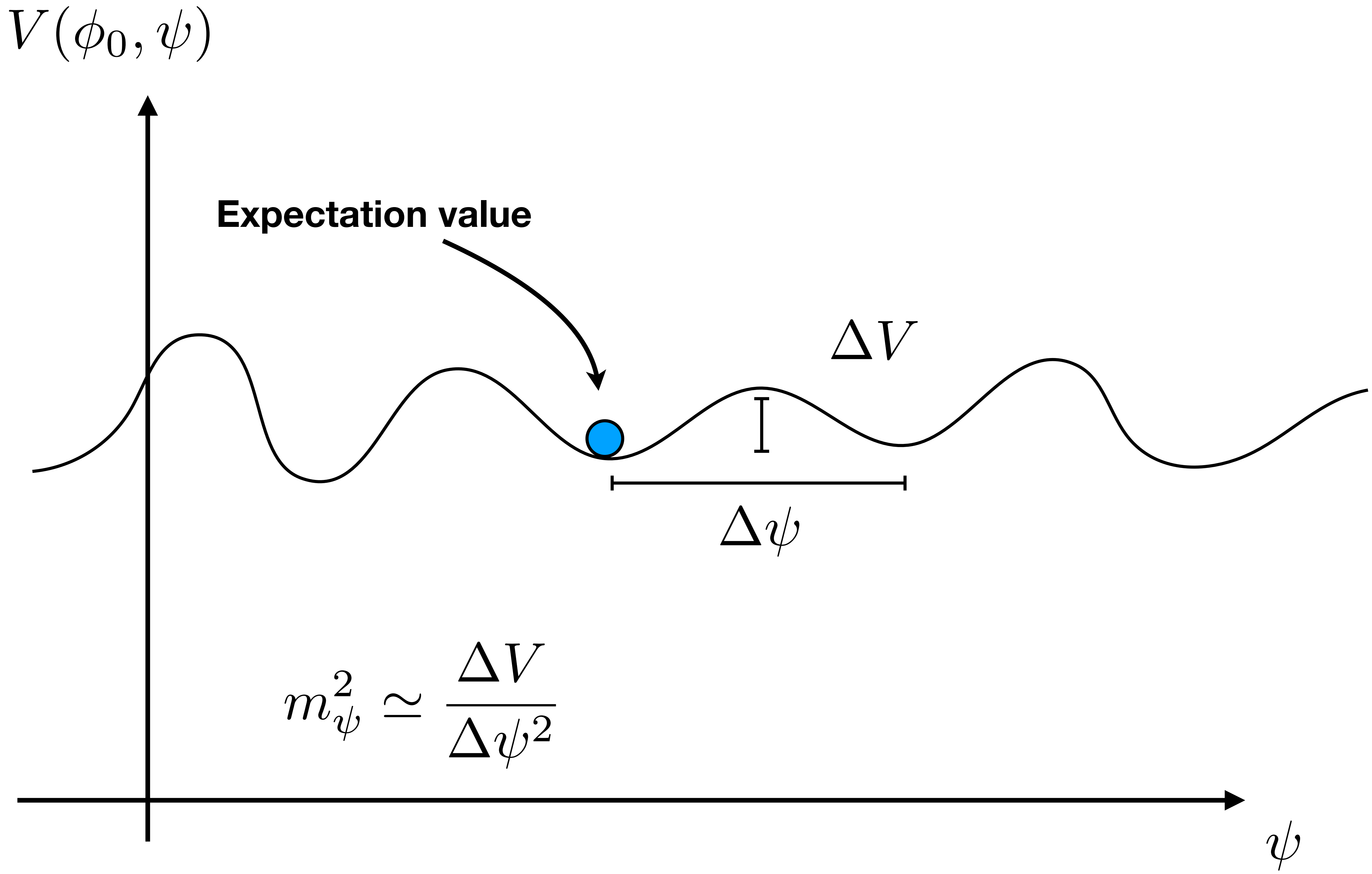
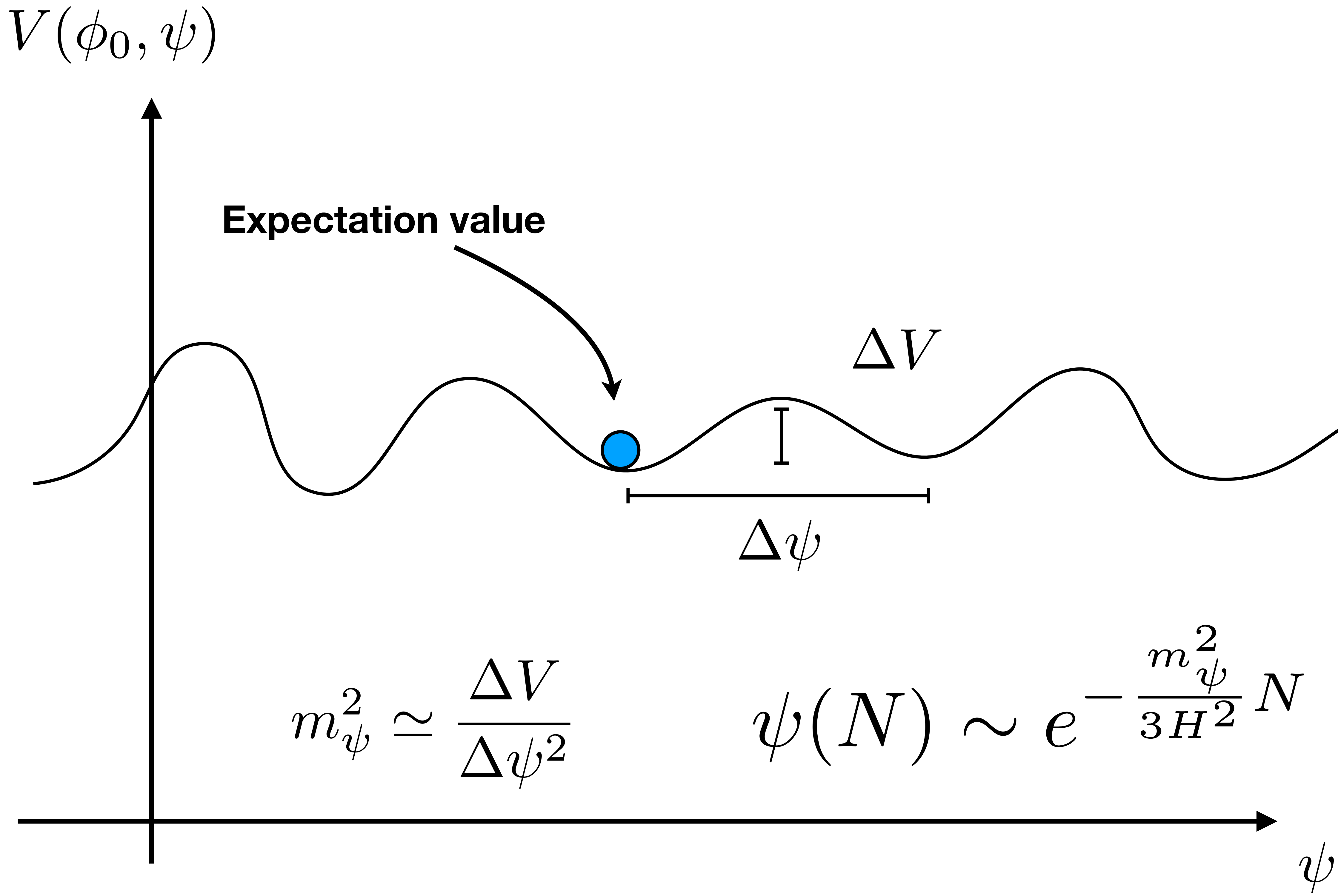
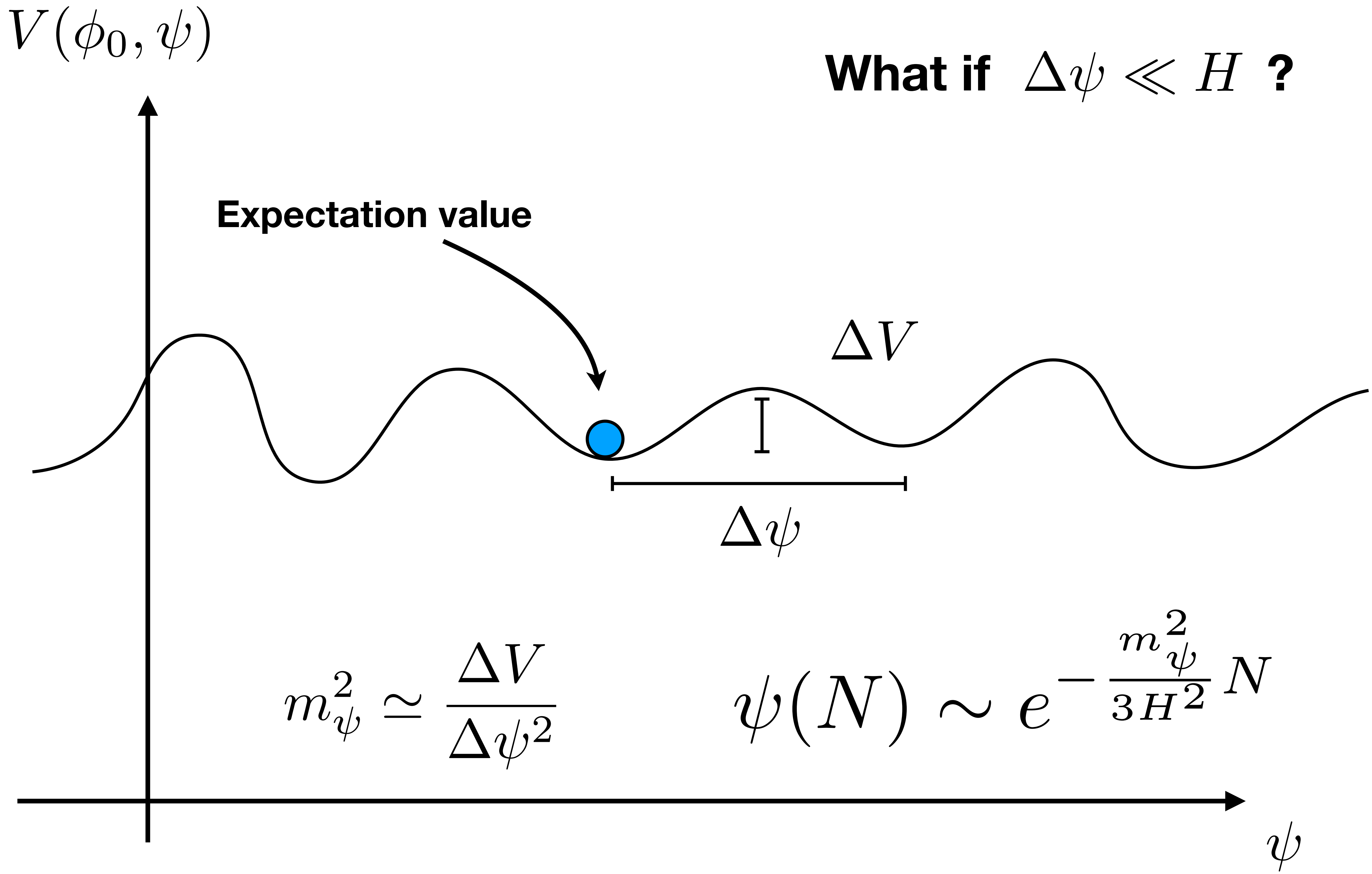


Motivation and idea



Motivation and idea

What if $\Delta\psi \ll H$?



$$m_\psi^2 \simeq \frac{\Delta V}{\Delta\psi^2}$$

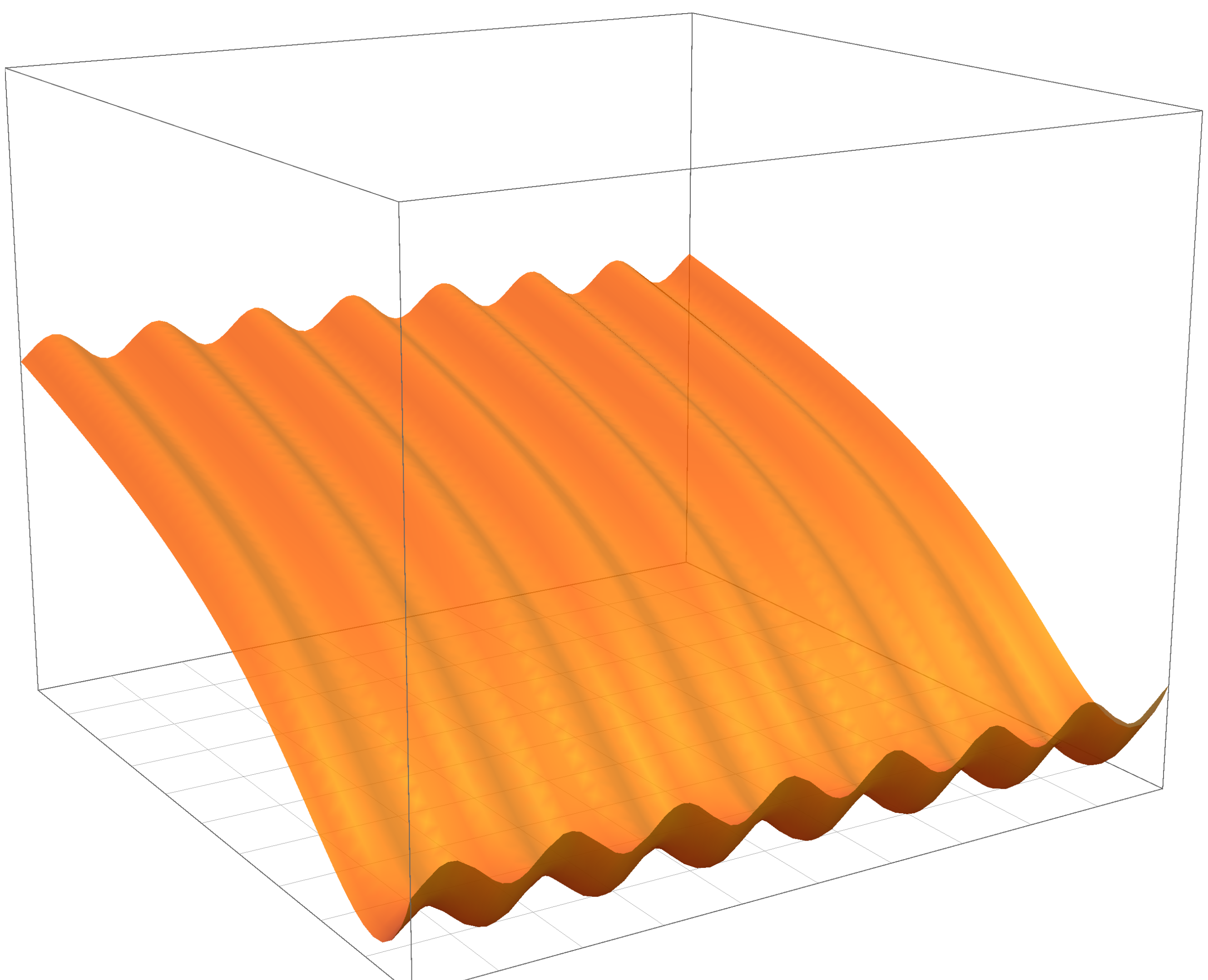
$$\psi(N) \sim e^{-\frac{m_\psi^2}{3H^2}N}$$

Axions in inflation

For simplicity I will consider an axion like potential

$$V(\phi,\psi) = V_0(\phi) + \Lambda^4\left[1-\cos\left(\frac{\psi}{f}\right)\right]$$

$V_0(\phi) =$ **Your favorite inflationary potential**



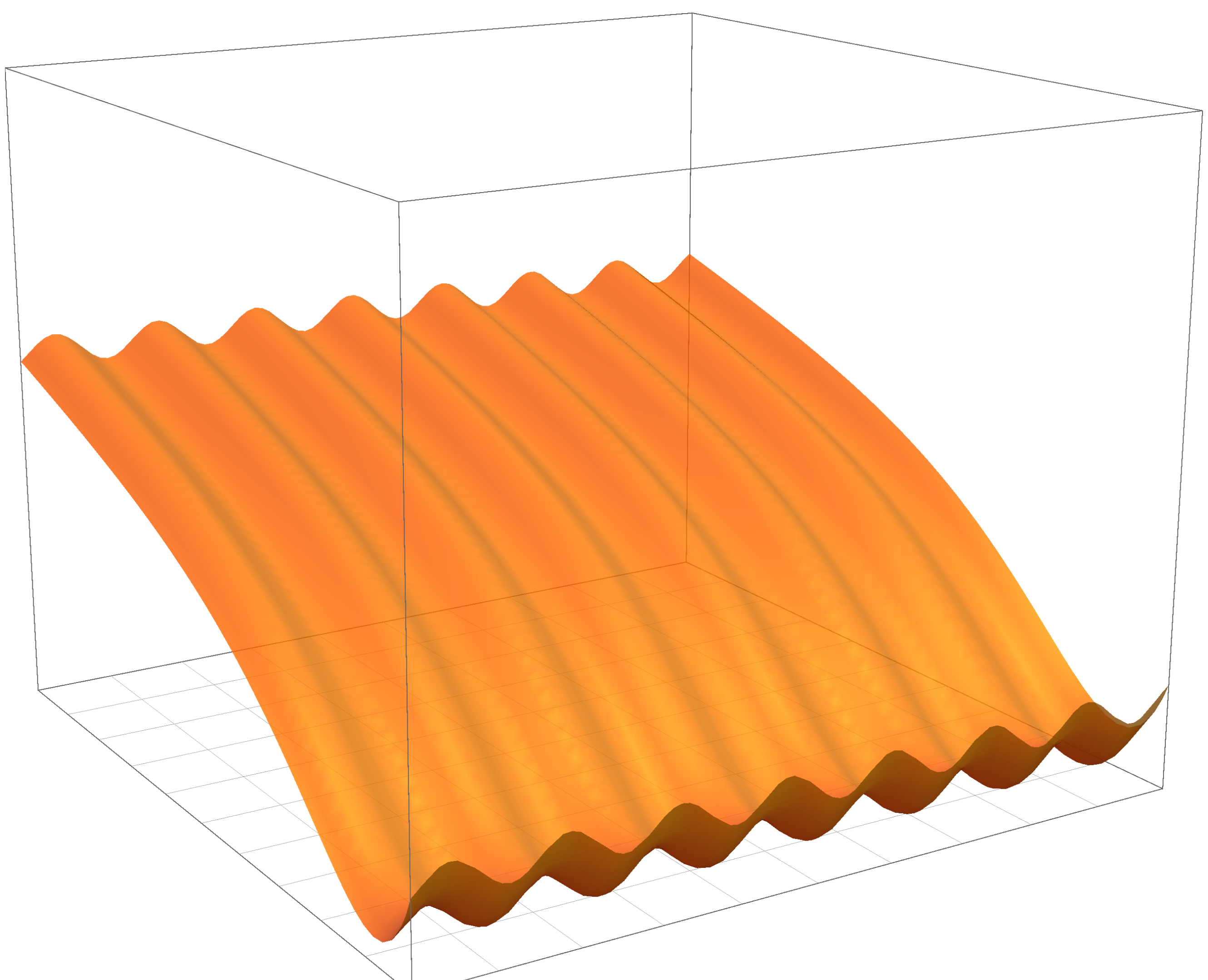
$$\Delta V = \Lambda^4$$

$$\Delta \psi = f$$

Axions in inflation

$$S_\psi = \int d^3x\, dt\, a^3 \left(\frac{1}{2}\dot{\psi}^2 + \frac{1}{2a^2}(\nabla\psi)^2 - \Lambda^4 \left[1 - \cos\left(\frac{\psi}{f}\right)\right]\right)$$

This is just an axion field in a de Sitter spacetime



We have an additional parameter:

$$H = \frac{\dot{a}}{a} \simeq \quad \textbf{Constant}$$

Linde (1991), Lyth & Stewart (1992)

Axions in inflation

A realistic axion model requires some care:

$$V(r,\psi) = \lambda(r^2 - f^2)^2 + \Lambda^4\left[1 - \cos\left(\frac{\psi}{f}\right)\right]$$

To have perturbative control one needs $\lambda \ll 1$

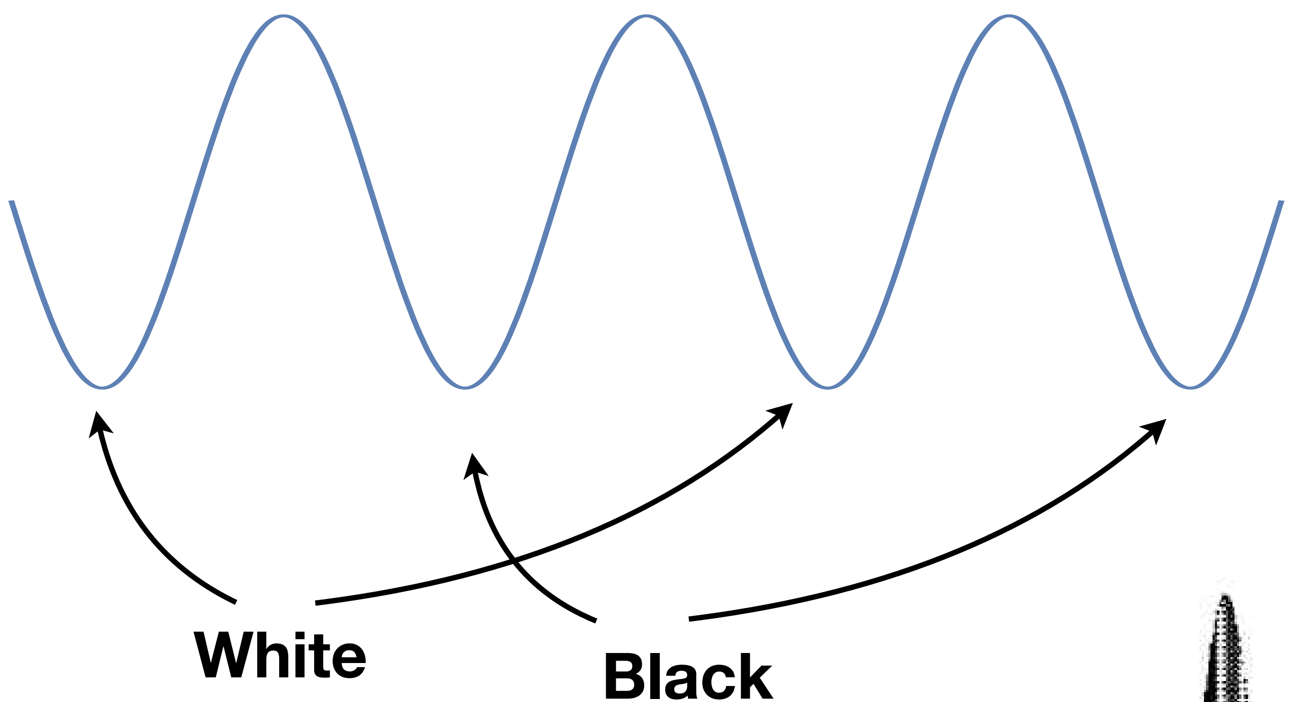
Then, if $H > f$ **the** r **field fluctuates during inflation**

It generates a potential domain wall problem

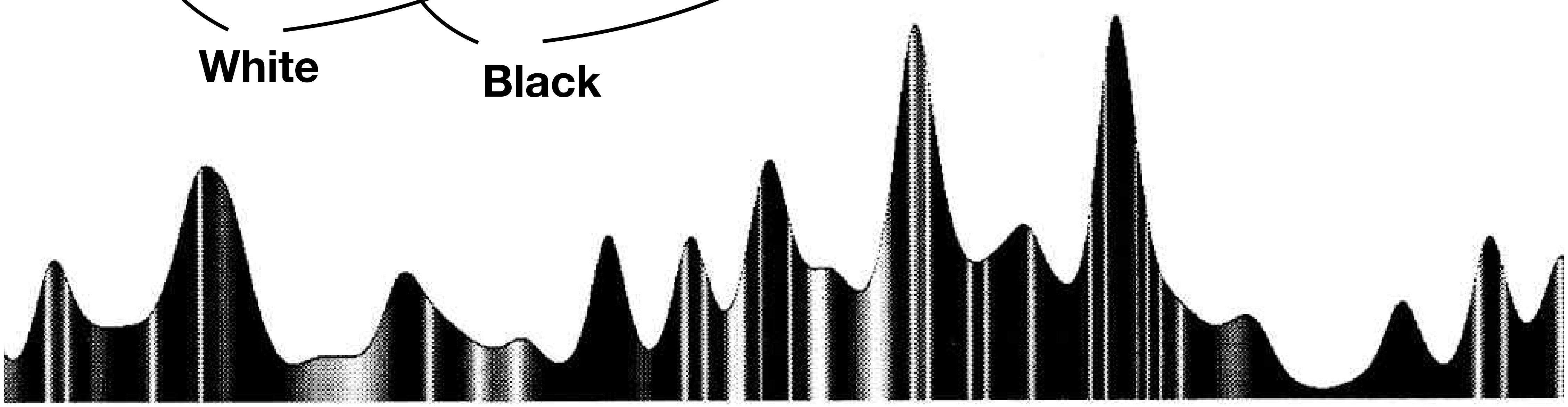
Lyth & Stewart (1992)

Axions in inflation

$$S_\psi = \int d^3x\, dt\, a^3 \left(\frac{1}{2}\dot{\psi}^2 + \frac{1}{2a^2}(\nabla\psi)^2 - \Lambda^4 \left[1 - \cos\left(\frac{\psi}{f}\right) \right] \right)$$

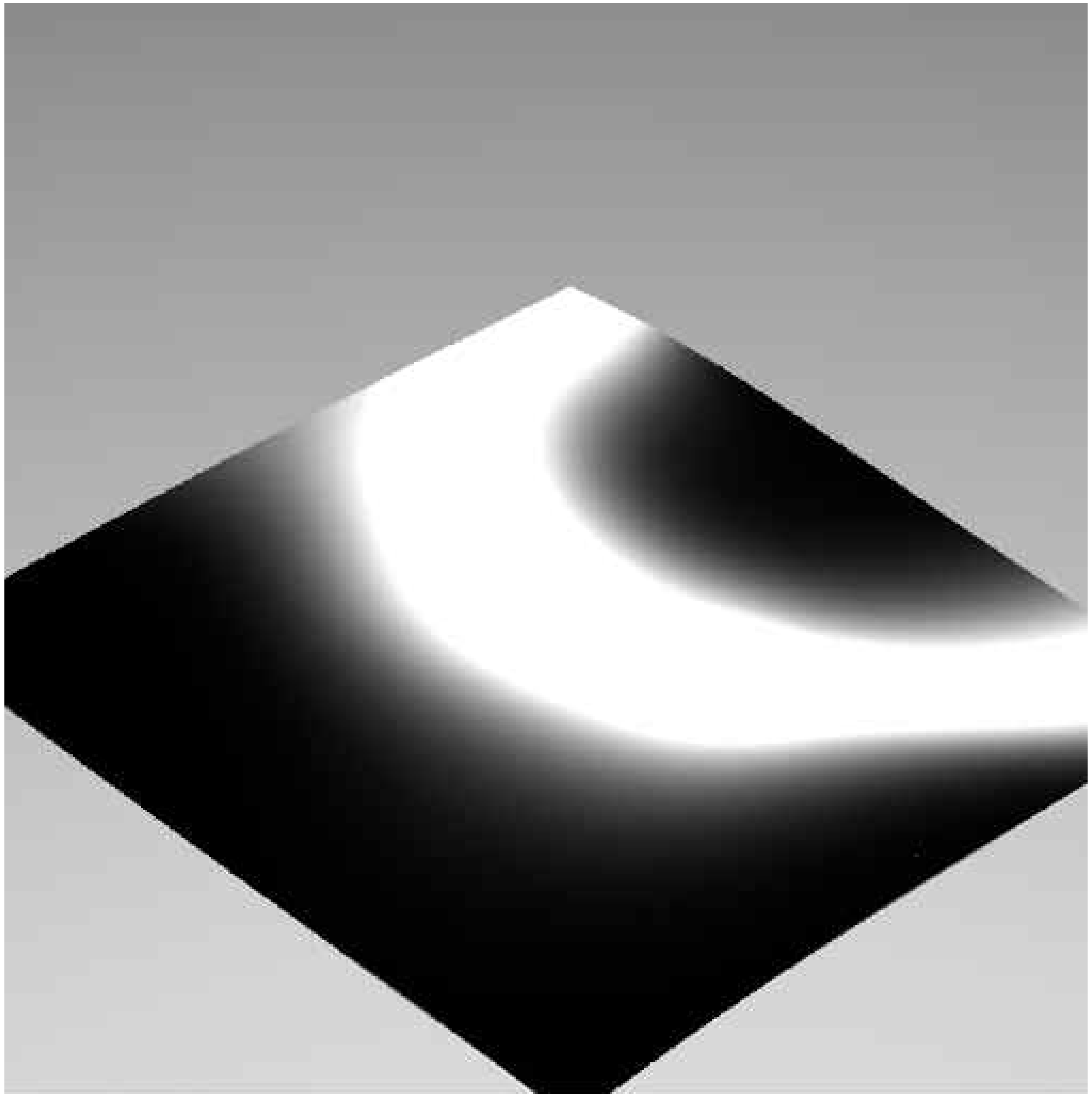


$$N_{DW} = 2$$



Linde, Linde & Mezhlumian (1993)

Axions in inflation



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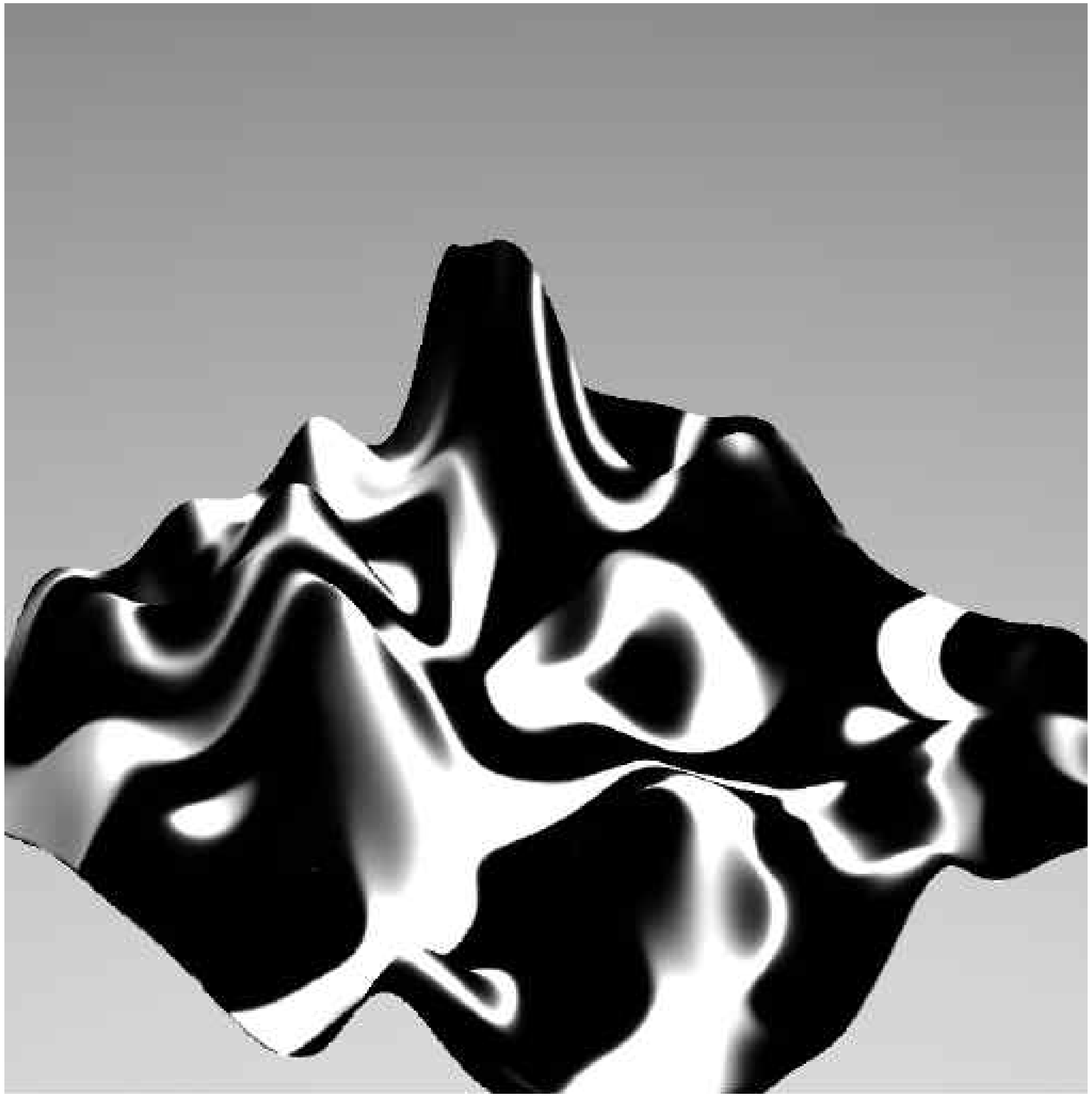

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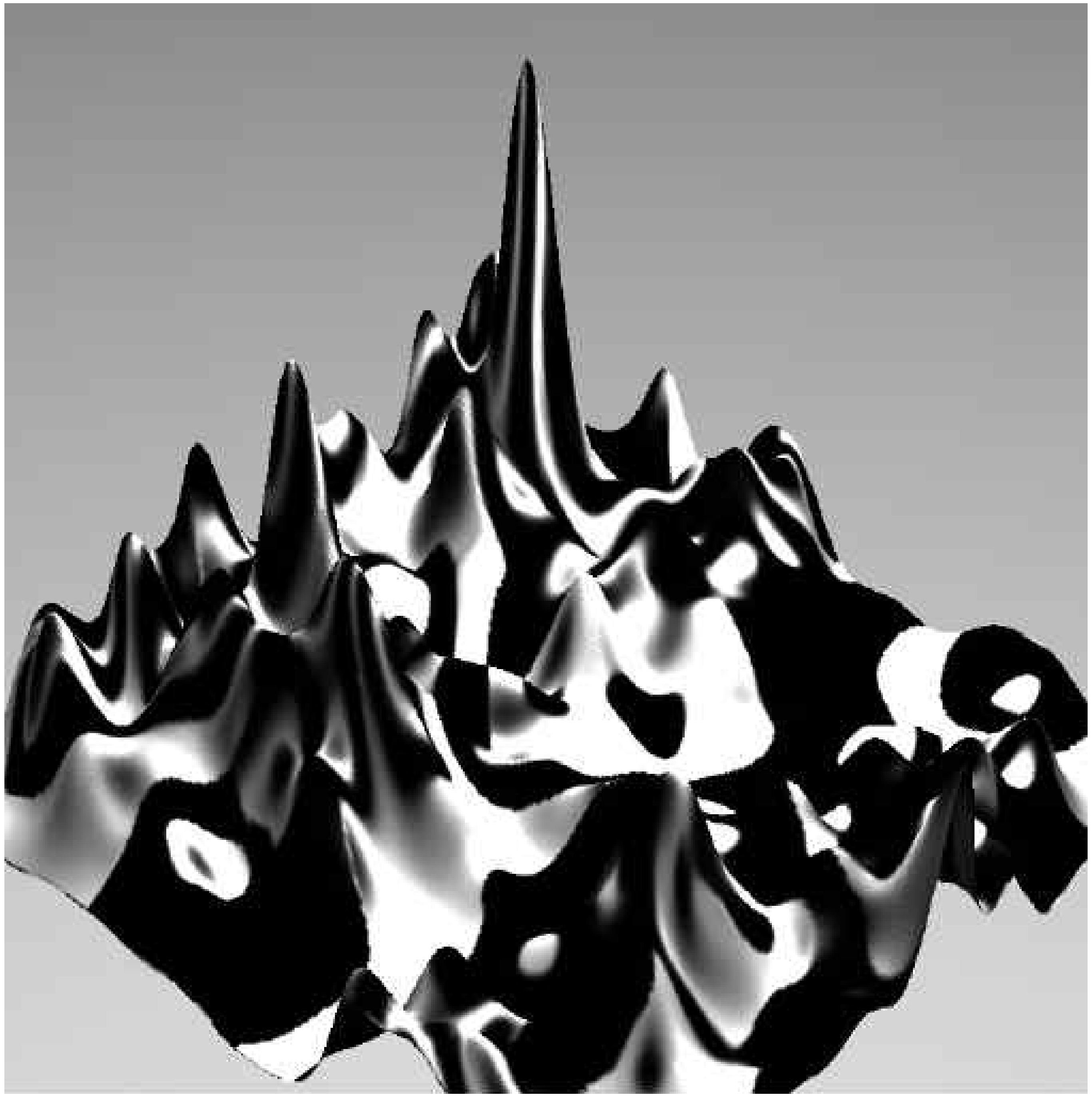

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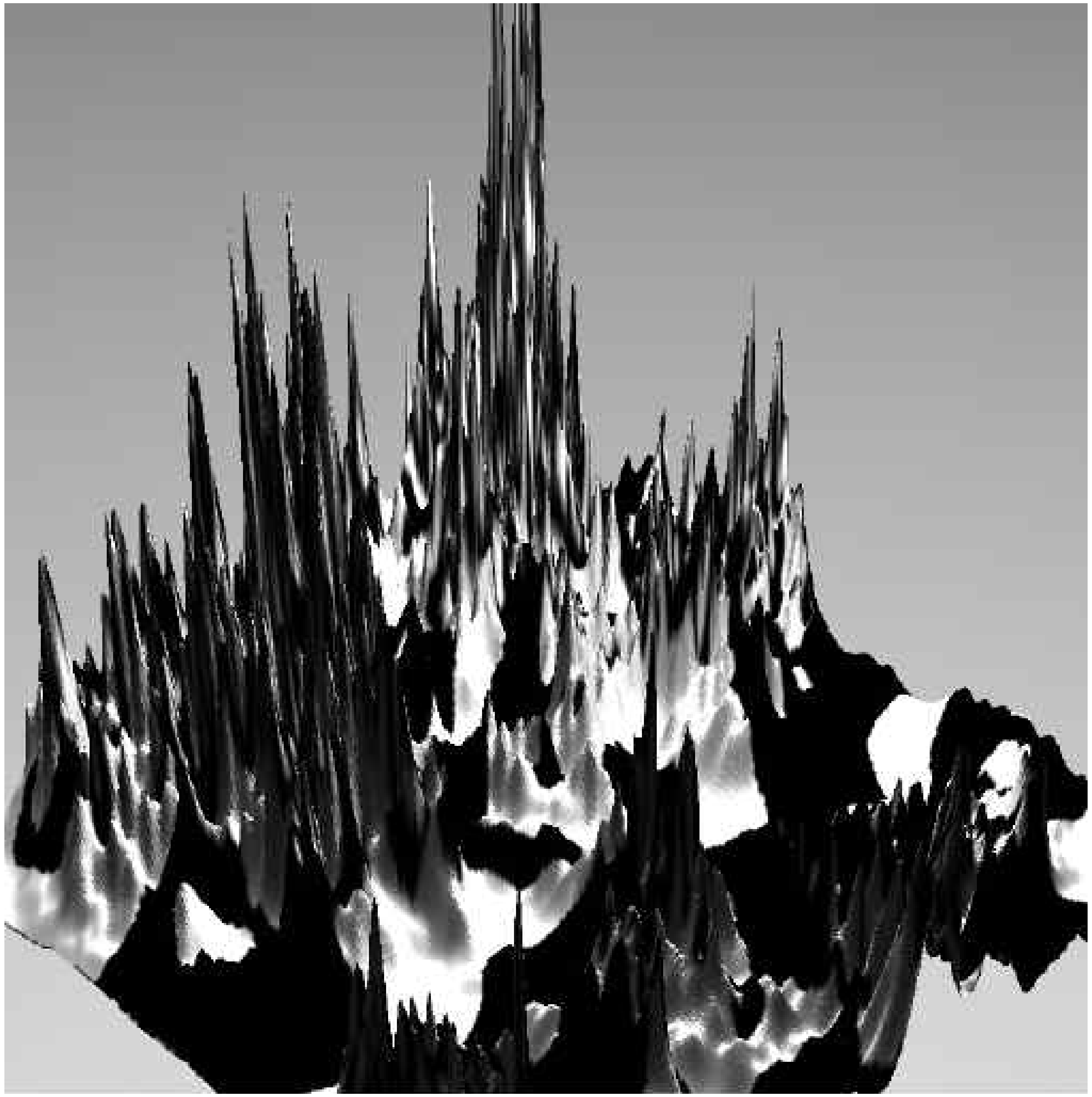


Linde, Linde & Mezhlumian (1993)

In-in formalism

$$S_\psi = \int d^3x\, dt\, a^3 \left[\frac{1}{2}\dot{\psi}^2 + \frac{1}{2a^2}(\nabla\psi)^2 - v(\psi)\right]$$

$$v(\psi) \equiv \Lambda^4 \left[1 - \cos\left(\frac{\psi}{f}\right)\right]$$

$$u = a\psi \qquad a = -\frac{1}{H\tau}$$

$$S_\psi = \int d^3x\, d\tau \left[\frac{1}{2}(u')^2 + \frac{1}{2}(\nabla u)^2 - a^4 v(u/a)\right]$$

I want to use the in-in formalism to compute correlations

$$\langle u(\mathbf{x}_1,\tau)\cdots u(\mathbf{x}_n,\tau)\rangle = \langle 0|U^\dagger u_I(\mathbf{x}_1,\tau)\cdots u_I(\mathbf{x}_n,\tau)U|0\rangle$$

$$u_I(\mathbf{x},\tau) = \int_k \hat{u}_I(\mathbf{k},\tau)e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$\hat{u}_I(\mathbf{k},\tau) \equiv a_{\mathbf{k}}u_k^I(\tau) + a^\dagger_{-\mathbf{k}}u_k^{I*}(\tau)$$

$$U(\tau) = \mathcal{T}\exp\left\{-i\int_{-\infty^+}^{\tau} d\tau' H_I(\tau')\right\}$$

I want to use the in-in formalism to compute correlations

$$1 - \cos(\psi) = -\sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \left(\frac{H\tau}{f} u_I \right)^{2m}$$

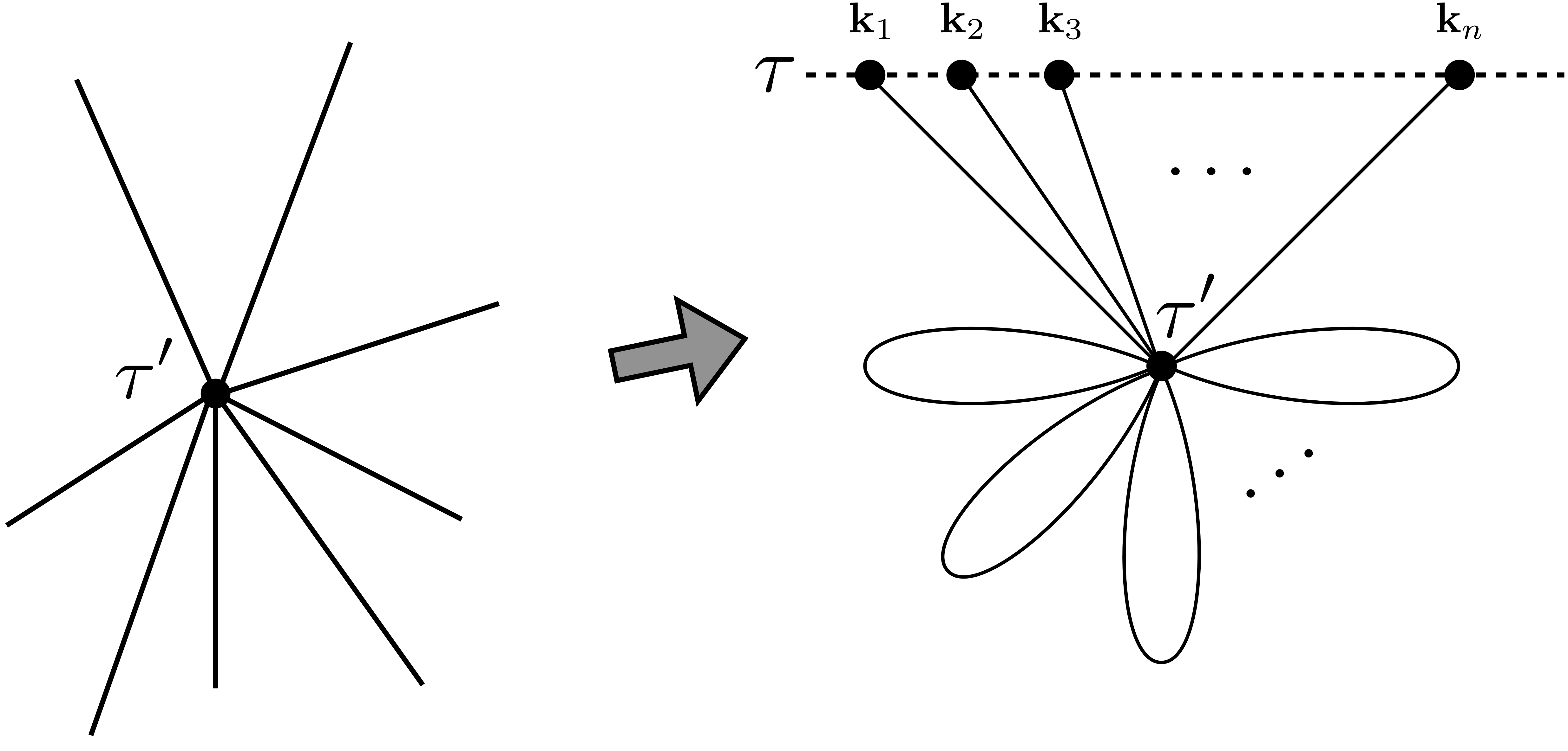
$$H_I(\tau) = -\frac{\Lambda^4}{H^4 \tau^4} \sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!} \int_z \left(\frac{H\tau}{f} u_I(\mathbf{z}, \tau) \right)^{2m}$$

$$\Delta(\tau', \tau, k) \equiv u_k^I(\tau') u_k^{I*}(\tau)$$

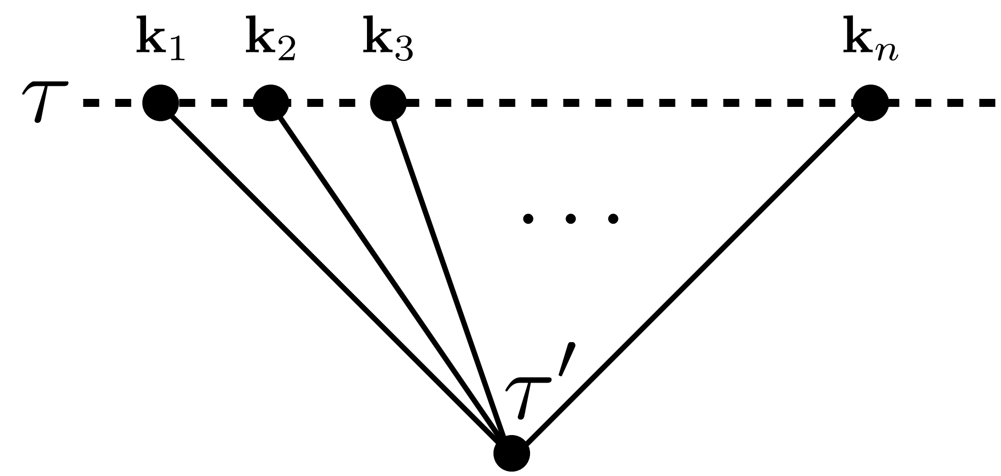
In-in formalism

Only one type of vertex at order Λ^4

$$\propto -\frac{\Lambda^4}{H^4\tau^4}\frac{(-1)^m}{(2m)!}\left(\frac{H\tau}{f}\right)^{2m}$$

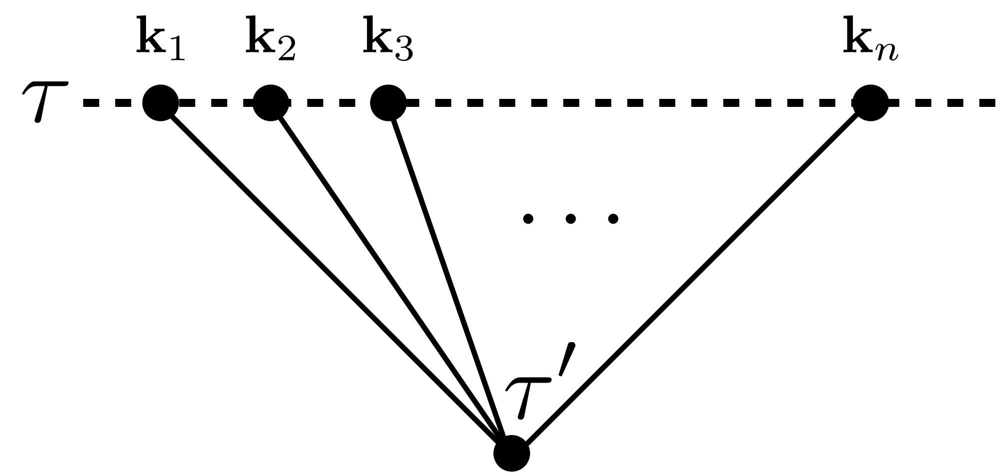


$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c =$$



In-in formalism

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c =$$



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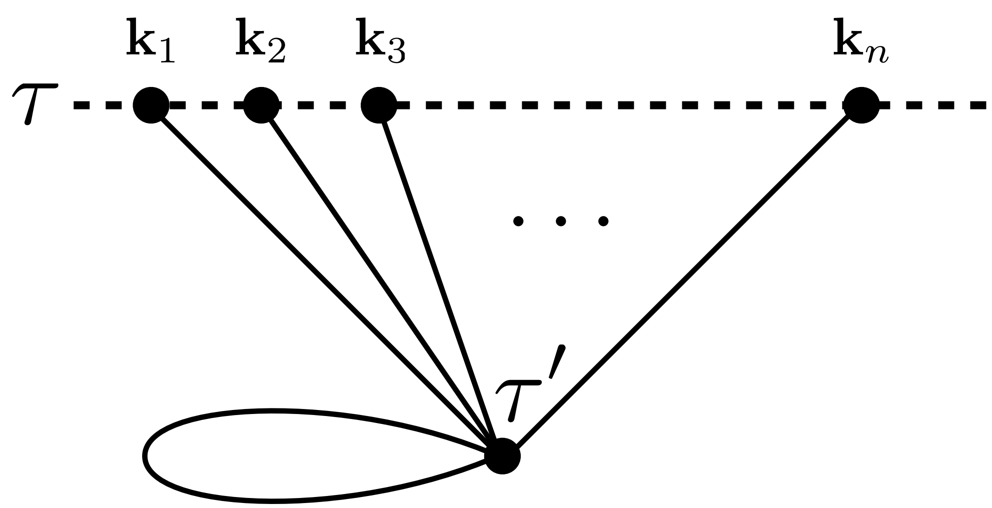
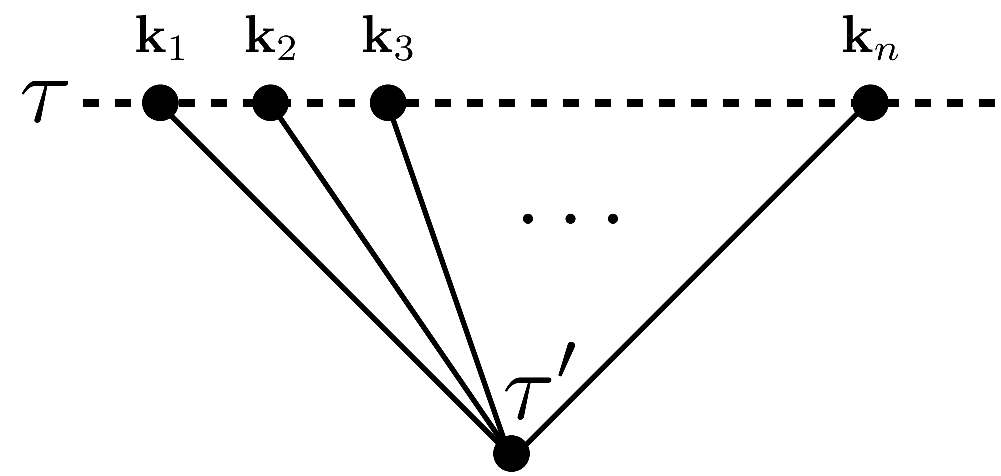


$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c =$$



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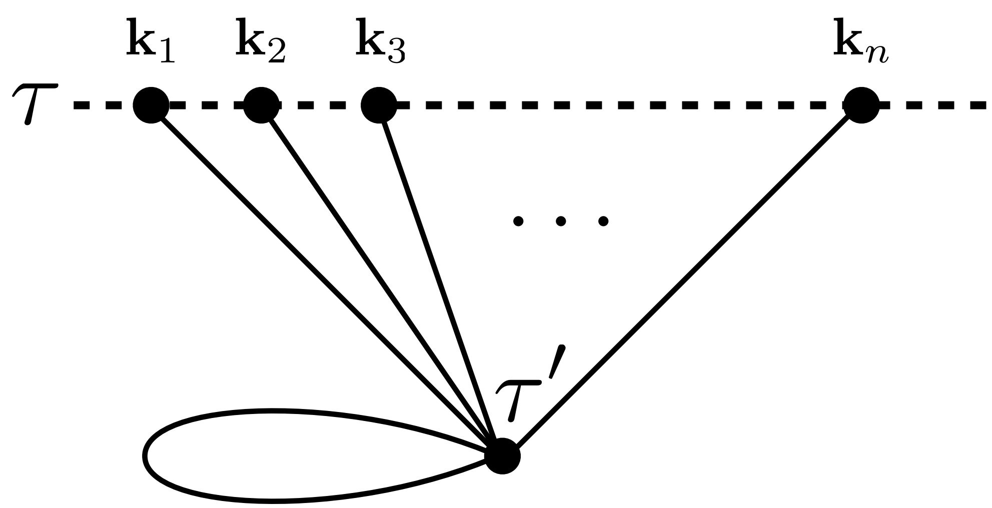
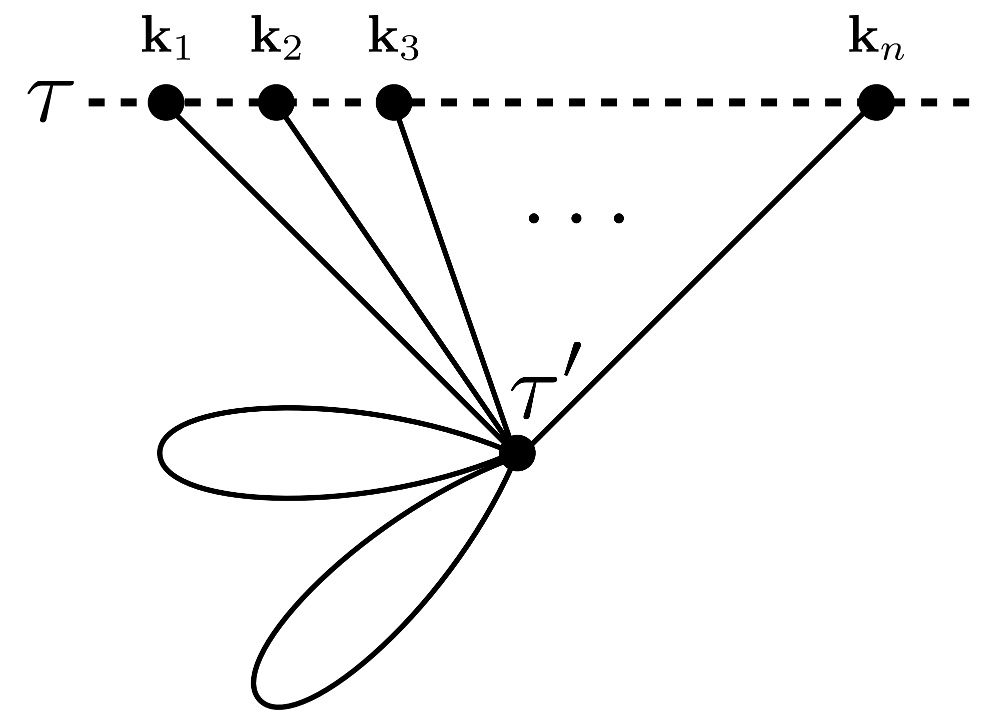
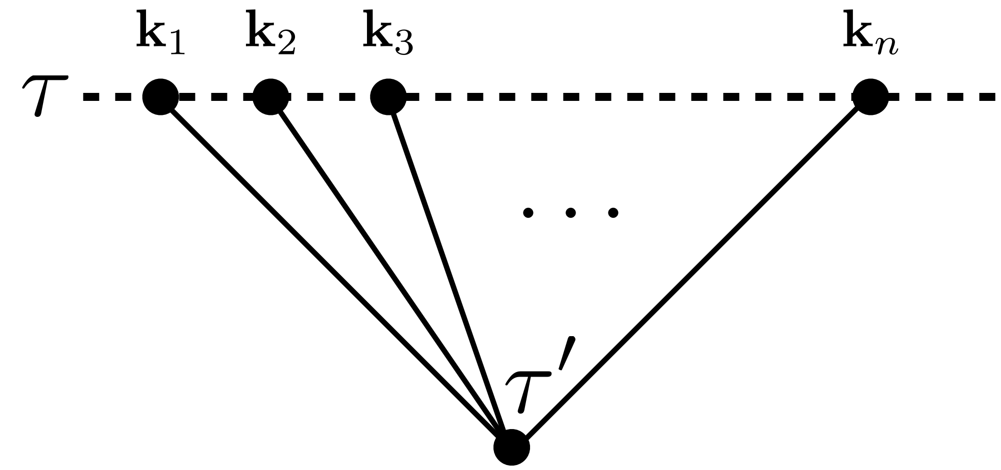


$$+$$



In-in formalism

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c =$$



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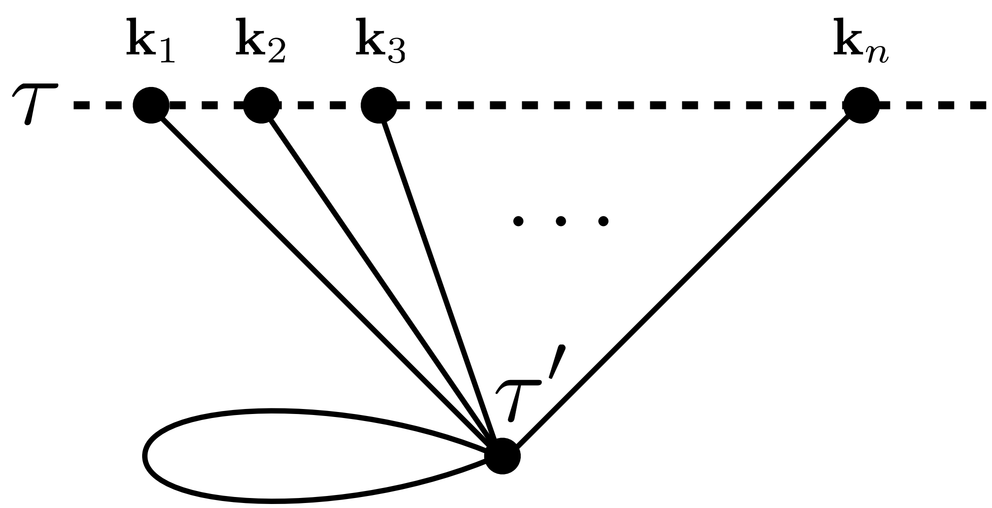

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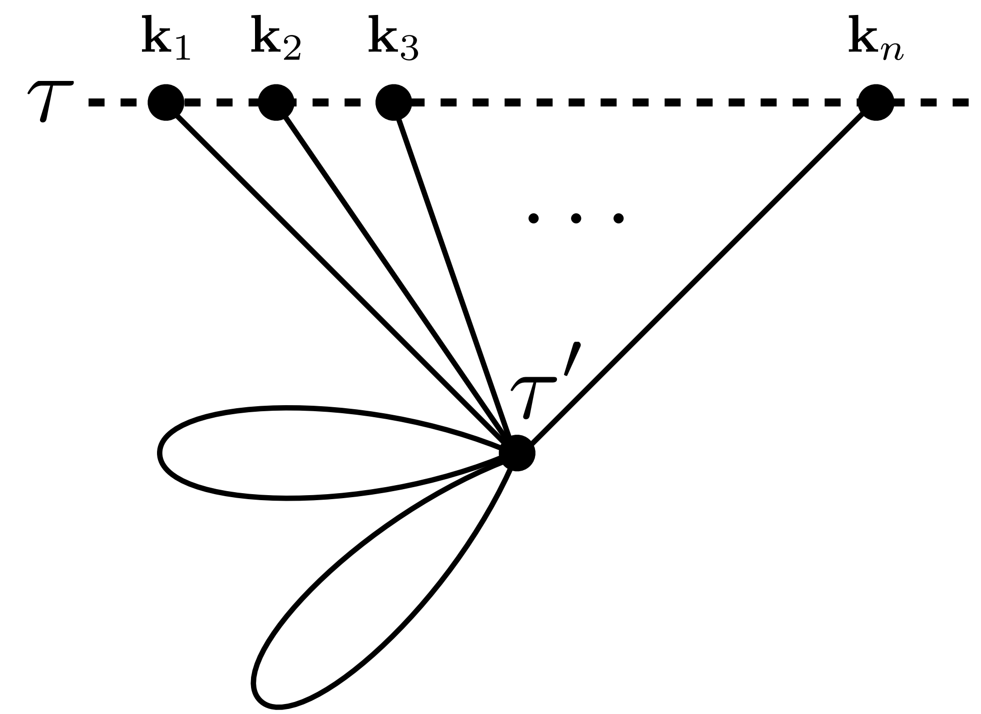

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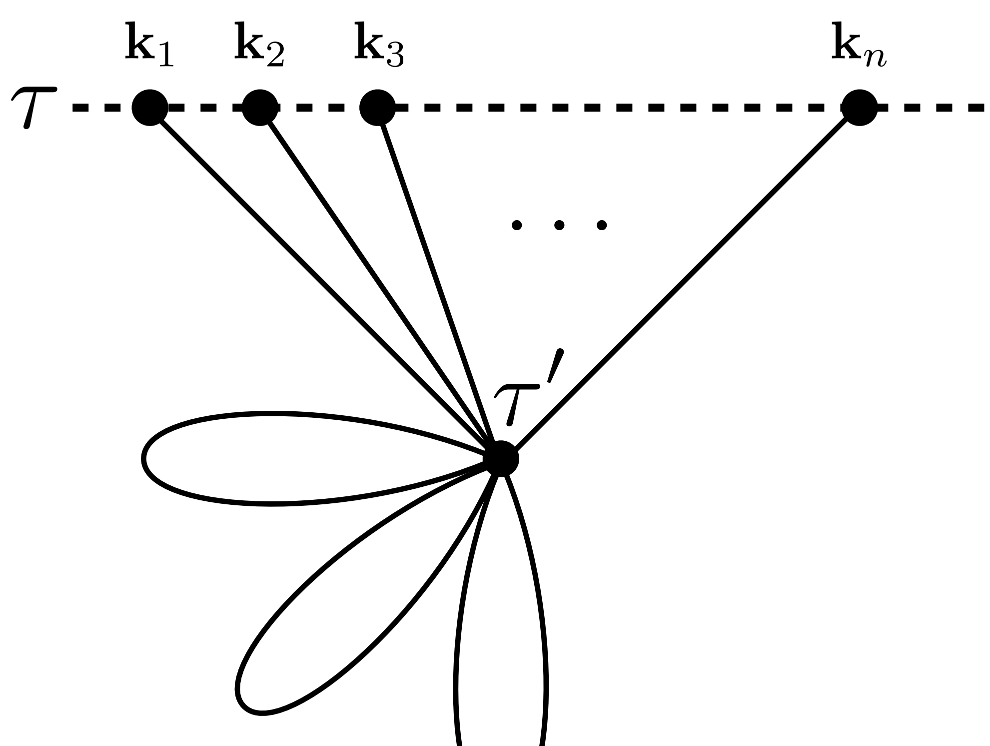

$+\cdots$

In-in formalism

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}\frac{\Lambda^4}{H^4}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big)$$

$$\sum_{m=n/2}^{\infty}\frac{1}{(m-n/2)!}\left[-\frac{1}{2}\left(\frac{H\tau'}{f}\right)^2\int_k \Delta(\tau',\tau',k)\right]^{m-n/2}$$

$$\int_{-\infty}^{\tau} d\tau'\frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\cdots k_n).$$

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}\frac{\Lambda^4}{H^4}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big)$$

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$$\int_{-\infty}^{\tau} d\tau' \frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\cdots k_n).$$

$$\sum_{m'}\frac{1}{m'!}\left[-\frac{1}{2}\left(\frac{H\tau'}{f}\right)^2\int_k \Delta(\tau',\tau',k)\right]^{m'} = e^{-\frac{\sigma_0^2}{2f^2}}$$

$$\sigma_0^2 = H^2\tau^2\int_k \Delta(\tau,\tau,k)$$

In-in formalism

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}\frac{\Lambda^4}{H^4}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big)$$

$$e^{-\frac{\sigma_0^2}{2f^2}}\int_{-\infty}^{\tau} d\tau' \frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\cdots k_n).$$

$$G_c(\tau',\tau,k_1,\cdots k_n) = i\sum_{l=1}^{n}\Delta(\tau,\tau',k_1)\cdots\Delta(\tau,\tau',k_{l-1})$$

$$[\Delta(\tau',\tau,k_l)-\Delta(\tau,\tau',k_l)]$$

$$\Delta(\tau',\tau,k_{l+1})\cdots\Delta(\tau',\tau,k_n)$$

In-in formalism

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}\frac{\Lambda^4}{H^4}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big)$$

$$e^{-\frac{\sigma_0^2}{2f^2}}\int_{-\infty}^{\tau} d\tau' \frac{1}{(\tau')^4}\left(\frac{H\tau'}{f}\right)^n G_c(\tau',\tau,k_1,\cdots k_n).$$

We consider a given time τ_0 **such that** $|\tau_0|k_i \ll 1$

$$\int_{\tau_0}^{\tau} d\tau' \tau^n \left(\tau'\right)^{n-4} G_c \rightarrow \frac{1}{3}\frac{k_1^3+\cdots+k_n^3}{2^{n-1}k_1^3\cdots k_n^3}\ln\left(\frac{\tau_0}{\tau}\right)$$

In-in formalism

$$\sigma_0^2 = \sigma_S^2 + \sigma_L^2 \qquad \sigma_L^2 = H^2\tau^2 \int_{k_L} u_k^I(\tau) u_k^{I*}(\tau)$$

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big) \times \frac{2A^2}{H^2} e^{-\frac{\sigma_L^2}{2f^2}} \left(\frac{H}{2f\tau}\right)^n \frac{k_1^3+\cdots+k_n^3}{k_1^3\cdots k_n^3}$$

$$A^2 \equiv \frac{\Lambda^4}{3H^2} e^{-\frac{\sigma_S^2}{2f^2}} \ln\left(\frac{\tau_0}{\tau}\right)$$

Main result

$$\sigma_0^2 = \sigma_S^2 + \sigma_L^2 \qquad \sigma_L^2 = H^2\tau^2 \int_{k_L} u_k^I(\tau) u_k^{I*}(\tau)$$

$$\langle u(\mathbf{k}_1,\tau)\cdots u(\mathbf{k}_n,\tau)\rangle_c = (-1)^{n/2}(2\pi)^3\delta^{(3)}\Big(\sum_j \mathbf{k}_j\Big) \times \frac{2A^2}{H^2} e^{-\frac{\sigma_L^2}{2f^2}} \left(\frac{H}{2f\tau}\right)^n \frac{k_1^3+\cdots+k_n^3}{k_1^3\cdots k_n^3}$$

$$A^2 \equiv \frac{\Lambda^4}{3H^2} e^{-\frac{\sigma_S^2}{2f^2}} \ln\left(\frac{\tau_0}{\tau}\right) \quad \Rightarrow \quad A^2 \equiv \frac{\Lambda_{\rm ph}^4}{3H^2} N$$

Application 1: The PDF

We may consider sub horizon distances

$$|\mathbf{x}_i - \mathbf{x}_j|/H|\tau| \ll H^{-1}$$

$$\langle \psi_{\rm L}^n \rangle_c = (-1)^{n/2} n \frac{A^2}{\sigma_{\rm L}^2} e^{-\frac{\sigma_{\rm L}^2}{2f^2}} \left(\frac{\sigma_{\rm L}^2}{f} \right)^n$$

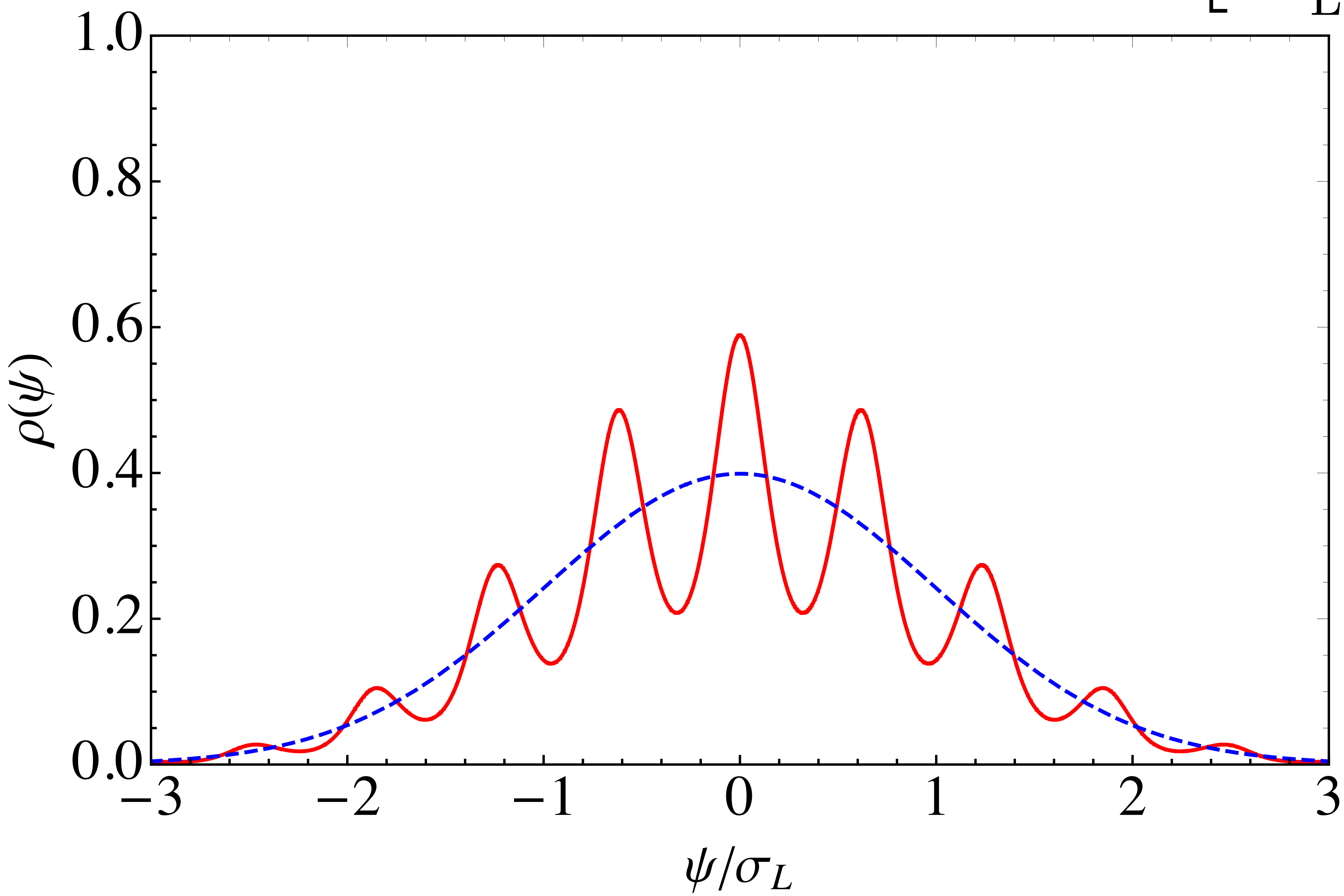
$$\langle \psi_{\rm L}^n \rangle = \int d\psi \; \psi^n \rho(\psi)$$

$$\rho(\psi) = \frac{e^{-\frac{\psi^2}{2\sigma_{\rm L}^2}}}{\sqrt{2\pi}\sigma_{\rm L}} \left[1 - A^2 \left(\frac{\sigma_{\rm L}^2 - \psi^2 - \sigma_{\rm L}^4/f^2}{2\sigma_{\rm L}^4} \right) \cos\left(\frac{\psi}{f} \right) \right]$$

Application 1: The PDF

$$\rho(\psi) = \frac{1}{\mathcal{N}} \frac{e^{-\frac{\psi^2}{2\sigma^2(\psi)}}}{\sqrt{2\pi}\sigma(\psi)} \exp\left[\frac{A^2}{2f^2}\cos(\psi/f)\right]$$

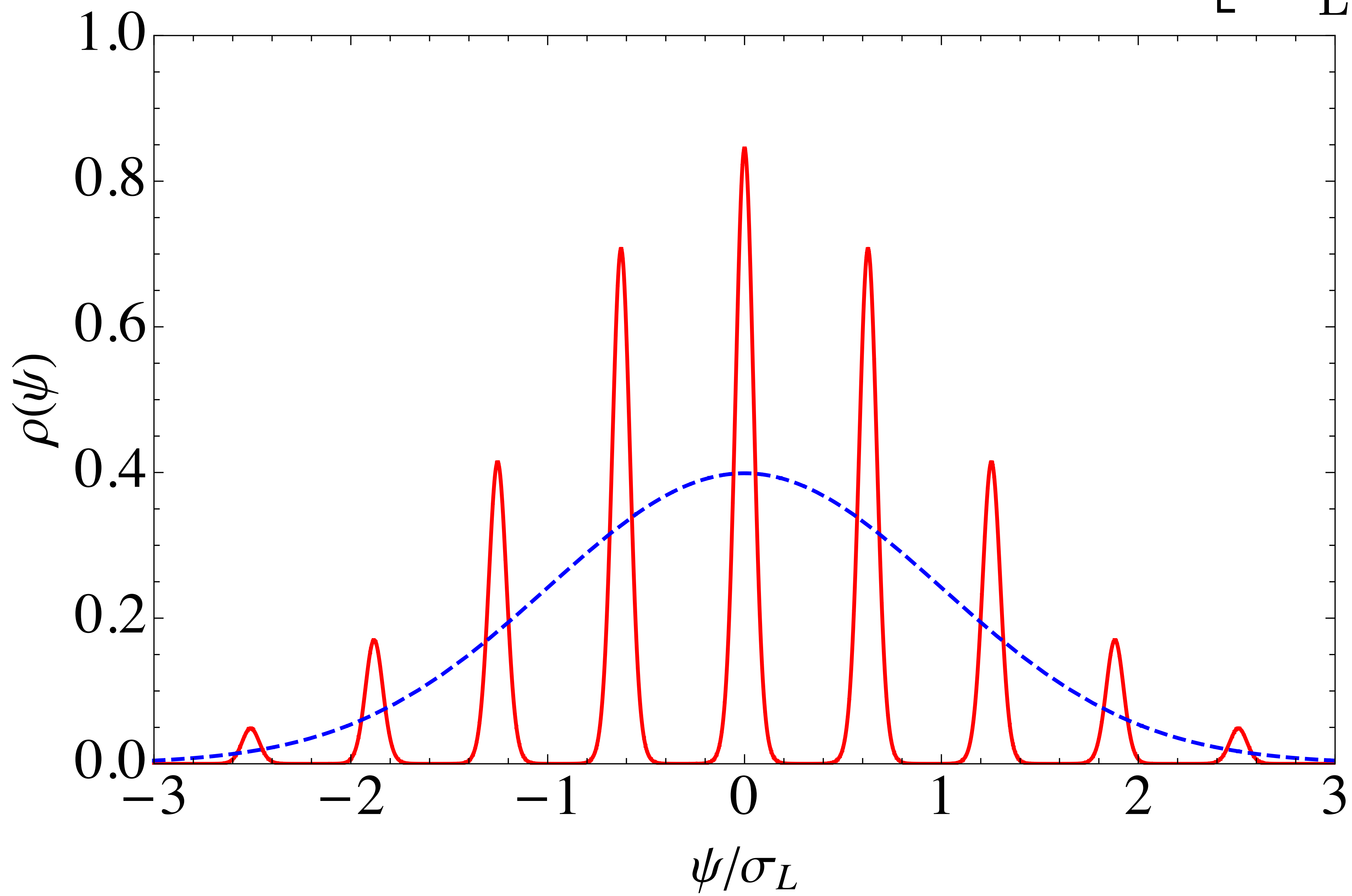
$$\sigma(\psi) \equiv \sigma_{\mathrm{L}} \exp\left[\frac{A^2}{2\sigma_{\mathrm{L}}^2}\cos(\psi/f)\right]$$



Application 1: The PDF

$$\rho(\psi) = \frac{1}{\mathcal{N}} \frac{e^{-\frac{\psi^2}{2\sigma^2(\psi)}}}{\sqrt{2\pi}\sigma(\psi)} \exp\left[\frac{A^2}{2f^2}\cos(\psi/f)\right]$$

$$\sigma(\psi) \equiv \sigma_\mathrm{L} \exp\left[\frac{A^2}{2\sigma_\mathrm{L}^2}\cos(\psi/f)\right]$$



Application 2: Non-Gaussianity

The idea of ultra-light fields:

$$\mathcal{L} = \epsilon\left(\dot{\zeta} - \alpha\psi\right)^2 - \frac{1}{a^2}(\nabla\zeta)^2 + \mathcal{L}_\psi$$

This coupling transfers the statistics of ψ to ζ

$$\zeta = \alpha\frac{N}{H}\psi_*$$

Achúcarro, Atal, Germani, Palma (2016)

In summary

- We were able to compute all n-point correlation functions of ψ (of order Λ^4)
- These required the resummation of all loops (of order Λ^4)
- The result reflects the tunneling between the degenerate vacua
- The statistic may be transferred to curvature perturbations (in progress)