Exotic Cosmologies and the Origin of the Null Energy Condition

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M. P., "No Open or Flat Bouncing Cosmologies in Einstein Gravity," arXiv: 1501.04606; JHEP.

M. P. and J.-P. van der Schaar, "Derivation of the Null Energy Condition," arXiv: 1406.5163; Phys. Rev. D.

Summary of Talk

- The null energy condition (NEC) can be derived from first principles.
- Consequently, we obtain a no-go theorem:

  There are no open or flat bouncing cosmologies in Einstein gravity.

- To find such solutions, one must therefore look beyond minimally-coupled Einstein gravity.

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The Null Energy Condition

The most basic of the energy conditions is the null energy condition (NEC):

$$T_{\mu\nu} v^{\mu} v^{\nu} \geq 0$$

where v is any light-like vector.

The weak, dominant, and strong energy conditions all imply the NEC.

The Null Energy Condition

For a fluid, the NEC translates to

$$\rho + p \geq 0$$

The NEC is violated by ghosts, phantoms, and most Horndeski theories.

The NEC is not violated by potential energy (including vacuum energy), tachyons, or Standard Model fields.

Importance of the NEC to Bouncing Cosmologies

For 3+1-dimensional FRW cosmology,

$$\dot{H} = -4\pi G_N(\rho + p) + \frac{k}{a^2}$$

where

$$\dot{H} = \frac{\ddot{a}}{a} - H^2$$

At or near a bounce, $\ddot{a} > 0$ and H vanishes. Hence $\dot{H} > 0$ which requires a violation of the NEC when $k \neq 1$.

Violation of the NEC is necessary for open or flat bouncing cosmologies in Einstein gravity and for bypassing the Hawking-Penrose singularity theorem.

Energy Conditions from Matter?

The energy conditions are conventionally viewed as properties of *matter*.

Indeed, they are defined in terms of the matter energy-momentum tensor.

Matter

We have a fantastic framework for understanding matter: quantum field theory.

However, it can be shown that the NEC cannot follow from QFT.

How the NEC is Used in Practice

In GR, the NEC is always used in conjunction with Einstein's equations:

$$T_{\mu\nu}v^\mu v^\nu \geq 0 \Rightarrow \left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right)v^\mu v^\nu \geq 0 \Rightarrow R_{\mu\nu}v^\mu v^\nu \geq 0$$

Why not just start with a condition on the Ricci tensor?

Perhaps the true null "energy condition" is

$$R_{\mu\nu}v^\mu v^\nu \geq 0$$

We will derive precisely this condition.

Calculation Goal of the Talk

Derive

$$R_{\mu\nu} v^\mu v^\nu \geq 0$$

We will continue to refer to this as the “null energy condition”. But note that this is now a condition on geometry.

Where Could the NEC Come From?

The geometric form of the NEC does not come from general relativity.

We have claimed that the matter form does not come from QFT either.

What is neither general relativity nor quantum field theory and contains both matter and gravity?

Two Possibilities

1. The null energy condition comes from string theory.

M. P. and J.-P. van der Schaar, "Derivation of the Null Energy Condition," arXiv: 1406.5163; Phys. Rev. D.

2. The null energy condition comes from thermodynamics.

M. P. and A. Svesko, "Thermodynamic Origin of the Null Energy Condition," arXiv: 1511.06460; Phys. Rev. D.

Worldsheet String Theory

Polyakov action

$$S[X^\mu, h_{ab}] = -\frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{-h} \left(g_{\mu\nu}(X) h^{ab} \partial_a X^\mu \partial_b X^\nu + \alpha' \Phi(X) R^{(2)} \right)$$

$X^\mu(\sigma, \tau)$	spacetime coordinates of the string
$h_{ab}(\sigma, \tau)$	worldsheet metric
$R^{(2)}$	two-dimensional Ricci scalar of worldsheet
$g_{\mu\nu}(X)$	spacetime metric
$\Phi(X)$	dilaton

Worldsheet string theory = two-dimensional conformal field theory of D fields + two-dimensional gravity

Strings in Minkowski Space

For a string propagating in flat space, the worldsheet CFT is free.

$$S[X^\mu, h_{ab}] = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}\left(\eta_{\mu\nu}h^{ab}\partial_a X^\mu\partial_b X^\nu + \lambda\alpha' R^{(2)}\right)$$

The equation of motion for the worldsheet metric is the 2D Einstein's equation:

$$-\frac{\lambda}{2\pi}\left(R^{(2)}_{ab} - \frac{1}{2}R^{(2)}h_{ab}\right) = T^{\mathrm{ws}}_{ab}$$

As the Einstein tensor vanishes in 2D, the *worldsheet* stress tensor must be 0.

Spacetime Null Vectors from the String Worldsheet

Consider the Virasoro constraint:

$$T_{ab}^{\text{ws}} = 0 \Rightarrow \eta_{\mu\nu}\partial_a X^\mu \partial_b X^\nu - \frac{1}{2}h_{ab}(\partial X)^2 = 0$$

In light cone coordinates, this becomes

$$\eta_{\mu\nu}\partial_+ X^\mu \partial_+ X^\nu = 0$$

In other words, $v^\mu \equiv \partial_+ X^\mu$ is a *spacetime* null vector field:

$$\eta_{\mu\nu} v^\mu v^\nu = 0$$

Strings in Curved Spacetime

For a string propagating in curved space, the action is

$$S[X^\mu, h_{ab}] = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}\left(g_{\mu\nu}(X)h^{ab}\partial_a X^\mu \partial_b X^\nu + \alpha'\Phi(X)R^{(2)}\right)$$

For any value of (τ, σ) choose Riemann normal coordinates about the spacetime point $X_0^\mu(\tau, \sigma)$:

$$g_{\mu\nu}(X) = \eta_{\mu\nu} - \frac{1}{3}R_{\mu\alpha\nu\beta}(X_0)Y^\alpha Y^\beta + \ldots$$

The action becomes

$$S_P[X, h] = S_P[X_0, h] + \int \left(\eta_{\mu\nu}(\partial Y^\mu \partial Y^\nu) + R_{\mu\alpha\nu\beta}(X_0)\partial X_0^\mu \partial X_0^\nu Y^\alpha Y^\beta + \ldots\right)$$

This is now a theory with a quartic interaction.

One-Loop Effective Action

After integrating out Y, we obtain a one-loop effective action.

The equation of motion for the worldsheet metric now reads

$$0 = \partial_+ X_0^\mu \partial_+ X_0^\nu \left(\eta_{\mu\nu} + C_\epsilon \alpha' R_{\mu\nu} + 2C_\epsilon \alpha' \nabla_\mu \nabla_\nu \Phi\right)$$

At zeroeth order in α' we recover our original equation:

$$\eta_{\mu\nu} v^\mu v^\nu = 0$$

But at first order in α' we miraculously discover that

$$v^\mu v^\nu (R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi) = 0$$

Einstein-Frame Metric

The spacetime metric appearing in the string action is the string-frame metric.

It is related to the usual ("Einstein-frame") metric by scaling:

$$g_{\mu\nu} = e^{+\frac{4\Phi}{D-2}} g^E_{\mu\nu}$$

Transforming the Virasoro condition to the Einstein-frame metric, we find

$$0 = v^\mu v^\nu (R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi) = v^\mu v^\nu \left(R^E_{\mu\nu} - \frac{4}{D-2} \nabla^E_\mu \Phi \nabla^E_\nu \Phi \right)$$

That is

$$R^E_{\mu\nu} v^\mu v^\nu = +\frac{4}{D-2} (v^\mu \nabla^E_\mu \Phi)^2 \geq 0$$

This is precisely the desired geometric form of the null energy condition!

Summary of Worldsheet Derivation

We have seen that a geometric constraint condition on the Ricci tensor comes straight from the Virasoro constraint in string theory.

$$R_{\mu\nu} v^{\mu} v^{\nu} \geq 0$$

Beautifully, that constraint is nothing other than Einstein's equation in 2D.

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String Theory Prohibits Open or Flat Bounces

Let us apply our geometric condition directly to FRW cosmology:

$$R_{\mu\nu}v^\mu v^\nu = \vec{v}^2 a^2(t)(d-1)\left(-\dot{H} + \frac{k}{a^2}\right) \geq 0$$

Thus, without assuming anything about the existence of NEC-violating matter, we see that for $k \neq 1$ bounce solutions cannot exist.

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Loopholes for String-Compatible Bounce Solutions

Quantum (semi-classical) modifications to Einstein's equations:

$$G_{\mu\nu} = 8\pi G_N \langle T_{\mu\nu} \rangle - \Lambda g_{\mu\nu}$$

Use NEC-violating *quantum* matter e.g. Casimir energy.

Higher-curvature corrections:

$$v^\mu v^\nu (R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi + \alpha' \times \text{higher-curvature}) = 0$$

Here, NEC-violating matter is neither necessary nor sufficient for a bounce.

Non-minimal coupling? Orientifolds?

Summary of Talk

- A geometric counterpart of the null energy condition can be derived from first principles.

- Consequently, we obtain a no-go theorem:

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- To find such solutions, one must therefore look beyond minimally-coupled Einstein gravity.

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