Stability of nonsingular cosmologies with limiting curvature

Based on work with Robert Brandenberger (McGill), Daisuke Yoshida (McGill), and Masahide Yamaguchi (Tokyo Tech.)

Phys. Rev. D **96**, 043502 (2017) [arXiv:1704.04184]

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COSMO17, Paris
August 31st, 2017

Motivation

- General Relativity (GR) with normal matter
  $\Longrightarrow$ cosmological singularities are inevitable
  (Penrose ['65], Hawking ['67])
- Even inflationary cosmology (within GR) is inevitably past incomplete
  (Borde & Vilenkin [gr-qc/9312022], Borde et al. [gr-qc/0110012])
- One would thus like to build a theory that is free of these bad singularities
  $\Longrightarrow$ one has to go beyond classical GR
- Ultimate goal: a theory of the very early universe embedded in a quantum theory of gravity without singularities

One approach to nonsingular cosmology

- Horndeski theories:

$$\mathcal{L} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$+G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right],$$

  where $X \equiv \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$ and $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$

- Choose the $G_i(\phi, X)$'s in order to violate the Null Energy Condition for a short period of time

- Is the resulting theory stable?

Perturbations and (in)stability

- 2nd-order perturbed actions (unitary gauge):

$$S^{(2)}_{\mathrm{tensor}} = \frac{1}{8}\int \mathrm{d}^4x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right] ,$$

$$S^{(2)}_{\mathrm{scalar}} = \int \mathrm{d}^4x\, a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla} \zeta)^2\right]$$

- Conditions for stability (e.g., scalar sector):

$$\mathcal{G}_S > 0 \Leftrightarrow \text{no ghost instability} ,$$

$$\mathcal{F}_S > 0 \Leftrightarrow \text{no gradient instability}$$

No-Go Theorems

- Within Horndeski theories, it is **not** possible to have a geodesically complete spacetime and be free of both ghost and gradient instabilities at all times

  (Libanov et al. [1605.05992], Kobayashi [1606.05831])

$$\mathcal{G}_S(t) > 0\,,\ \mathcal{F}_S(t) > 0\,,\ \mathcal{G}_T(t) > 0\,,\ \mathcal{F}_T(t) > 0\,,\ \forall t \in (-\infty, \infty)$$

- Can also be shown in effective field theory (EFT) (Cai et al. [1610.03400, 1701.04330], Creminelli et al. [1610.04207])
- The no-go can be evaded only if:
  - In EFT, include higher-order operators

    (Cai & Piao [1705.03401, 1707.01017])
  - Work with beyond-Horndeski theories

    (Kolevatov et al. [1705.06626])

Other approach to nonsingular cosmology: limiting curvature

- The idea of limiting curvature: there should exist a fundamental length scale ℓ_f (possibly $\sim \ell_{\mathrm{Pl}}$) such that

$$|R| < \ell_f^{-2},\ |R_{\mu\nu}R^{\mu\nu}| < \ell_f^{-4},\ |\nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}| < \ell_f^{-6},\ \mathrm{etc.}$$

- Difficulty: one could have $|R_{\mu\nu}R^{\mu\nu}| < \ell_f^{-4}$, but still $|\nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}| \to \infty$.

- Limiting curvature hypothesis: find a theory with a finite number of curvature invariants bounded, e.g., $|R| \leq \ell_f^{-2}$, $|R_{\mu\nu}R^{\mu\nu}| \leq \ell_f^{-4}$. Then, when these invariants take on their limiting values, any solution of the field equations reduces to a definite nonsingular solution.

Mukhanov & Brandenberger [PRL, 1992]

Limiting curvature implementation: example in special relativity

- Action for a point particle in special relativity:

$$S = m \int \mathrm{d}t \left[\frac{1}{2}\dot{x}^2 + \varphi\dot{x}^2 - V(\varphi) \right] , \qquad V(\varphi) = \frac{2\varphi^2}{1+2\varphi} .$$

- $\delta_\varphi S = 0 \implies \dot{x}^2 = \frac{\mathrm{d}V}{\mathrm{d}\varphi} = 1 - \frac{1}{(1+2\varphi)^2} \implies \dot{x}^2 \leq 1 \ \forall\varphi \in (-\infty, \infty)$

- Solving for φ in terms of $\dot{x}^2$ and substituting in the action above, one finds

$$S = m \int \mathrm{d}t \sqrt{1 - \dot{x}^2} ,$$

as expected.

Limiting curvature implementation

- Naturally constructed to avoid singularities (contrary to, e.g., Horndeski theories)
- Used to construct nonsingular black holes (Frolov et al. ['89, '90], Morgan ['91], Trodden et al. [hep-th/9305111], Bogojevic & Stojkovic [gr-qc/9804070], Easson [hep-th/0210016], Frolov [1609.01758], Chamseddine & Mukhanov [1612.05861])
- In cosmology, the action is (Mukhanov & Brandenberger ['92], Brandenberger et al. [gr-qc/9303001])

$$S = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\big[R + \chi_1 I_1(R, R_{\mu\nu}R^{\mu\nu}, ...) - V_1(\chi_1) \\ + \chi_2 I_2(R, R_{\mu\nu}R^{\mu\nu}, ...) - V_2(\chi_2)\big] + S_{\mathrm{m}}\,,$$

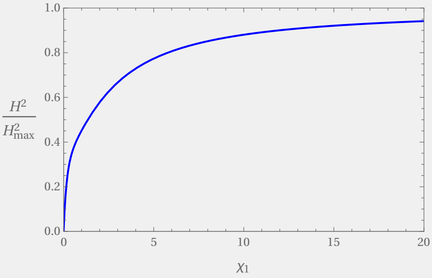
- Assume $I_1,\ I_2 \sim \mathcal{O}(R)$. Since in FRW $\bar{R} = 12H^2 + 6\dot{H}$, a natural choice is

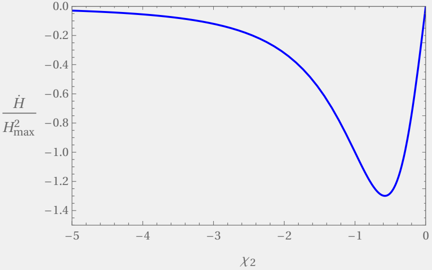
$$\bar{I}_1 = 12H^2\,, \qquad \bar{I}_2 = -6\dot{H}\,.$$

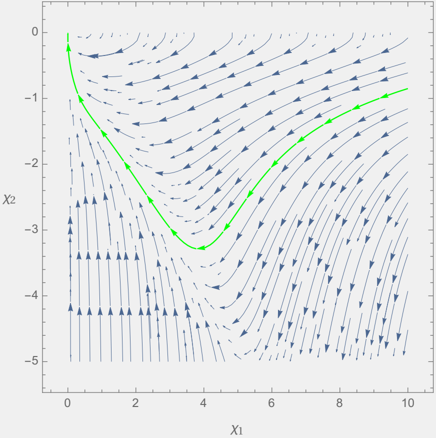
- So $\delta_{\chi_1}S = 0$ and $\delta_{\chi_2}S = 0$ gives the constraint equations

$$\bar{I}_1 = 12H^2 = \frac{\mathrm{d}V_1}{\mathrm{d}\chi_1}\,, \qquad \bar{I}_2 = -6\dot{H} = \frac{\mathrm{d}V_2}{\mathrm{d}\chi_2}\,.$$

Example: de Sitter to Minkowski (inflation)







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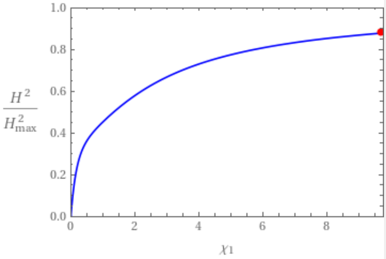
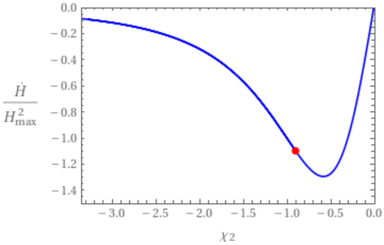
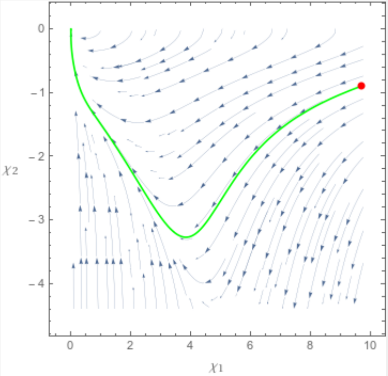
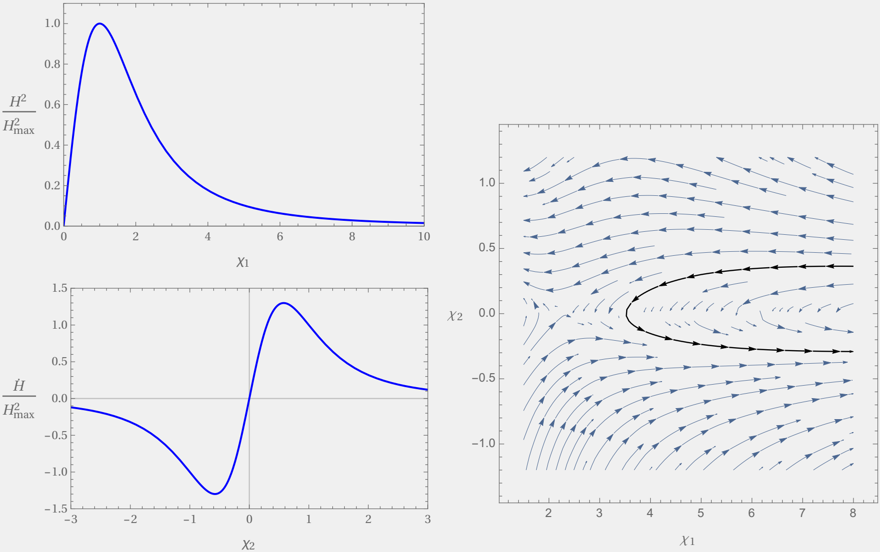


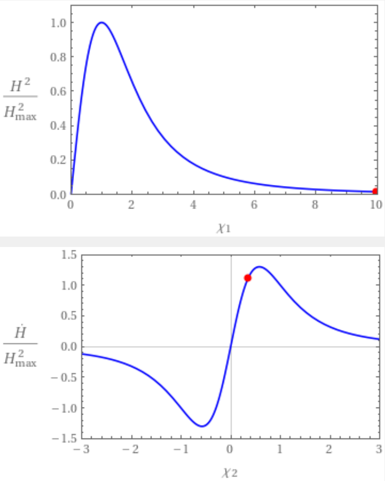


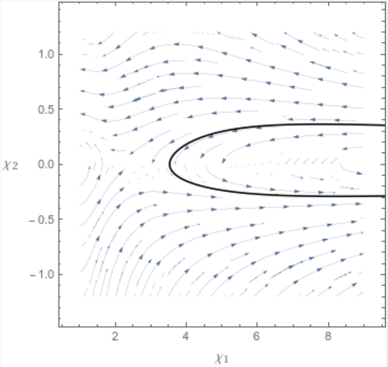


Example: Minkowski to Minkowski (genesis)



Example: Minkowski to Minkowski (genesis)





Perturbations and stability

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Mukhanov & Brandenberger ['92] took

$$I_2 \equiv \sqrt{12R_{\mu\nu}R^{\mu\nu} - 3R^2}\,, \qquad I_1 \equiv I_2 + R\,.$$

- In FLRW, $\bar{I}_1 = 12H^2$ and $\bar{I}_2 = -6\dot{H}$ as wanted
- Consider tensor modes. 2nd-order perturbed action (in Fourier space):

$$S^{(2)}_{\mathrm{tensor}} \supset \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{G}_T\ddot{h}_k^2 + \mathcal{K}_T\dot{h}_k^2 - \mathcal{M}_T\frac{k^2}{a^2}h_k^2\right)\,,$$

$$\mathcal{G}_T = -\frac{\chi_1 + \chi_2}{2\dot{H}}\,.$$

$\longrightarrow$ Ostrogradski instability (corresponds to a linearly unstable Hamiltonian)

Possible resolution

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Include the Weyl tensor squared:

$$I_2 \equiv \sqrt{12 R_{\mu\nu} R^{\mu\nu} - 3R^2 + 3\kappa C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}}\,, \qquad I_1 \equiv I_2 + R\,.$$

- Perturbing the action in the tensor sector:

$$S^{(2)}_{\mathrm{tensor}} \supset \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{G}_T \ddot{h}_k^2 + \mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2 \right)\,,$$

$$\mathcal{G}_T = -(2+\kappa) \frac{\chi_1 + \chi_2}{4\dot{H}}\,,$$

$\longrightarrow \kappa = -2 \implies \mathcal{G}_T = 0 \longrightarrow$ no Ostrogradski ghost

- Is it valid at higher order ($S^{(3)}$, $S^{(4)}$, ...)? Possibly only up to $H_{\max}$

Perturbations modes

$$\mathcal{L} = R + \chi_1 \left(R + \sqrt{12R_{\mu\nu}R^{\mu\nu} - 3R^2 - 6C^2} \right) - V_1(\chi_1)$$
$$+ \chi_2 \sqrt{12R_{\mu\nu}R^{\mu\nu} - 3R^2 - 6C^2} - V_2(\chi_2)\,, \qquad C^2 \equiv C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}$$

- No propagating vector modes
- Tensor and scalar modes:

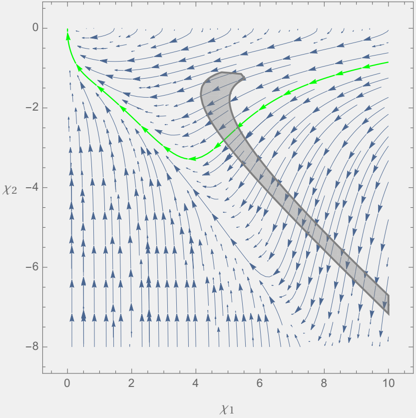
$$S^{(2)}_{\mathrm{tensor}} \sim \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2 \right)$$

$$S^{(2)}_{\mathrm{scalar}} \sim \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{K}_S \dot{\Phi}_k^2 - \mathcal{M}_S(k) \Phi_k^2 \right)$$

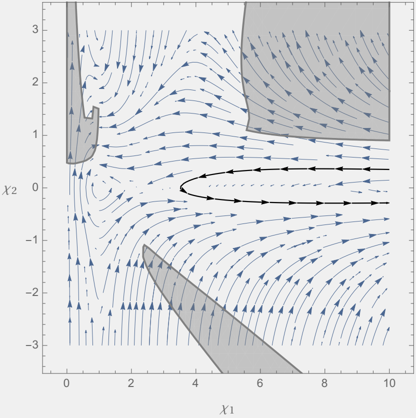
- $\mathcal{M}_S(k) \sim \mathcal{O}[(k/a)^8]$ for $k/a \gg 1 \longrightarrow$ modified dispersion relation
- $\mathcal{K}_T$, $\mathcal{M}_T$, $\mathcal{K}_S$, $\mathcal{M}_S$ are complicated functions of χ_n and $V_n(\chi_n)$ that can be positive or negative depending on the background trajectory $\longrightarrow$ possible ghost and gradient instabilities

Stability during inflation and genesis

Inflation



Genesis



Construct another curvature invariant function

$$\mathcal{L} = R + \chi_1 I_1 - V_1(\chi_1) + \chi_2 I_2 - V_2(\chi_2)$$

- Consider first derivatives of R:

$$X^\mu{}_\nu \equiv \nabla^\mu R \nabla_\nu R\,, \qquad X = \nabla^\mu R \nabla_\mu R$$

- For a flat FLRW background,

$$\bar{X} = -36(4H\dot{H} + \ddot{H})^2$$

- Want to construct I_1 such that $\bar{I}_1 = 12H^2$:

$$I_1 \equiv -\frac{1}{X^3}[4X^2(\nabla_\mu\nabla_\nu R)^2 - 2X(\nabla_\mu X)^2 + (\nabla_\mu R \nabla_\nu X)^2]$$

- Then, $I_2 \equiv I_1 - R$ and $\bar{I}_2 = -6\dot{H}$.

Perturbations for the new curvature invariant function

- No propagating vector modes
- Tensor modes:

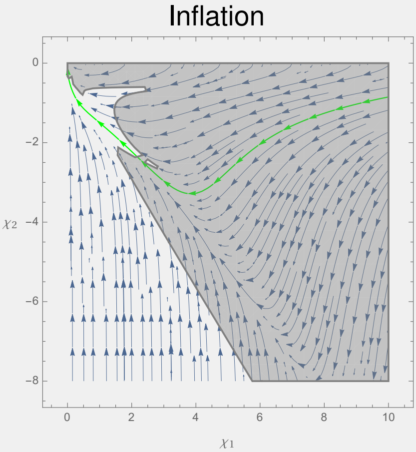
$$S^{(2)}_{\mathrm{tensor}} \sim \int \mathrm{d}t\mathrm{d}^3k\, a^3 \left(\mathcal{K}_T \dot{h}_k^2 - \mathcal{M}_T \frac{k^2}{a^2} h_k^2\right)$$

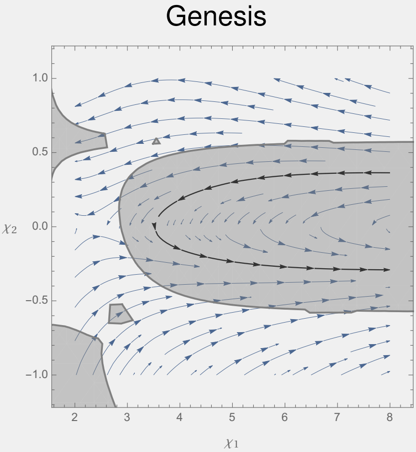
where

$$\mathcal{K}_T = \frac{1+4\chi_1+3\chi_2}{2}\,, \qquad \mathcal{M}_T = \frac{1-\chi_2}{2}$$

- No Ostrogradski instability. No ghost or gradient instabilities as long as $\chi_2 < 1$ and $\chi_1 > -(1+3\chi_2)/2$.
- No superluminality $\implies c_\mathrm{s}^2 \equiv \mathcal{M}_T/\mathcal{K}_T \leq 1 \implies \chi_1 \geq -\chi_2$.
- Similar story in the scalar sector, though the conditions on χ_1 and χ_2 are slightly more non-trivial

Stability during inflation and genesis





Conclusions

- Simple nonsingular cosmologies within Horndeski theories are unstable (no-go theorem)
  $\longrightarrow$ one needs to consider beyond-Horndeski theories
- Other approach: limiting curvature
- Old model of Mukhanov & Brandenberger has Ostrogradski instabilities
- Can be cured by including the Weyl tensor squared
  $\longrightarrow$ still important ghost and gradient instabilities
- New curvature invariant constructed with derivatives of R leads to no apparent Ostrogradski instability
- Inflationary and genesis scenarios are mostly stable with regards to ghost and gradient instabilities
- Still very hard to construct stable nonsingular cosmologies

What's next?

- Construct viable nonsingular inflationary and genesis scenarios, and explore cosmological observables
- Explore nonsingular bouncing cosmology
(E.g., Chamseddine & Mukhanov [1612.05860], Bodendorfer et al. [1703.10670], Liu et al. [1703.10812])
- How does the theory with limiting curvature fit in the greater picture of scalar-tensor theories of gravity?
$\longrightarrow$ included in beyond-Horndeski theories? in Degenerate Higher-Order Scalar-Tensor (DHOST) theories?

Acknowledgments

Thank you for your attention!

I acknowledge support from the following agencies:

Bourses d'études supérieures du Canada
Vanier
Canada Graduate Scholarships

NSERC
CRSNG

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