Particle production at sharp turns of the inflationary trajectory

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Outline

Sudden turning trajectory $|\dot{\theta}| \gg H$
(i.e. the turn lasts a fraction of an e-fold):

- Modification of $\mathcal{P}_{\mathcal{R}}$

T. Noumi, M. Yamaguchi arXiv:1307.7110v2

X. Gao, D. Langlois, S. Mizuno arXiv:1205.5275v3

S. Cespedes, V. Atal, G. A. Palma arXiv:1201.4848v3

G. Shiu, J. Xu arXiv:1108.0981v2

A. Achucarro, J.-O. Gong, et al. arXiv:1010.3693v4

X. Chen, Y. Wang arXiv:0911.3380

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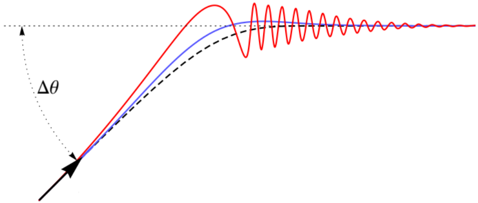

X. Gao, D. Langlois, S. Mizuno arXiv:1205.5275v3

- Particle production

M. Konieczka, R. H. Ribeiro, K. Turzyński arXiv:1401.6163v3

(see D. Battefeld, T. Battefeld, C. Byrnes, D. Langlois arXiv:1106.1891v2 for Extra Symmetry Point)

(also: primordial clocks, see X. Chen's talk)

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V$$

$$V = \frac{1}{2}m^2\phi^2 + \frac{1}{2}M^2\cos^2\left(\frac{\Delta\theta}{2}\right)\left[\chi - (\phi - \phi_0)\,\mathrm{tg}\left(\frac{\Delta\theta}{\pi}\,\mathrm{tg}^{-1}(s(\phi - \phi_0))\right)\right]^2$$

- $\Delta\theta$ - the angle variation
- s - the sharpness of the turn
- ϕ_0 - the position of the turn

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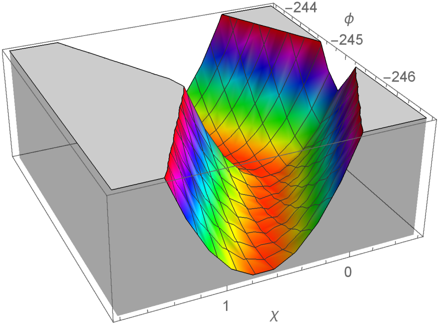

M. Mijić arXiv:gr-qc/9801094v1

One-field

$$u(\tau) = a(\tau)\delta\phi(\tau)$$

$$u_0 = \frac{\exp(-ik\tau)}{\sqrt{2k}}$$

$$u'' + \omega^2 u = 0$$

$$\omega^2 = k^2 + a^2 m^2 - \frac{a''}{a}$$

Bogolyubov transformation:

- new basis -
  solutions in the adiabatic approximation $|\dot{\omega}| \ll \omega^2$

$$\{f \equiv \frac{1}{\sqrt{2\omega(\tau)}} \exp(-i\int^{\tau} \omega(\eta)\mathrm{d}\eta),$$

$$f^* \equiv \frac{1}{\sqrt{2\omega(\tau)}} \exp(+i\int^{\tau} \omega(\eta)\mathrm{d}\eta)\}$$

$$u(\tau) = \frac{1}{\sqrt{2\omega(\tau)}}\left[\underbrace{A(\tau)f}_{\alpha} + \underbrace{B(\tau)f^*}_{\beta}\right]$$

- additional condition

$$u'(\tau) = -i\sqrt{\frac{\omega(\tau)}{2}}\left[\alpha(\tau) - \beta(\tau)\right]$$

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$$\begin{cases} \alpha' = & -\mathrm{i}\omega\alpha + \frac{\omega'}{2\omega}\beta \\ \beta' = & \mathrm{i}\omega\beta + \frac{\omega'}{2\omega}\alpha \end{cases}$$

Bogolyubov transformation:

- definition of a new annihilation operator

$$a^{\text{out}} = A a^{\text{in}} + B a^{\text{in}\dagger}$$

$$a^{\text{out}\dagger} = A^* a^{\text{in}\dagger} + B^* a^{\text{in}}$$

- normalization condition

$$|A|^2 - |B|^2 = 1$$

$$|\alpha|^2 - |\beta|^2 = 1$$

- occupation number of produced particles

$$n \equiv \int \frac{\mathrm{d}^3 k}{(2\pi)^3} a^{\text{out}\dagger} a^{\text{out}}$$

$$N \equiv \langle 0^{\text{in}} | n | 0^{\text{in}} \rangle = |B|^2 = |\beta|^2$$

H. P. Nilles, M. Peloso, L. Sorbo arXiv:hep-th/0103202v3

Multi-field

$$u = \begin{pmatrix} u_1^{\text{init}} & u_1^{\text{ind}(2)} & \cdots \\ u_2^{\text{ind}(1)} & u_2^{\text{init}} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

$$u_{0ij} = \delta_{ij} \frac{\exp(-ik\tau)}{\sqrt{2k}}$$

$$u_i'' + \Omega_{ij}^2 u_j = 0$$

$$(\Omega^2)_{ij} = (k^2 - \frac{a''}{a})\delta_{ij} + a^2 \mathbb{M}_{ij}$$

$$\omega^2 = \mathbb{C}^{\mathsf{T}} \Omega^2 \mathbb{C} = k^2 \mathbf{1} + \mathbb{M}_{\mathrm{d}}^2$$

$$\begin{cases} \alpha' = & -i\omega\alpha + \frac{1}{2}\omega'\omega^{-1}\beta - \mathbb{I}\alpha - \mathbb{J}\beta \\ \beta' = & i\omega\beta + \frac{1}{2}\omega'\omega^{-1}\alpha - \mathbb{I}\beta - \mathbb{J}\alpha \end{cases}$$

$$\mathbb{I} = \frac{1}{2}\left(\sqrt{\omega}\mathbb{C}^{\mathsf{T}}\mathbb{C}'\sqrt{\omega}^{-1} + \sqrt{\omega}^{-1}\mathbb{C}^{\mathsf{T}}\mathbb{C}'\sqrt{\omega}\right)$$

$$\mathbb{J} = \frac{1}{2}\left(\sqrt{\omega}\mathbb{C}^{\mathsf{T}}\mathbb{C}'\sqrt{\omega}^{-1} - \sqrt{\omega}^{-1}\mathbb{C}^{\mathsf{T}}\mathbb{C}'\sqrt{\omega}\right)$$

$$N_i = (\beta\beta^\dagger)_{ii}$$

M. Konieczka, R. H. Ribeiro, K. Turzyński arXiv:1401.6163v3

Assumptions:

(A) Two-field inflation models with one light and one heavy mode (i.e. $m^2 \ll H^2$ and $M^2 \gg H^2$)

M. Konieczka, R. H. Ribeiro, K. Turzyński arXiv:1401.6163v3

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(B) Before and after the turn fields are constant or they evolve adiabatically (i.e. the trajectory is straight)
$\rightarrow$ the orthogonal matrix is time-independent $\mathbb{C}' = 0$
$\rightarrow$ the mass matrix is time-independent $(\mathbb{M}_\mathrm{d})' = 0$
and eigenvalues are m^2 and M^2

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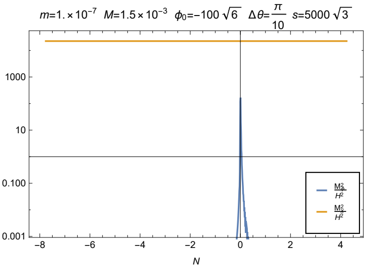
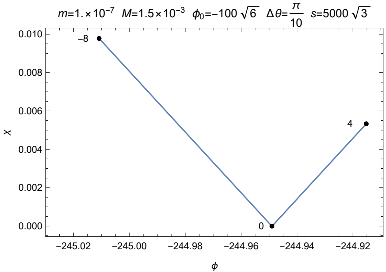
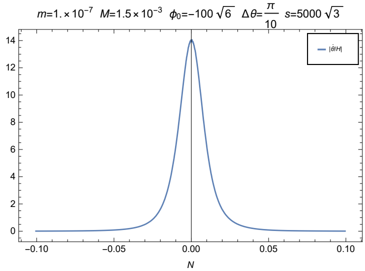
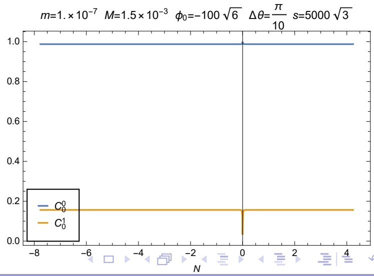
(C) The turn is sharp (i.e. $|\dot{\theta}| \ll H$ except for a fraction of an e-fold)
$\longrightarrow$ the expansion of the universe can be neglected during the turn
$\longrightarrow$ the calculations can be done in Minkowski spacetime without distinguishing between the cosmic time t and the conformal time τ

Simplification for two-field case









Under the assumptions:

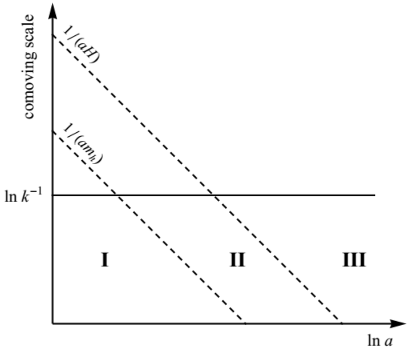
$$\begin{cases} \alpha = \frac{1}{2}\left(\sqrt{r} + \frac{1}{\sqrt{r}}\right)\sin(\Delta\theta)\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \cos(\Delta\theta)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \beta = \frac{1}{2}\left(\sqrt{r} - \frac{1}{\sqrt{r}}\right)\sin(\Delta\theta)\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \end{cases}$$

$$|\beta|^2 = \frac{1}{4}\left(\sqrt{r} - \frac{1}{\sqrt{r}}\right)^2 \sin^2(\Delta\theta)$$

where

$$r = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k^2 + M^2}{k^2 + m^2}} = \text{const}$$

$\Delta\theta$ - the angle by which the inflationary trajectory turns



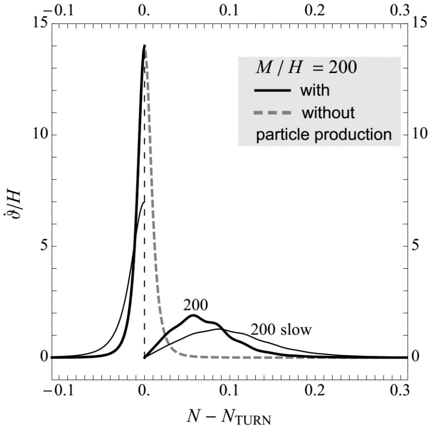
X. Gao, D. Langlois, S. Mizuno arXiv:1205.5275v3

Energy density of particles produced at the turn:

$$\rho_i = \int_H^M \frac{\mathrm{d}^3 k}{(2\pi)^3} \sqrt{k^2 + m_i^2} |\beta|^2$$

$$\rho_l \approx \frac{M^4}{24\pi^2} \sin^2(\Delta\theta)$$

$$\rho_h \approx \frac{M^4}{16\pi^2} \sin^2(\Delta\theta)$$



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Energy density of particles produced in the section of the turn:

$$\mathrm{d}|\beta|^2 = \frac{1}{4}\left(\sqrt{r} - \frac{1}{\sqrt{r}}\right)^2 2\sin(\Delta\theta)\cos(\Delta\theta)\mathrm{d}\theta$$

$\Delta\theta$ - from the beginning of the turn to the end of the section
$\mathrm{d}\theta$ - in the section

$$\rho_l \approx \frac{M^4}{24\pi^2} 2\sin(\Delta\theta)\cos(\Delta\theta)\mathrm{d}\theta$$

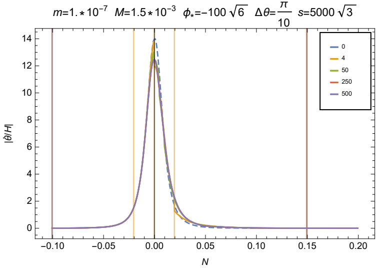
$$\rho_h \approx \frac{M^4}{16\pi^2} 2\sin(\Delta\theta)\cos(\Delta\theta)\mathrm{d}\theta$$

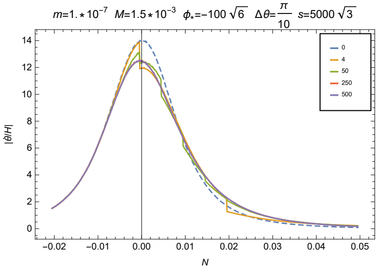
Continuity conditions for the background:

- Values of the fields remain unchanged
- Values of the velocities are changed

Final state	Initial state
$\phi(t_0), \ \ \chi(t_0)$	$\phi(t_0), \ \ \chi(t_0)$
$\frac{1}{2}\dot{\phi}^2(t_0), \ \ \frac{1}{2}\dot{\chi}^2(t_0)$	$\frac{1}{2}\dot{\phi}^2(t_0) - \rho_l(t_0), \ \ \frac{1}{2}\dot{\chi}^2(t_0) - \rho_h(t_0)$
$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V\right)$ + old particles	$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V + \right.$ $\left. + \rho_l(t_0)\frac{a^4(t_0)}{a^4} + \rho_h(t_0)\frac{a^3(t_0)}{a^3}\right)$ + old particles

On background





Kinetic energy:

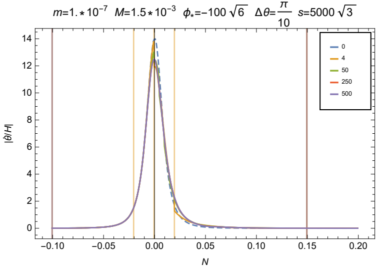
$$\frac{1}{2}(\dot{\phi}^2 + \dot{\chi}^2) \approx 3.2 * 10^{-15}$$

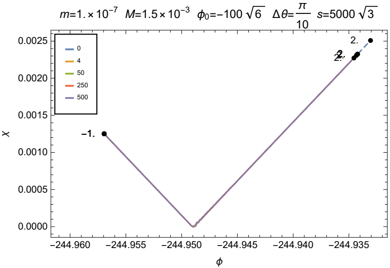
Energy density of particles produced at the turn ($\frac{M}{H} = 150$):

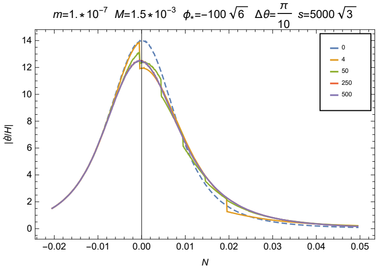
$$\rho_l \approx 2.04 * 10^{-15}$$

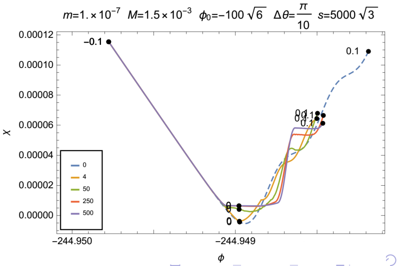
$$\rho_h \approx 3.06 * 10^{-15}$$

On background









Continuity conditions for the perturbations:

- gauge-invariant Mukhanov-Sasaki variables remain unchanged

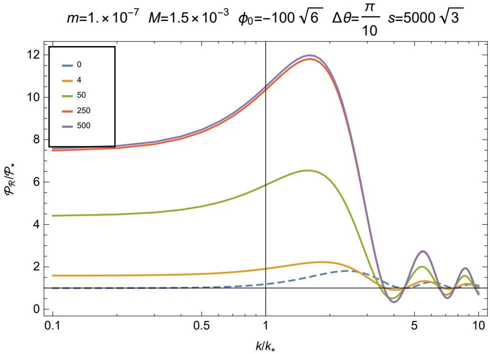
$$Q_\phi = \delta\phi + \frac{\dot{\phi}}{H}\Phi \quad \text{and} \quad Q_\chi = \delta\chi + \frac{\dot{\chi}}{H}\Phi$$

- as well as their derivatives remain unchanged

$$\ddot{Q}_I + 3H\dot{Q}_I + Q_I\Big(\frac{k^2}{a^2} + \frac{2\dot{\varphi}_I V_I}{H} - \frac{\dot{\varphi}_I{}^4}{2H^2} - \frac{\dot{\varphi}_I{}^2\dot{\varphi}_J{}^2}{2H^2} + 3\dot{\varphi}_I{}^2 + V_{II}\Big) + \\ + Q_J\Big(\frac{\dot{\varphi}_J V_I}{H} + \frac{\dot{\varphi}_I V_J}{H} - \frac{\dot{\varphi}_I\dot{\varphi}_J{}^3}{2H^2} - \frac{\dot{\varphi}_I{}^3\dot{\varphi}_J}{2H^2} + 3\dot{\varphi}_I\dot{\varphi}_J + V_{IJ}\Big) = 0$$

On perturbations

preliminary



Thank you