Post-Newtonian Cosmology

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VAAS and T. Clifton
Based on **Arxiv:** *1503.08747 , Phys. Rev. D 91, 103532*
Based on **Arxiv:** *1604.06345, Phys. Rev. D 94, 023505*
Based on **Arxiv:** *1610.08039, Class. Quan. Grav. 34 (2017) 065003*
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Based on **Arxiv:** *1705.02328, JCAP 07 (2017) 028 .*

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Weak field regions

Arxiv: 1503.08747

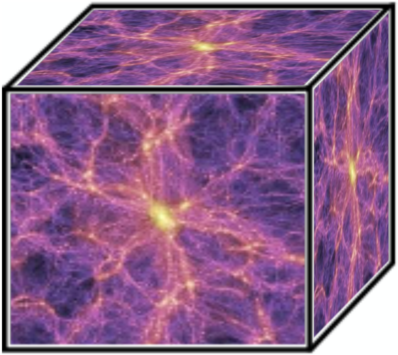


Figure: This figure was produced using an image from D. J. Croton *et al.*, 2005

Parameterized post-Newtonian (PPN) Formalism

At leading order the PPN metric is given by

$$ds^2 \equiv (-1 + 2\alpha U)dt^2 + (1 + 2\gamma U)\delta_{\mu\nu} dx^\mu dx^\nu \ ,$$

where

- applicable to isolated astrophysical systems.
- $U \sim \rho_M \sim \frac{v^2}{c^2} \equiv \epsilon^2$ is the Newtonian gravitational potential.
- $\nabla^2 U \equiv -4\pi G \rho_M$.
- We assume U has asymptotically flat solutions.
- α and γ are constant PPN parameters.

(Will, 1993)

Parametrized post-Newtonian Cosmology (PPNC)

Arxiv: 1610.08039

In cosmology, we need to account for

- Time dependence of barotropic fluids
- Time dependence of additional degrees of freedom: ϕ, A_a

The metric potentials satisfy Poisson-like equations of the form

$$\nabla^2\Phi \equiv -4\pi G\alpha\rho_M + \alpha_C \ ,$$
$$\nabla^2\Psi \equiv -4\pi G\gamma\rho_M + \gamma_C \ ,$$

where $\alpha(t)$, $\alpha_C(t)$, $\gamma(t)$ and $\gamma_C(t)$ are four functions of time.

For solutions to the potentials, $\nabla^2 U = -4\pi G\rho_M$ we do not assume asymptotic flatness.

Cosmological Expansion

Arxiv: 1610.08039

Acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\alpha}{3}\langle\rho_M\rangle + \frac{\alpha_C}{3},$$

Constraint equation

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\gamma}{3}\langle\rho_M\rangle - \frac{2\gamma_C}{3}.$$

Additional constraint

$$4\pi G\langle\rho_M\rangle = \left(\alpha_C + 2\gamma_C + \frac{d\gamma_C}{d\ln a}\right)\Big/\left(\alpha - \gamma + \frac{d\gamma}{d\ln a}\right).$$

Cosmological perturbations

Arxiv: 1610.08039

Poisson-like equations in an FLRW background

$$\hat{\nabla}^2 \hat{\Phi} = -4\pi G a^2 \alpha \delta\rho \ ,$$
$$\hat{\nabla}^2 \hat{\Psi} = -4\pi G a^2 \gamma \delta\rho \ .$$

Gravitational constant parameter, μ, and gravitational slip parameter, ζ

$$\mu = \gamma \ ,$$
$$\zeta = 1 - \frac{\alpha}{\gamma} \ .$$

ΛCDM

Arxiv: 1610.08039

The PPNC parameters are given by

$$\alpha = \gamma = 1,$$

$$\alpha_C = \Lambda,$$

$$\gamma_C = -\frac{\Lambda}{2} \, .$$

Scalar-tensor theories of gravity

Arxiv: 1610.08039

The Lagrangian of such a theory is given by

$$L = \frac{1}{16\pi G}\left[\phi R - \frac{\omega(\phi)}{\phi} g^{ab}\phi_{;a}\phi_{;b} - 2\phi\Lambda(\phi)\right] + L_m(\psi, g_{ab}) \ ,$$

The PPNC parameters are given by

$$\alpha = \left(\frac{2\omega+4}{2\omega+3}\right)\frac{1}{\bar{\phi}} \ , \quad \gamma = \left(\frac{2\omega+2}{2\omega+3}\right)\frac{1}{\bar{\phi}}$$

$$\alpha_C = -\left(\frac{2\omega+4}{2\omega+3}\right)\sum_I \frac{4\pi G\rho_I}{\bar{\phi}} + \left(\frac{2\omega+2}{2\omega+3}\right)\left(-\sum_I \frac{12\pi G p_I}{\bar{\phi}} + \Lambda(\bar{\phi})\right) - \frac{\omega(\bar{\phi})}{\bar{\phi}^2}\dot{\bar{\phi}}^2 - \frac{\ddot{\bar{\phi}}}{\bar{\phi}}$$
$$+ \frac{1}{2\omega+3}\left(\frac{\omega'\dot{\bar{\phi}}^2}{2\bar{\phi}} + \bar{\phi}\Lambda'(\bar{\phi})\right) \ ,$$

$$\gamma_C = -\left(\frac{2\omega+2}{2\omega+3}\right)\sum_I \frac{4\pi G\rho_I}{\bar{\phi}} - \left(\frac{1}{2\omega+3}\right)\left(\sum_I \frac{12\pi G p_I}{\bar{\phi}} + \frac{\omega'}{2\bar{\phi}}\dot{\bar{\phi}}^2 + \bar{\phi}\Lambda'(\bar{\phi})\right)$$
$$- \frac{\omega(\bar{\phi})}{4\bar{\phi}^2}\dot{\bar{\phi}}^2 - \left(\frac{2\omega+1}{4\omega+6}\right)\Lambda(\bar{\phi}) - \frac{\ddot{\bar{\phi}}}{2\bar{\phi}} \ .$$

Scalar-tensor theories of gravity

Arxiv: 1610.08039

The cosmological expansion equations are given by

$$\frac{\ddot{a}}{a} = -\left(\frac{\omega+3}{6\omega+9}\right)\frac{8\pi G}{\bar{\phi}}\langle\rho_M\rangle - \left(\frac{\omega+3}{6\omega+9}\right)\frac{8\pi G}{\bar{\phi}}\sum_I \rho_I - \frac{8\pi G}{\bar{\phi}}\sum_I p_I\left(\frac{\omega}{2\omega+3}\right)$$
$$-\frac{\omega(\bar{\phi})}{3\bar{\phi}^2}\dot{\bar{\phi}}^2 + \frac{\dot{\bar{\phi}}\dot{a}}{\bar{\phi}a} + \Lambda(\bar{\phi})\left(\frac{2\omega}{6\omega+9}\right) + \frac{1}{2\omega+3}\left(\frac{\omega'}{2\bar{\phi}}\dot{\bar{\phi}}^2 + \Lambda'(\bar{\phi})\right) ,$$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3\bar{\phi}}\langle\rho_M\rangle + \frac{8\pi G}{3\bar{\phi}}\sum_I \rho_I + \frac{\omega(\bar{\phi})}{6\bar{\phi}^2}\dot{\bar{\phi}}^2 - \frac{\dot{\bar{\phi}}\dot{a}}{\bar{\phi}a} + \frac{\Lambda(\bar{\phi})}{3} ,$$

$$\frac{\ddot{\bar{\phi}}}{\bar{\phi}} = \frac{1}{2\omega+3}\left(\frac{8\pi G}{\bar{\phi}}\left(\langle\rho_M\rangle + \sum_I(\rho_I - 3p_I)\right) - \frac{\omega'\dot{\bar{\phi}}^2}{\bar{\phi}} + 2\Lambda(\bar{\phi}) - 2\bar{\phi}\Lambda'(\bar{\phi})\right) - 3\frac{\dot{a}\dot{\bar{\phi}}}{a\bar{\phi}}$$

Other examples

Arxiv: 1610.08039

- Dark energy models like quintessence

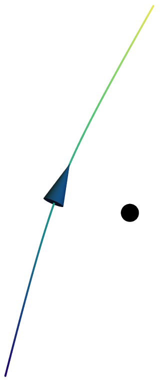
$$\rho_I = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p_I = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

- Vector-tensor theories of gravity

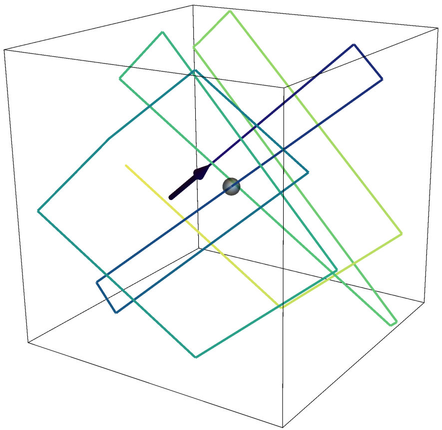
$$L = \frac{1}{16\pi G}\left[R + \omega A_a A^a R + \eta A^a A^b R_{ab} - \epsilon F^{ab} F_{ab} + \tau A_{a;b} A^{a;b}\right] + L_m(\psi, g_{ab})$$

Illustrations

Arxiv: 1705.02328



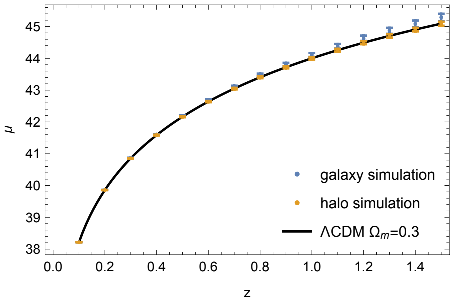
: Point Mass

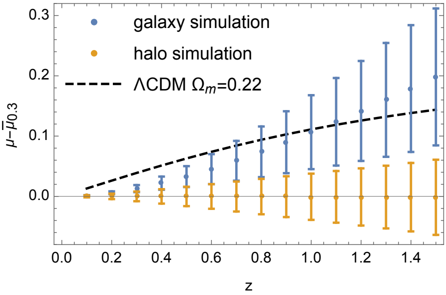


: Homogeneous Halo

Hubble Diagrams

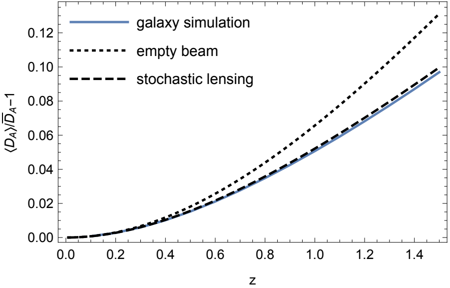
Arxiv: 1705.02328

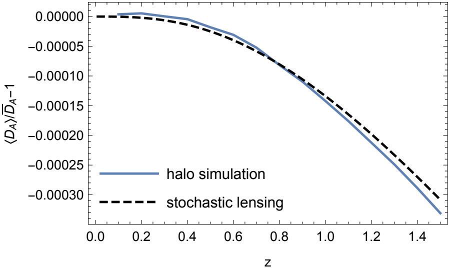




Comparison with Stochastic Lensing Formalism

Arxiv: 1705.02328





Summary and Future Work

- We have constructed a parametrization that requires only 4 functions of time to parameterize a large class of metric theories of gravity in cosmology.
- In GR, we have calculated the effect of small-scale inhomogeneities on observables in these type of models.
- Apply it to more general distributions of matter
- Constrain the parameters using observations

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