Preheating after Multi-field Inflation with Non-minimal Couplings

Evangelos Sfakianakis

from: University of Illinois at Urbana-Champaign
to: Nikhef & University of Leiden

COSMO 2017

with M. P. DeCross, D. I. Kaiser, A. Prabhu, C. Prescod-Weinstein

based mostly on arXiv:1510.08553, 1610.08868, 1610.08916

Conformal Transformations

$$S_{\mathrm{Jordan}} = \int d^4x \sqrt{-\tilde{g}} \left[f(\phi^I)\tilde{R} - \frac{1}{2}\tilde{\mathcal{G}}_{IJ}\tilde{g}^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - \tilde{V}(\phi^I) \right]$$

$$\boxed{g_{\mu\nu}(x) = \frac{2}{M_{\mathrm{pl}}^2} f(\phi^I(x))\, \tilde{g}_{\mu\nu}(x)}$$

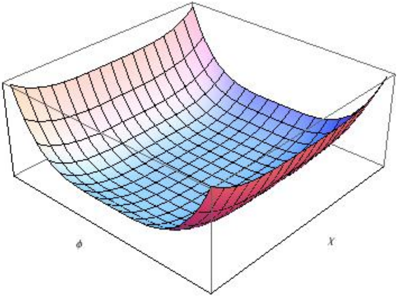
$$S_{\mathrm{Einstein}} = \int d^4x \sqrt{-g} \left[\frac{M_{\mathrm{pl}}^2}{2} R - \frac{1}{2}\mathcal{G}_{IJ} g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I) \right]$$

$$\boxed{V(\phi^I) = \frac{M_{\mathrm{pl}}^4}{4f^2(\phi^I)}\tilde{V}(\phi^I)}$$

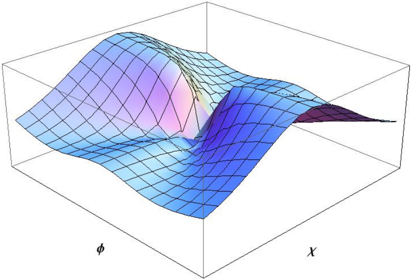
We specify $f(\phi^I) = \frac{1}{2}[M_{Pl}^2 + \xi_I(\phi^I)^2]$ (e.g. from RG counter-terms)

Potential: Jordan vs Einstein

Potential "stretching" factor: $f(\phi,\chi) = \frac{1}{2}\left[M_{Pl}^2 + \xi_\phi \phi^2 + \xi_\chi \chi^2\right]$



$V(\phi,\chi) = \frac{\lambda_\phi}{4}\phi^4 + \frac{\lambda_\chi}{4}\chi^4 + \frac{g}{2}\phi^2\chi^2$

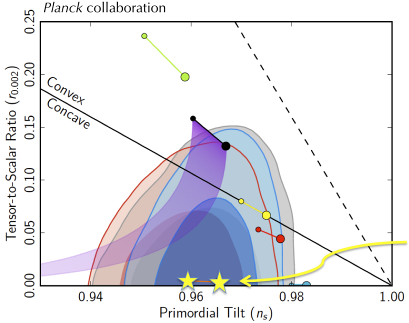


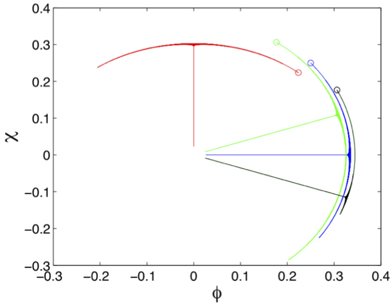
concave (flat) potential

$$V(\phi^I) \to \frac{M_{Pl}^2}{4}\frac{\lambda_I}{\xi_I} \Rightarrow H = \frac{M_{Pl}}{\sqrt{12}}\sqrt{\frac{\lambda}{\xi^2}}$$

Observables

Starting from **generic** initial conditions, the inflaton quickly settles to an **attractor** solution.





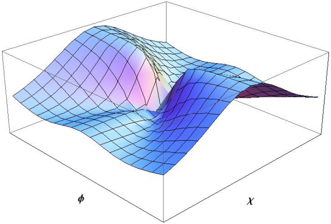
Results for the spectral tilt, running of the tilt and tensor to scalar ratio, insensitive to **initial conditions** AND **couplings**.

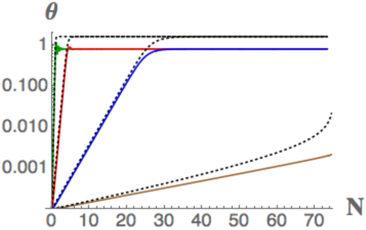
D.I. Kaiser & E.I.S., **PRL** 2014

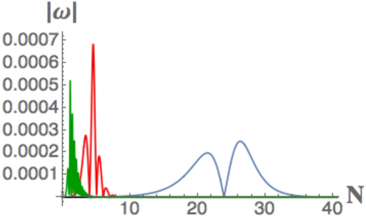
Quantifying the attractor

$$\frac{d^2\chi}{dN^2} + 3\frac{d\chi}{dN} - 12\,\tilde{\Lambda}_\phi \xi_\phi\,\chi = 0$$

$$\tilde{\Lambda}_\phi \equiv \left(\frac{\xi_\chi}{\xi_\phi} - \frac{g}{\lambda_\phi}\right)$$







Larger ξ_ϕ leads to a stronger attractor

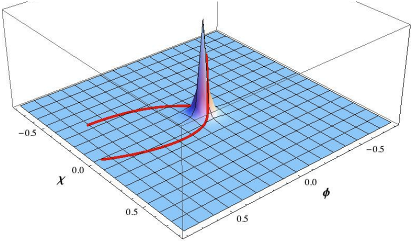
The need for a covariant formalism

In the Einstein frame, the field-space manifold is curved:

$$\mathcal{G}_{IJ}(\phi^K) = \left(\frac{M_{Pl}^2}{2f(\phi^K)}\right)\left[\delta_{IJ} + \frac{3}{f(\phi^K)} f_{,I} f_{,J}\right] \neq F(\phi^K)\delta_{IJ}$$

ϕ^I: coordinates in field space $\longleftrightarrow x^\mu$
$\mathcal{G}_{IJ}$: metric on field space $\longleftrightarrow g_{\mu\nu}$
$\mathcal{D}_J A^I = \partial_J A^I + \Gamma^I_{JK} A^K$

We can "turn off" the potential and visualize the effects of the field-space metric alone.



D.I. Kaiser. E.A. Mazenc & E.I.S., **PRD** 2013

The need for a covariant formalism

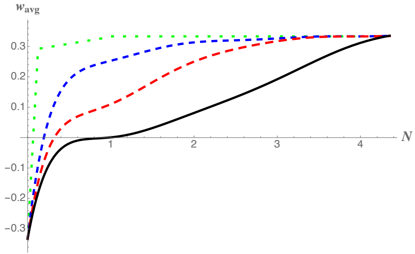
$$\boxed{\mathcal{G}_{IJ}(\phi^K) = \left(\frac{M_{Pl}^2}{2f(\phi^K)}\right)\left[\delta_{IJ} + \frac{3}{f(\phi^K)} f_{,I} f_{,J}\right] \neq F(\phi^K)\delta_{IJ}}$$

Modified Virial Theorem:

$$\langle \dot{\sigma}^2 \rangle + \frac{1}{2}\langle \dot{\sigma}^2 \cdot \phi \partial_\phi \ln \mathcal{G}_{\phi\phi} \rangle = 2 M_{\rm Pl}^2 \left\langle \frac{V}{f} \right\rangle$$

$\xi_\phi = 0.1, 1, 10, 100$

$\Rightarrow$ Prolonged period of $w \approx 0$



Effective Mass-squared: Ingredients

$$\partial_\tau^2 \delta\phi_k + (k^2 + a^2 m^2_{\mathrm{eff},\phi})\delta\phi_k = 0 \quad , \quad \partial_\tau^2 \delta\chi_k + (k^2 + a^2 m^2_{\mathrm{eff},\chi})\delta\chi_k = 0$$

$$\boxed{m^2_{\mathrm{eff},\phi} = m^2_{1,\phi} + m^2_{2,\phi} + m^3_{3,\phi} + m^2_{4,\phi}}$$

$$m^2_{1,\phi} \equiv \mathcal{G}^{\phi K}\left(\mathcal{D}_\phi \mathcal{D}_K V\right) \longleftrightarrow \text{potential gradient}$$

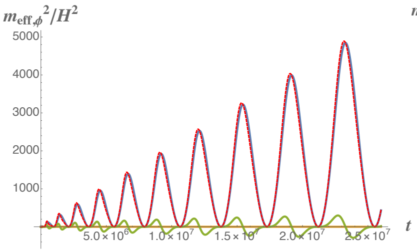
$$m^2_{2,\phi} \equiv -\mathcal{R}^{\phi}{}_{LM\phi}\dot{\varphi}^L \dot{\varphi}^M \longleftrightarrow \text{non-trivial field-space manifold}$$

$$m^3_{3,\phi} \equiv -\frac{\delta^\phi_I \delta^J_\phi}{M_{\mathrm{pl}}^2 a^3}\mathcal{D}_t\left(\frac{a^3}{H}\dot{\varphi}^I \dot{\varphi}_J\right) \longleftrightarrow \text{coupled metric perturbations}$$

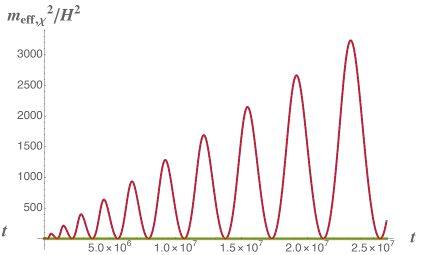
$$m^2_{4,\phi} \equiv -\frac{1}{6}R \longleftrightarrow \text{changes in the background spacetime}$$

Effective Mass-squared: $\xi = 0.1 \ll 1$

Adiabatic



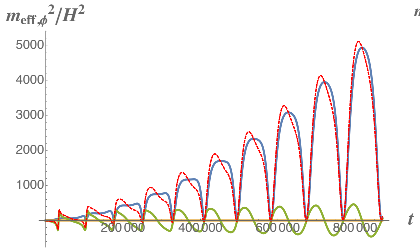
Isocurvature



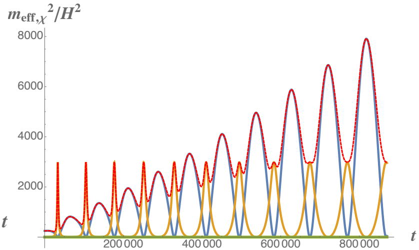
$$m^2_{\text{eff},\phi} \approx m^2_{1,\phi} + m^2_{2,\phi} + m^3_{3,\phi}$$

$$m^2_{\text{eff},\phi} \approx \textit{potential} + \textit{fieldspace} + \textit{metric}$$

Adiabatic



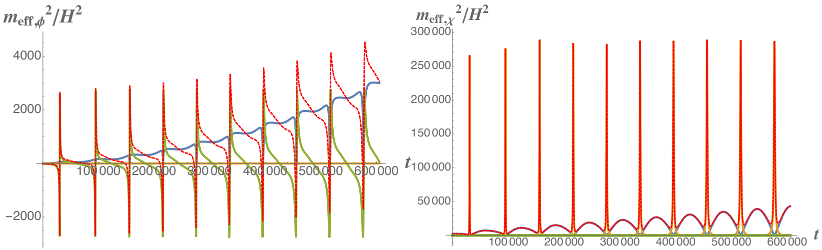
Isocurvature



$$m^2_{\text{eff},\phi} \approx m^2_{1,\phi} + m^2_{2,\phi} + m^3_{3,\phi}$$

$$m^2_{\text{eff},\phi} \approx \textit{potential} + \textit{fieldspace} + \textit{metric}$$

Adiabatic — Isocurvature



$$m^2_{\text{eff},\phi} \approx m^2_{1,\phi} + m^2_{2,\phi} + m^3_{3,\phi}$$

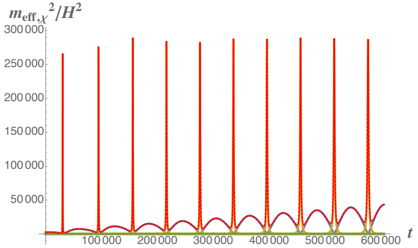
$$m^2_{\text{eff},\phi} \approx \textit{potential} + \textit{fieldspace} + \textit{metric}$$

Isocurvature

A new way to violate adiabaticity

$$\frac{\dot{\omega}}{\omega^2} > 1$$

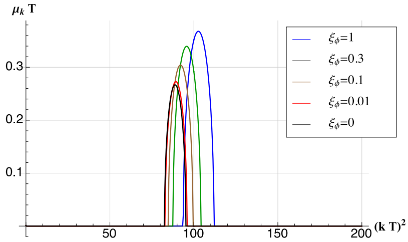
through $\dot{\omega} \gg 1$ rather than $\omega \sim 0$

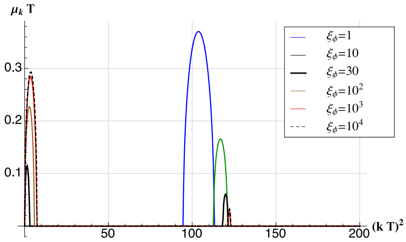


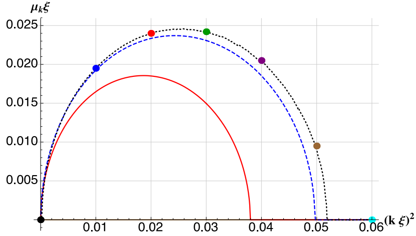
$$m^2_{\text{eff},\phi} \approx m^2_{1,\phi} + m^2_{2,\phi} + m^3_{3,\phi}$$

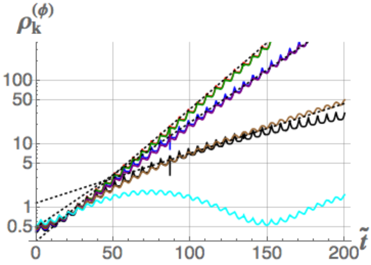
$$m^2_{\text{eff},\phi} \approx \textit{potential} + \textit{fieldspace} + \textit{metric}$$

Adiabatic Modes: Resonance Structure



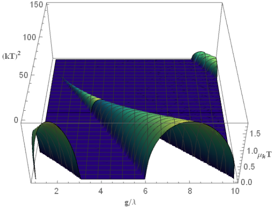




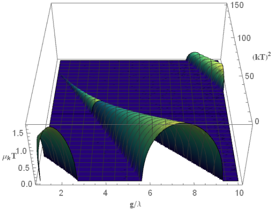


Isocurvature Modes: Resonance Structure (small ξ)

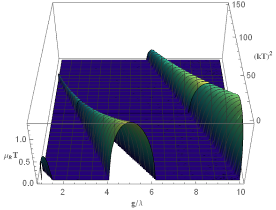
$\xi_\phi = 0$



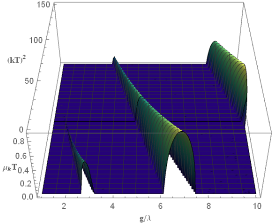
$\xi_\phi = 0.01$



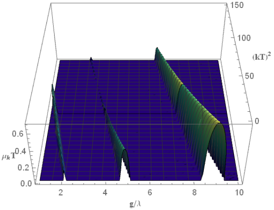
$\xi_\phi = 0.1$



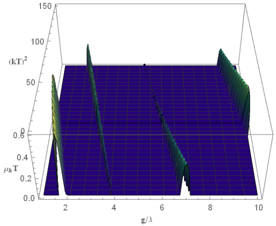
$\xi_\phi = 0.4$



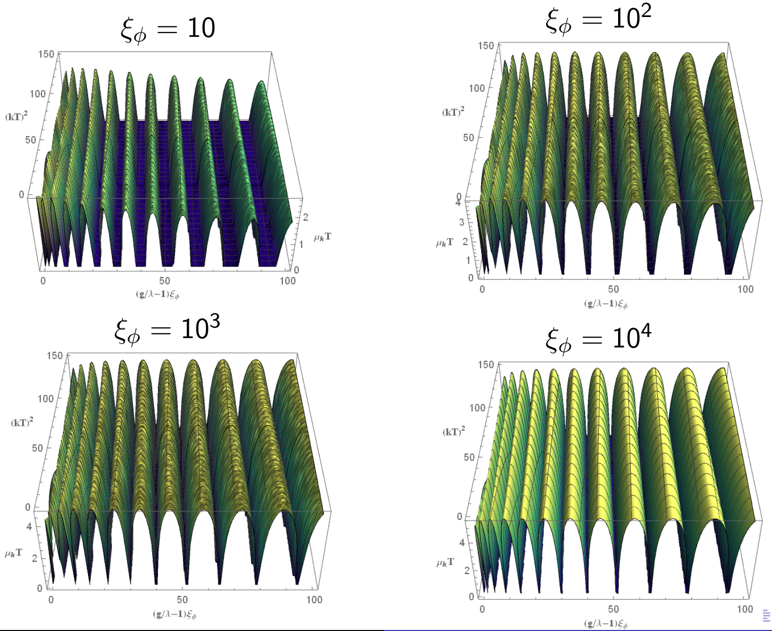
$\xi_\phi = 0.7$



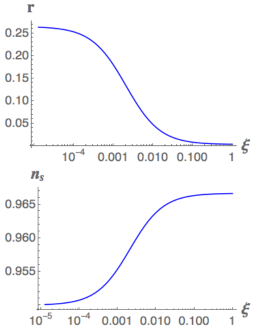
$\xi_\phi = 1$

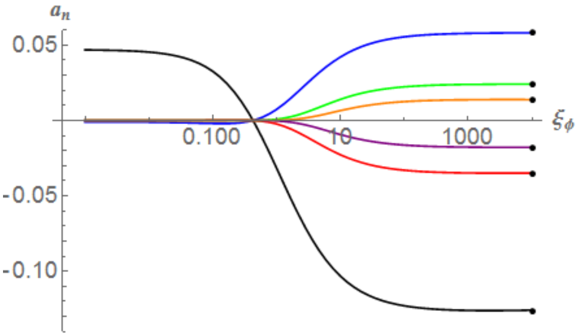


Isocurvature Modes: Resonance Structure (large ξ)



Background Evidence for Intermediate ξ





CMB observables show **two** regimes: $\xi_\phi \ll 1$ and $\xi_\phi > 1$.

BUT

The background spectral content exhibits **three** distinct regimes $\xi_\phi \ll 1$, $\xi_\phi = \mathcal{O}(1)$ and $\xi_\phi > 10$.

Summary

Increasingly strong **attractor**

$\Downarrow$

Simple e.o.m. for **perturbations**

$\Downarrow$

Novel **Field-space** effects

$\Downarrow$ Initial **tachyonic** burst & simple μ_k for low k **adiabatic** modes

$\Downarrow$ **Efficient isocurvature** production for large ξ

$\Downarrow$ **Suppressed isocurvature** production for $1 \lesssim \xi \lesssim 10$

Based (mostly) on the preprints

- M. P. DeCross, D. I. Kaiser, A. Prabhu, C. Prescod-Weinstein and E. I. S. ,
  "Preheating after Multifield Inflation with Nonminimal Couplings, I: Covariant Formalism Attractor Behavior,"
  arXiv:1510.08553 [astro-ph.CO].
- M. P. DeCross, D. I. Kaiser, A. Prabhu, C. Prescod-Weinstein and E. I. S. ,
  "Preheating after Multifield Inflation with Nonminimal Couplings, II: Resonance Structure,"
  arXiv:1610.08868 [astro-ph.CO].
- M. P. DeCross, D. I. Kaiser, A. Prabhu, C. Prescod-Weinstein and E. I. S. ,
  "Preheating after multifield inflation with nonminimal couplings, III: Dynamical spacetime results,"
  arXiv:1610.08916 [astro-ph.CO].

- D. I. Kaiser, **"Conformal transformations with multiple scalar fields,"** Phys. Rev. D **81**, 084044 (2010) [arXiv:1003.1159 [gr-qc]].
- D. I. Kaiser, E. A. Mazenc, E.I.S., **"Primordial bispectrum from multifield inflation with nonminimal couplings,"** Phys. Rev. D **87**, 064004 (2013) [arXiv:1210.7487 [astro-ph.CO]].
- D. I. Kaiser and E.I.S., **"Multifield inflation after Planck: The case for nonminimal couplings,"** Phys. Rev. Lett. **112**, no. 1, 011302 (2014) [arXiv:1304.0363 [astro-ph.CO]].
- K. Schutz, E.I.S. and D. I. Kaiser, **"Multifield Inflation after Planck: Isocurvature Modes from Nonminimal Couplings,"** Phys. Rev. D **89**, no. 6, 064044 (2014) [arXiv:1310.8285 [astro-ph.CO]].

Equations of motion

Backrgound fields:

$$\mathcal{D}_t \dot{\phi}^I + 3H\dot{\phi}^I + \mathcal{G}^{IK} V_{,K} = 0$$

Perturbations:

$$\mathcal{D}_t^2 Q_k^I + 3H\mathcal{D}_t Q_k^I + \left[\frac{k^2}{a^2}\delta^I_J + \mathcal{M}^I_J - \frac{1}{M_{Pl}^2 a^3}\mathcal{D}_t \left(\frac{a^3}{H}\dot{\phi}^I \dot{\phi}_J \right) \right] Q_k^I = 0$$

where

$$\mathcal{M}^I_J = \mathcal{G}^{IK}\mathcal{D}_J \mathcal{D}_K V - \mathcal{R}^I_{LMJ}\dot{\phi}^L \dot{\phi}^M$$

A tale of three trajectories

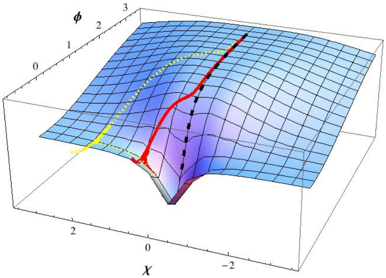
Parameters:

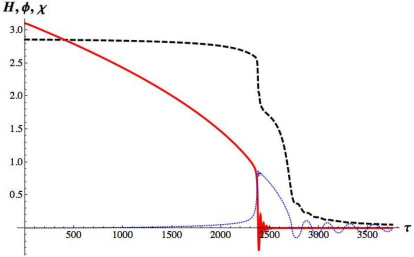
$\xi_\phi = 10, \xi_\chi = 10.02, \frac{\lambda_\chi}{\lambda_\phi} = 0.5, \frac{g}{\lambda_\phi} = 1$

Initial Conditions:

$\phi(0) = 3.1 \;,\; \dot{\phi}(0) = \dot{\chi}(0) = 0$

- Yellow ($\chi = 1.1 \times 10^{-2}$)
  - No isocurvature
  - No f_{NL}
- Red ($\chi = 1.1 \times 10^{-3}$)
  - Some isocurvature
  - Large f_{NL}
- Black ($\chi = 1.1 \times 10^{-4}$)
  - Large isocurvature
  - No f_{NL}





Fluctuations

$$\hat{X}^\phi(x^\mu) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(v_k e_1^\phi \hat{b}_\mathbf{k} + w_k e_2^\phi \hat{c}_\mathbf{k} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \right.$$
$$\left. + \left(v_k^* e_1^\phi \hat{b}_\mathbf{k}^\dagger + w_k^* e_2^\phi \hat{c}_\mathbf{k}^\dagger \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right],$$
$$\hat{X}^\chi(x^\mu) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(y_k e_1^\chi \hat{b}_\mathbf{k} + z_k e_2^\chi \hat{c}_\mathbf{k} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \right.$$
$$\left. + \left(y_k^* e_1^\chi \hat{b}_\mathbf{k}^\dagger + z_k^* e_2^\chi \hat{c}_\mathbf{k}^\dagger \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

$$\left(v_k'' + \Omega^2_{(\phi)} v_k \right) e_1^\phi = -a^2 \mathcal{M}^\phi{}_\chi y_k e_1^\chi,$$
$$\left(w_k'' + \Omega^2_{(\phi)} w_k \right) e_2^\phi = -a^2 \mathcal{M}^\phi{}_\chi z_k e_2^\chi,$$
$$\left(y_k'' + \Omega^2_{(\chi)} y_k \right) e_1^\chi = -a^2 \mathcal{M}^\chi{}_\phi v_k e_1^\phi,$$
$$\left(z_k'' + \Omega^2_{(\chi)} z_k \right) e_2^\chi = -a^2 \mathcal{M}^\chi{}_\phi w_k e_2^\phi,$$

Vielbeins + Attractor = Simplicity

- Vielbeins hide all field-space structure $\delta^{bc} e_b^I(\eta) e_c^J(\eta) = \mathcal{G}^{IJ}(\eta)$
  Note: the previous slide was already a simplification of the original.

- The attractor makes vielbeins almost trivial

$$e_b^I \to \begin{pmatrix} \sqrt{\mathcal{G}^{\chi\chi}} & 0 \\ 0 & \sqrt{\mathcal{G}^{\chi\chi}} \end{pmatrix}$$

$$\hat{X}^\phi(x^\mu) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(v_k e_1^{\phi} \hat{b}_{\mathbf{k}} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \left(v_k^* e_1^{\phi} \hat{b}_{\mathbf{k}}^\dagger \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

$$\hat{X}^\chi(x^\mu) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\left(z_k e_2^{\chi} \hat{c}_{\mathbf{k}} \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \left(z_k^* e_2^{\chi} \hat{c}_{\mathbf{k}}^\dagger \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

$$v_k'' + \Omega_{(\phi)}^2(k,\eta)\, v_k \simeq 0\,, \quad v_k = \delta\phi_k$$

$$z_k'' + \Omega_{(\chi)}^2(k,\eta)\, z_k \simeq 0\,, \quad z_k = \delta\chi_k$$

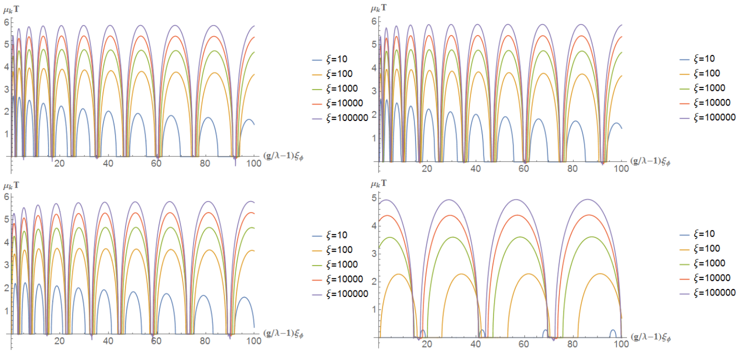
Can we find a scaling solution for the large ξ_ϕ region, as in the adiabatic modes??

Remember that the attractor strength is governed by

$$\xi_\phi \tilde{\Lambda}_\phi = \xi_\phi \left[\frac{\xi_\chi}{\xi_\phi} - \frac{g}{\lambda_\phi} \right] \to \xi_\phi \left[1 - \frac{g}{\lambda_\phi} \right]$$

Choose the combination $-\xi_\phi \tilde{\Lambda}_\phi$ as the "coupling" axis in our Floquet chart for large ξ_ϕ, instead of the minimal coupling g

Isocurvature Floquet chart - large $\xi_\phi \to$ 2D slices



→ **Dense forest of large, almost parallel instability bands at large ξ_ϕ.**

→ **Slow approach to a scaling solution.**