Stochastic Formalism in Curved Field Space

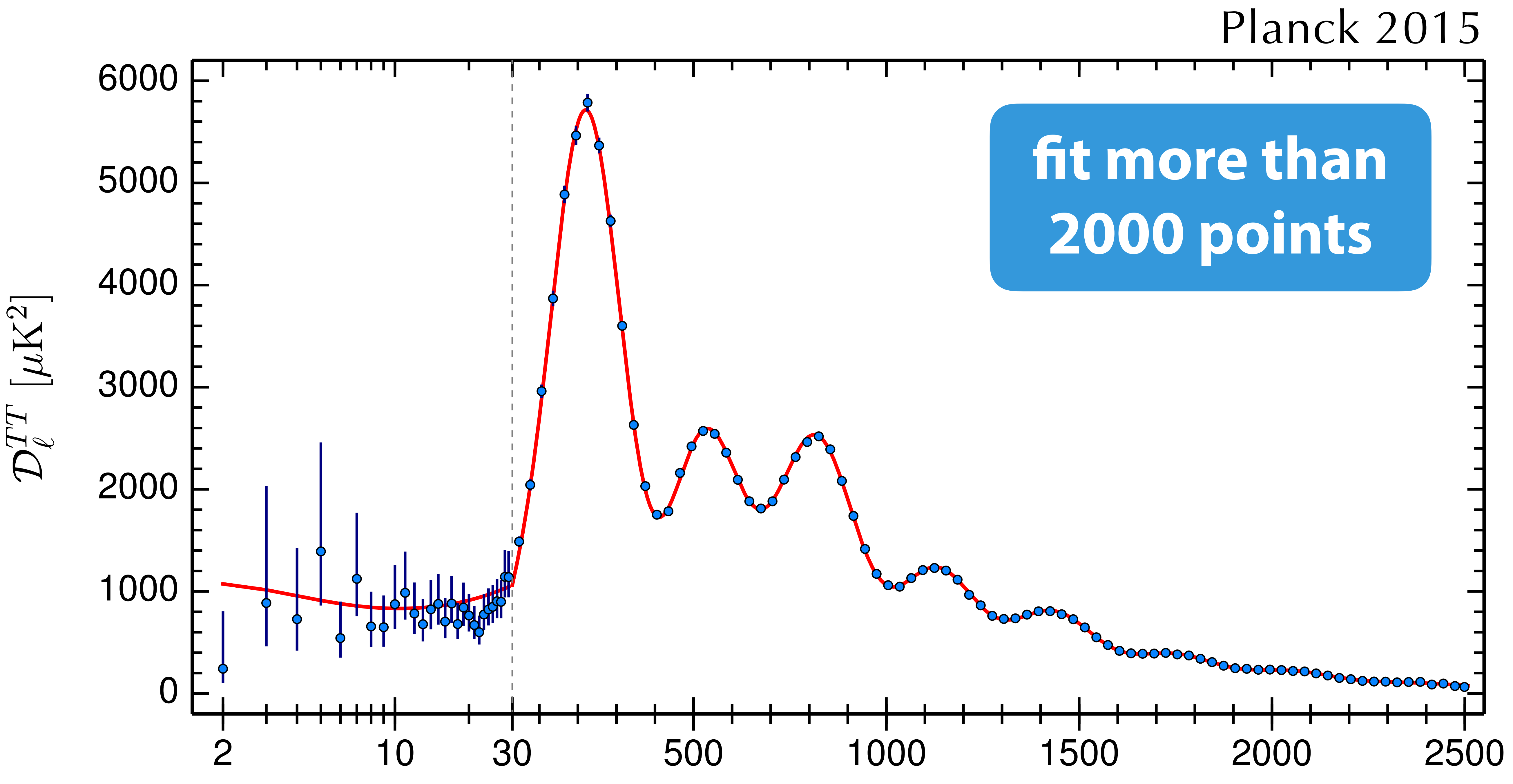
Yuichiro Tada (IAP)

w/ Lucas Pinol and Sébastien Renaux-Petel

in preparation

Current situation of inflation

Currently *inflation mechanism* has achieved great success, but...



w/ only 6 paras. $\Omega_b h^2$, $\Omega_c h^2$, H_0, τ, n_s, A_s

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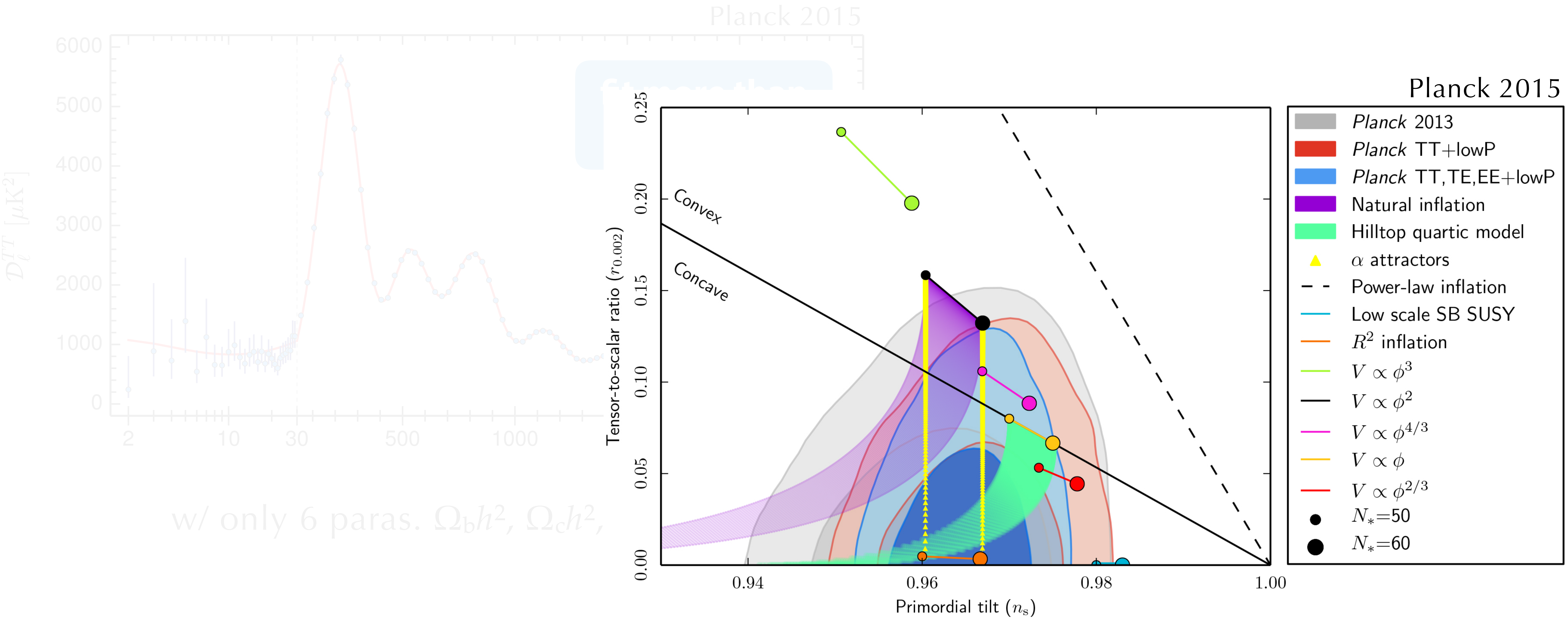

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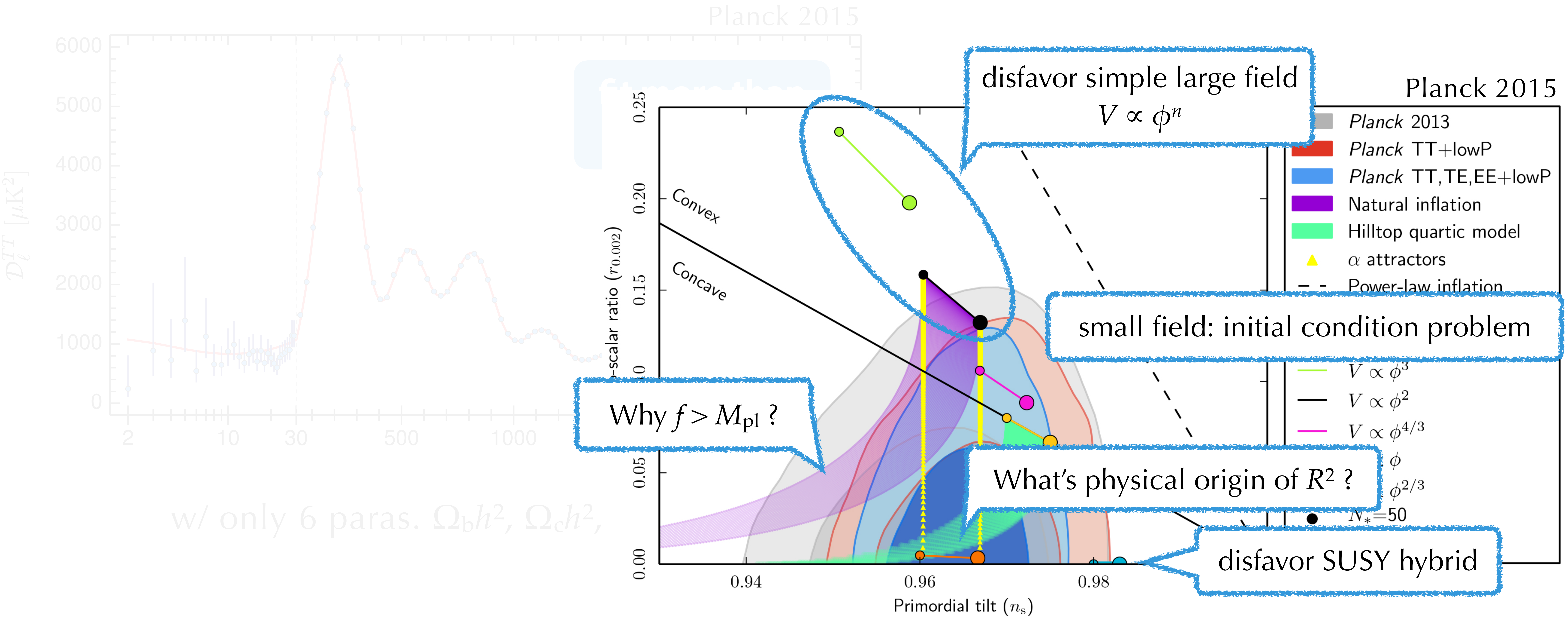
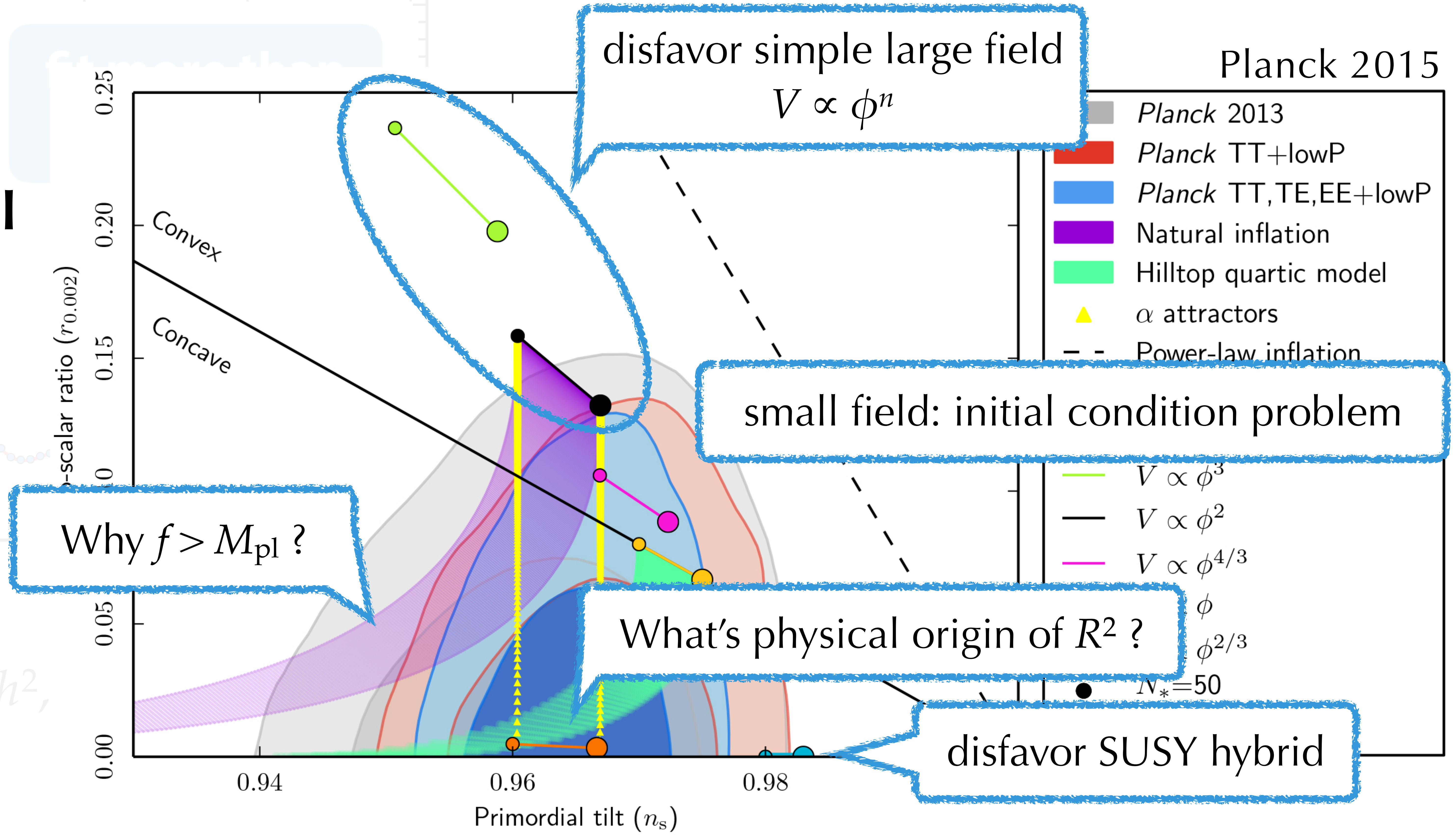


Current situation of inflation

Currently *inflation mechanism* has achieved great success, but…

- **No (positively) favored model**
- **String landscape…**
  → **multi-scalar field?**



Current situation of inflation

- **How to multiplex?**

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

Current situation of inflation

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$$\to V(\phi^1, \phi^2, \cdots)$$

- Multi-natural inflation, Kim, Nilles, Peloso 2005
- Quasi-single field, Chen & Wang 2009

⋮

Current situation of inflation

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$$-\frac{1}{2}g^{\mu\nu}G_{IJ}(\phi)\partial_\mu\phi^I\partial_\nu\phi^J$$

G_{IJ} : non-flat field space metric

Current situation of inflation

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Geometrical destabilization

Renaux-Petel & Turzyński 2015 (Sébastien's talk 17:20–)

$$m_{s,\mathrm{eff}}^2 \supset \epsilon_H H^2 R_{\mathrm{field}} M_{\mathrm{Pl}}^2$$

$$\sim \pm(\text{compact scale})^{-2}$$

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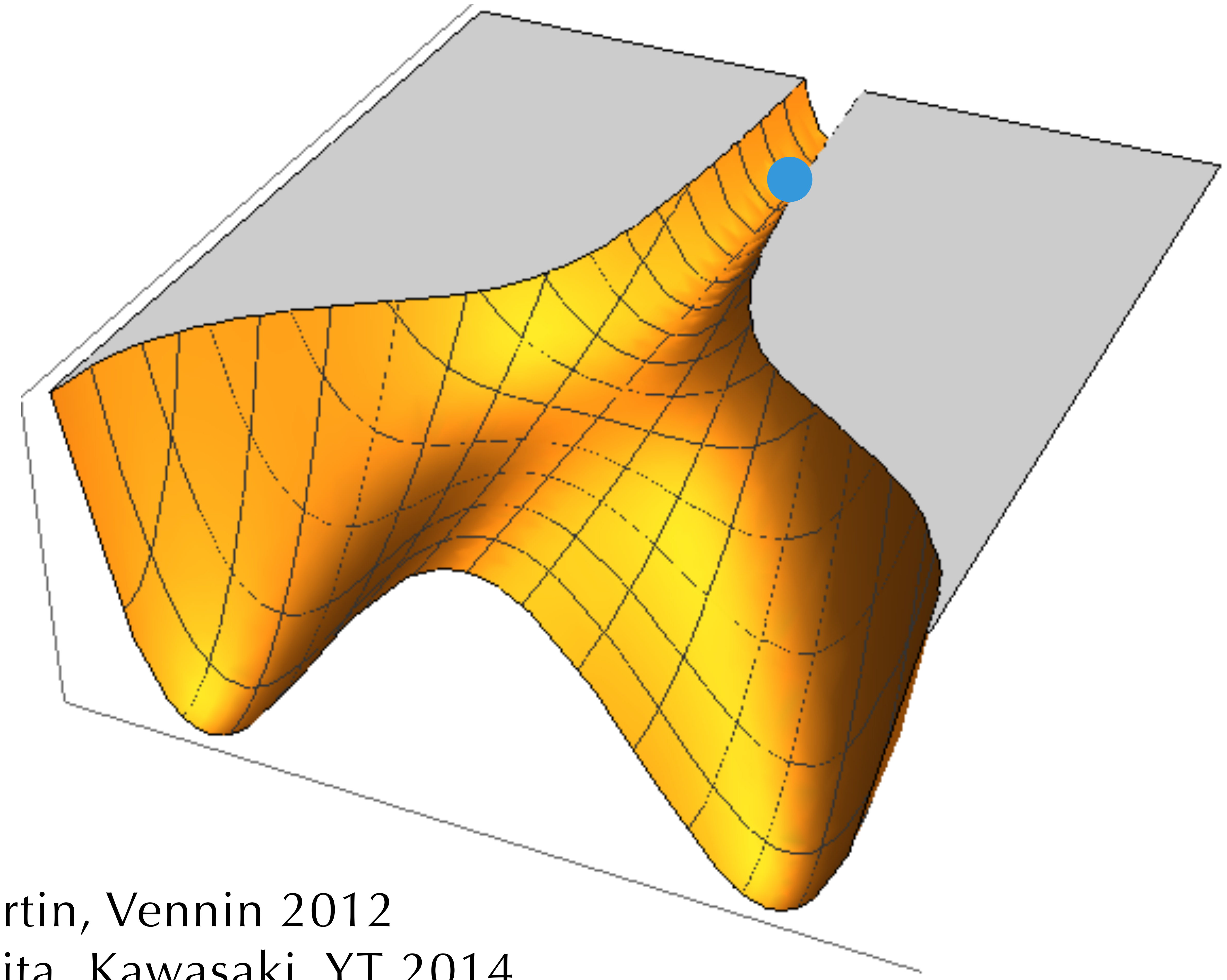
- Hyperinflation: hyperbolic space (const. negative R)

Brown 2017, Mizuno & Mukohyama 2017
(Shuntaro's talk 17:40–)

After destabilization, there might be another slow-roll phase!

Analyze destabilization



Martin, Vennin 2012
Fujita, Kawasaki, YT 2014

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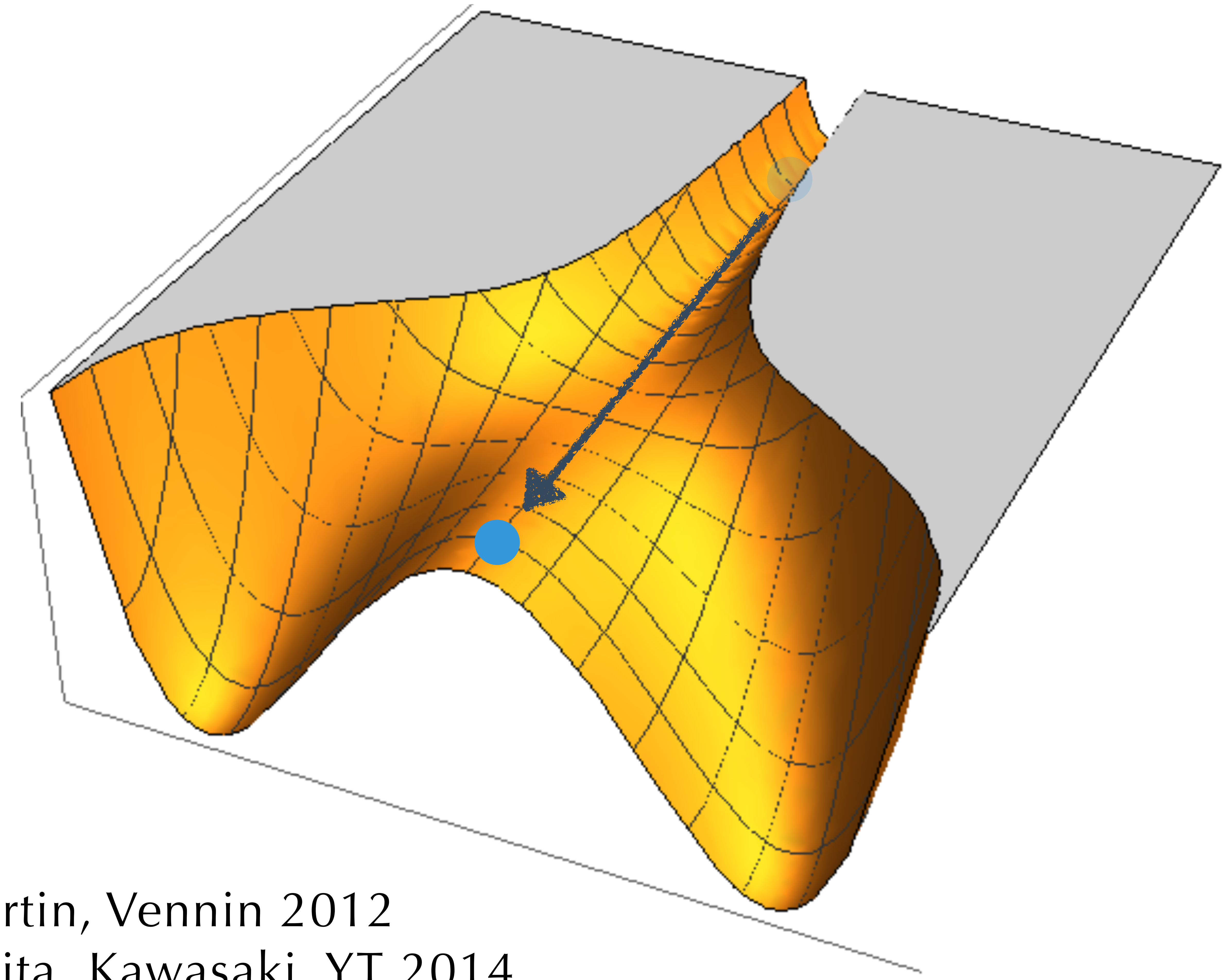

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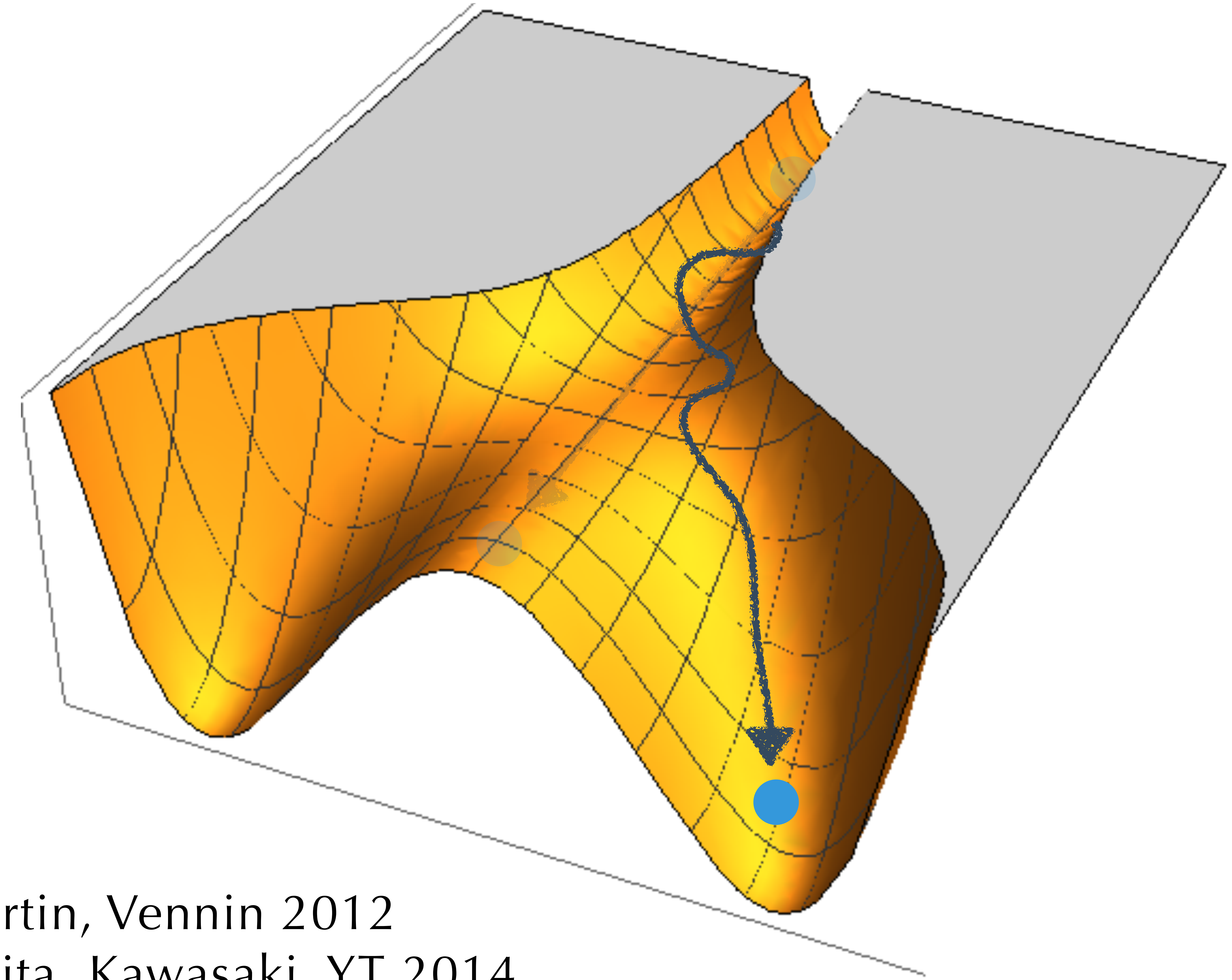

The perturbative approach is broken around the critical point

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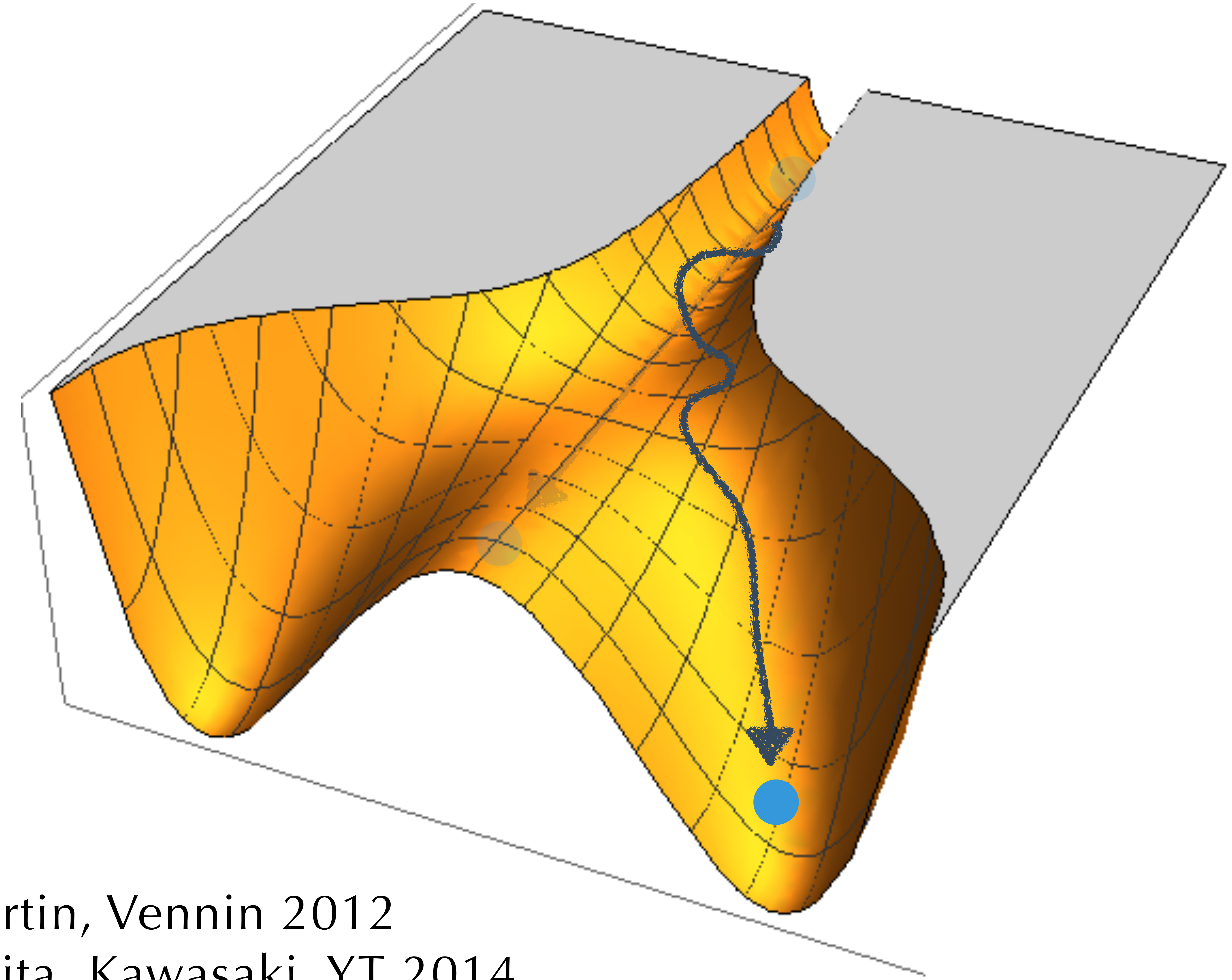

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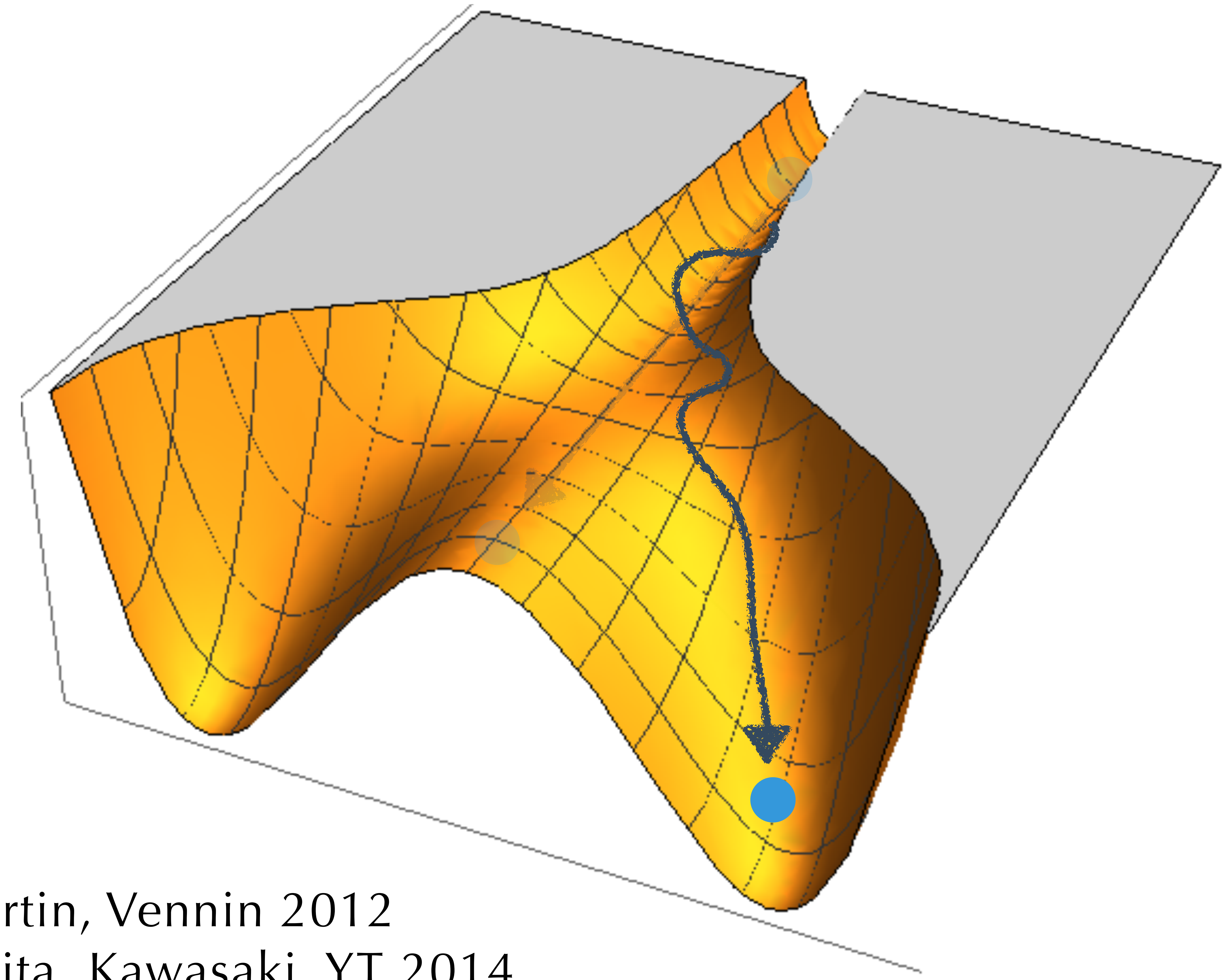
The perturbative approach is broken around the critical point

- standard

b.g.

pert.

Analyze destabilization



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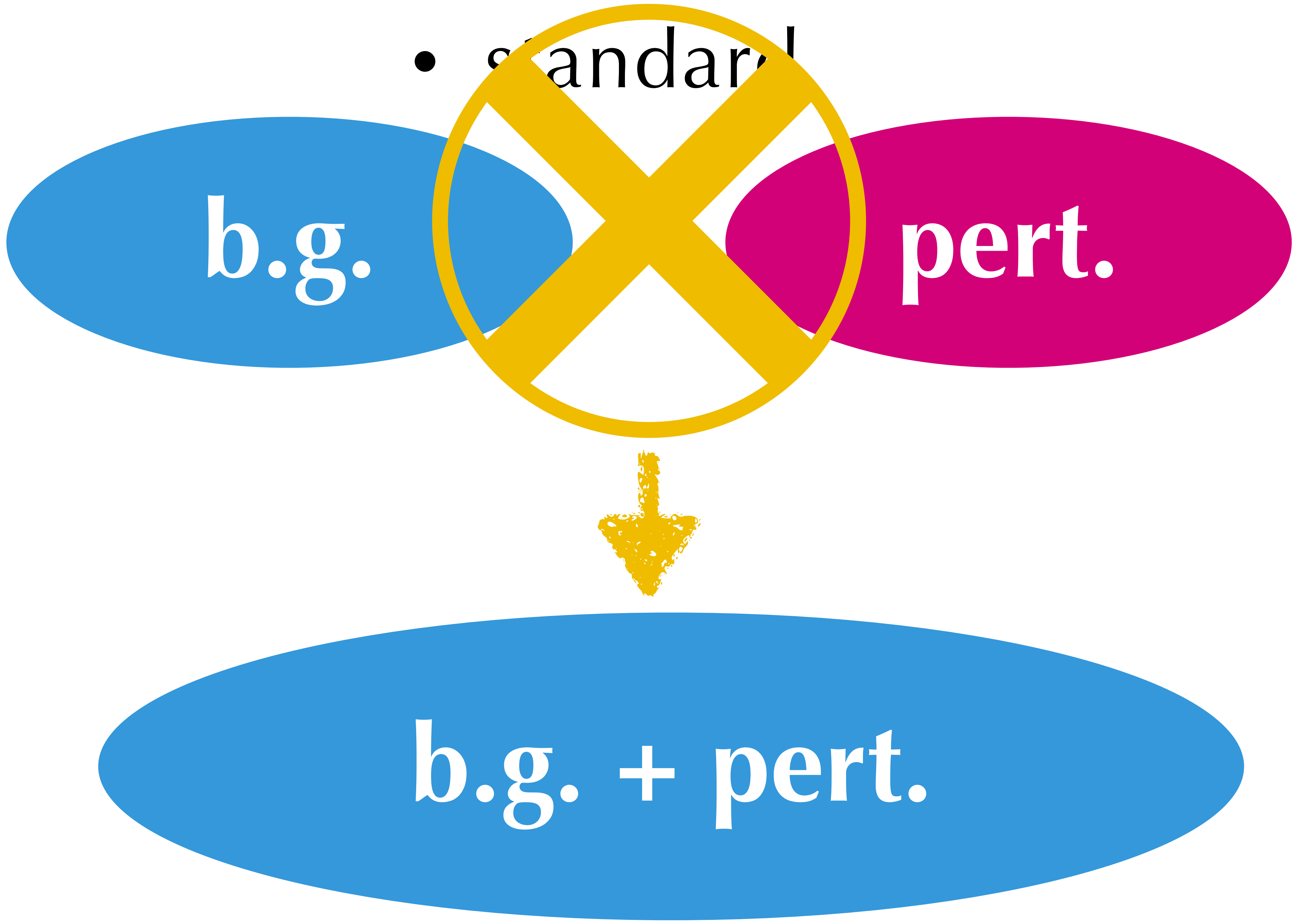
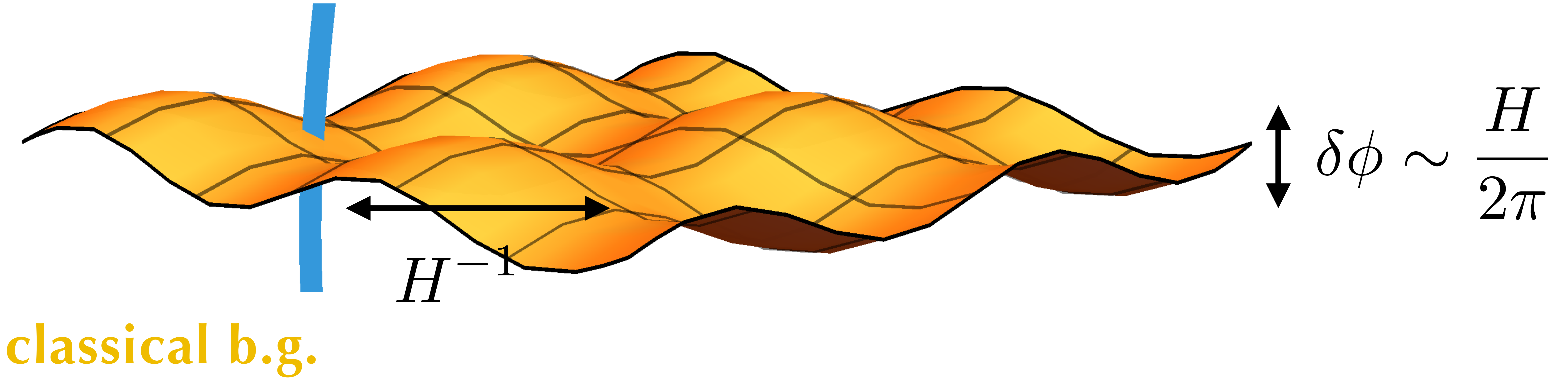


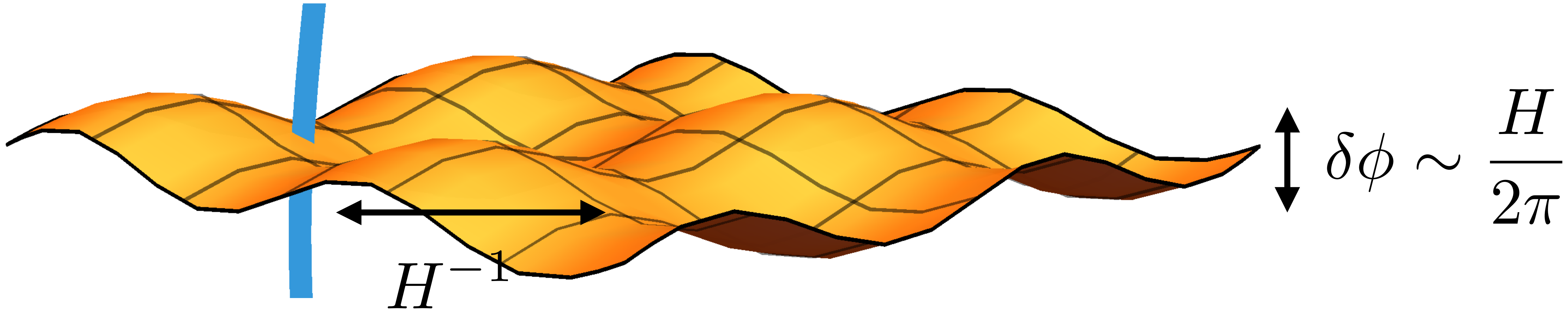
Stochastic formalism

Stochastic formalism Starobinsky 1986



$$\phi_{\rm IR}(t,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\theta(\sigma aH - k)\phi_{\mathbf{k}}(t)\mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad \sigma \ll 1$$

Stochastic formalism Starobinsky 1986



classical b.g.

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- rough expectation EoM for IR modes

$$\begin{cases} \dfrac{\mathrm{d}\phi_0}{\mathrm{d}N} = \dfrac{\pi_0}{a^3 H} \\ \dfrac{\mathrm{d}\pi_0}{\mathrm{d}N} = -\dfrac{a^3}{H}V' \end{cases}$$

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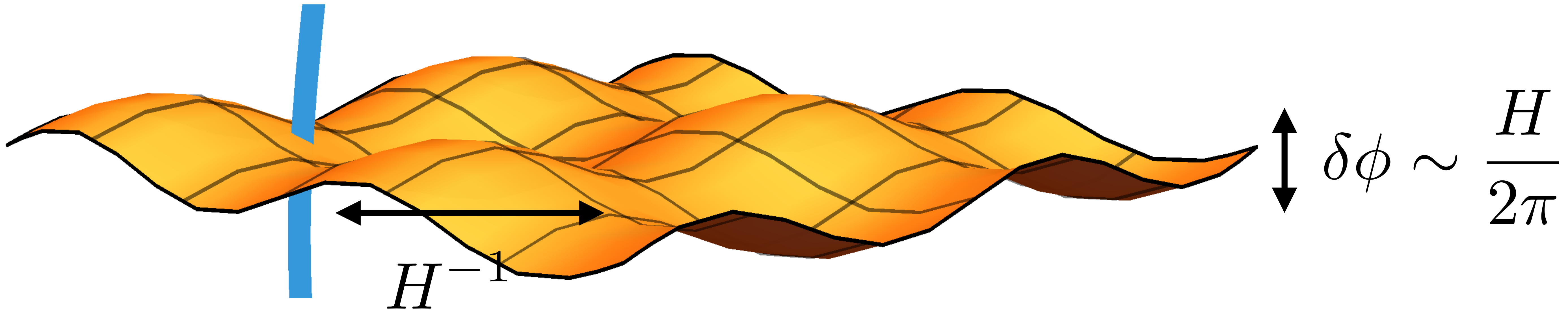

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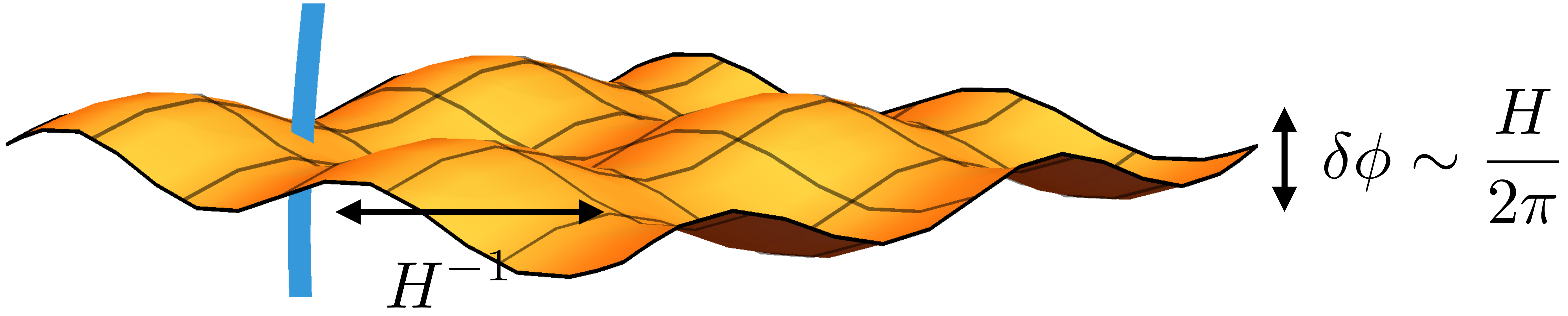

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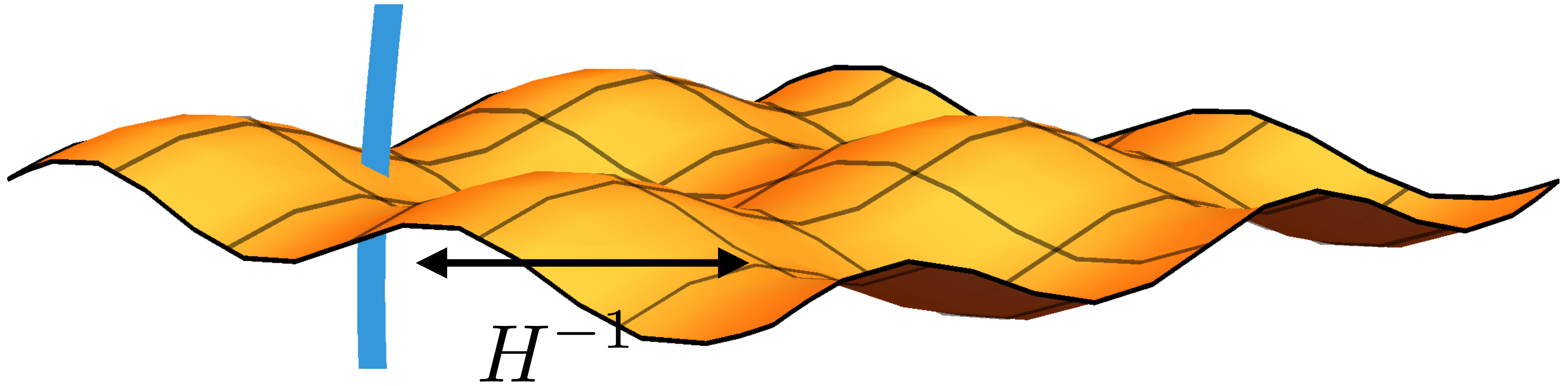
white and Hubble-patch-independent Gaussian noise

$$\begin{cases} \langle \xi_\phi \rangle = 0 \\ \langle \xi_\phi(N,\mathbf{x})\xi_\phi(N',\mathbf{x}')\rangle \simeq \mathcal{P}_\phi \delta(N-N')\theta(1-\sigma a H|\mathbf{x}-\mathbf{x}'|) \end{cases}$$

Stochastic formalism Starobinsky 1986



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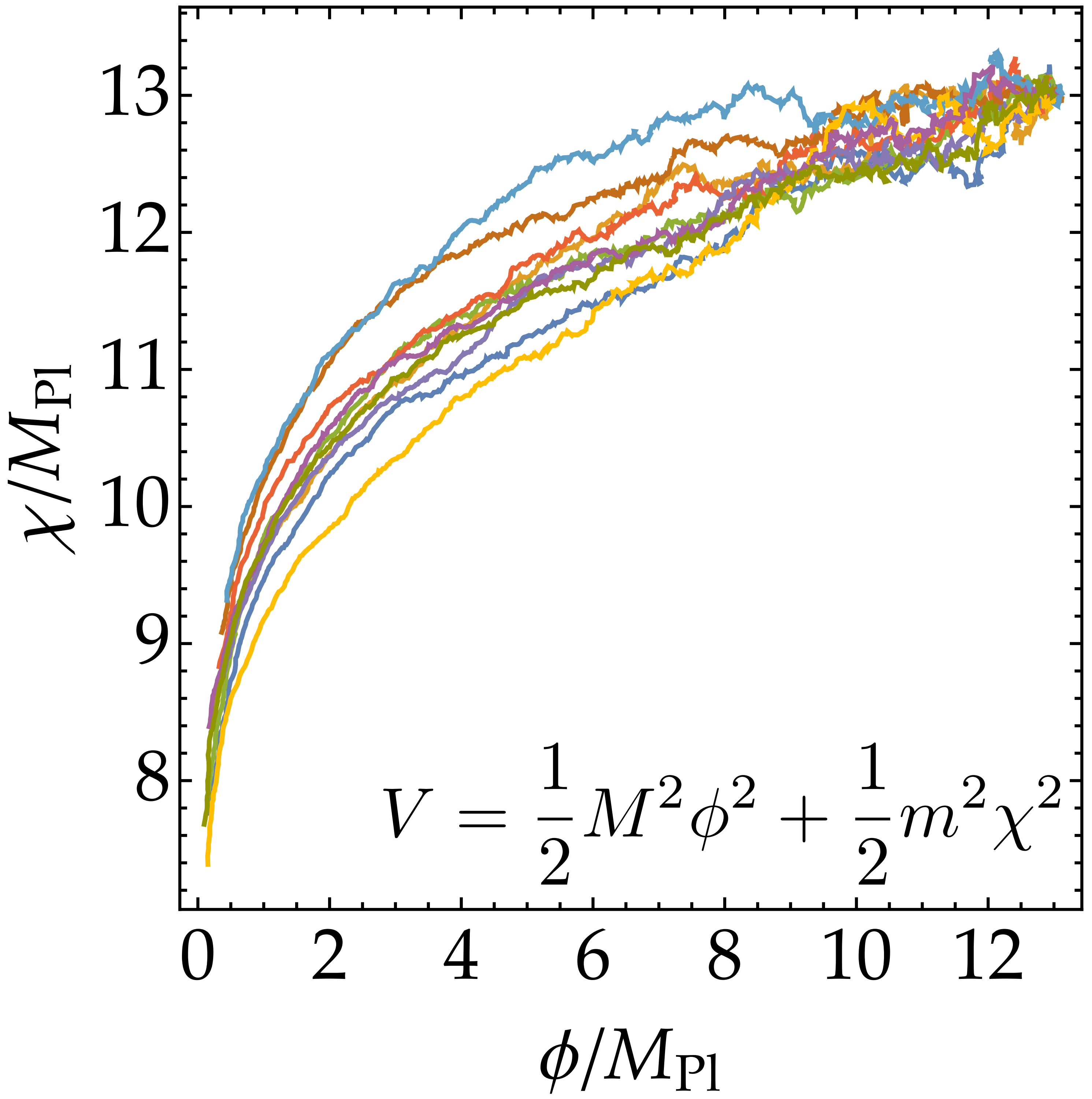

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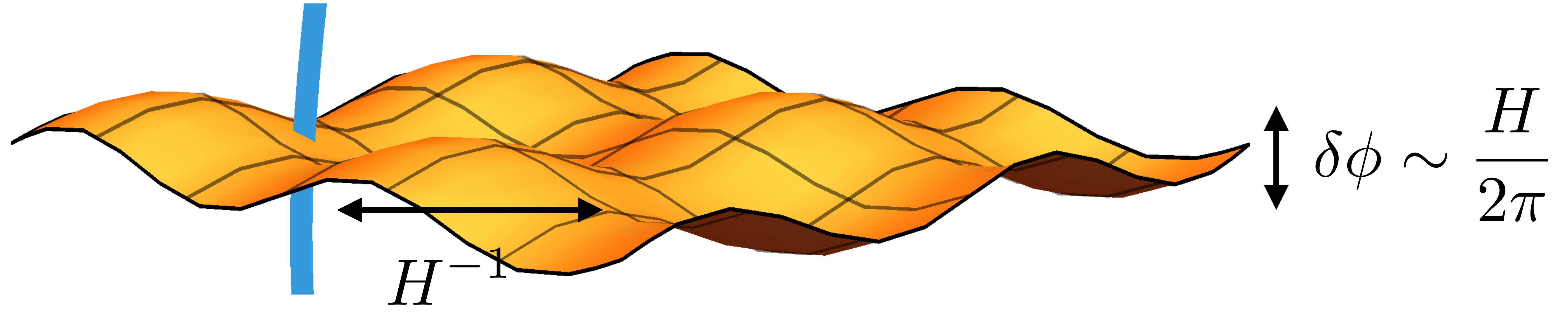

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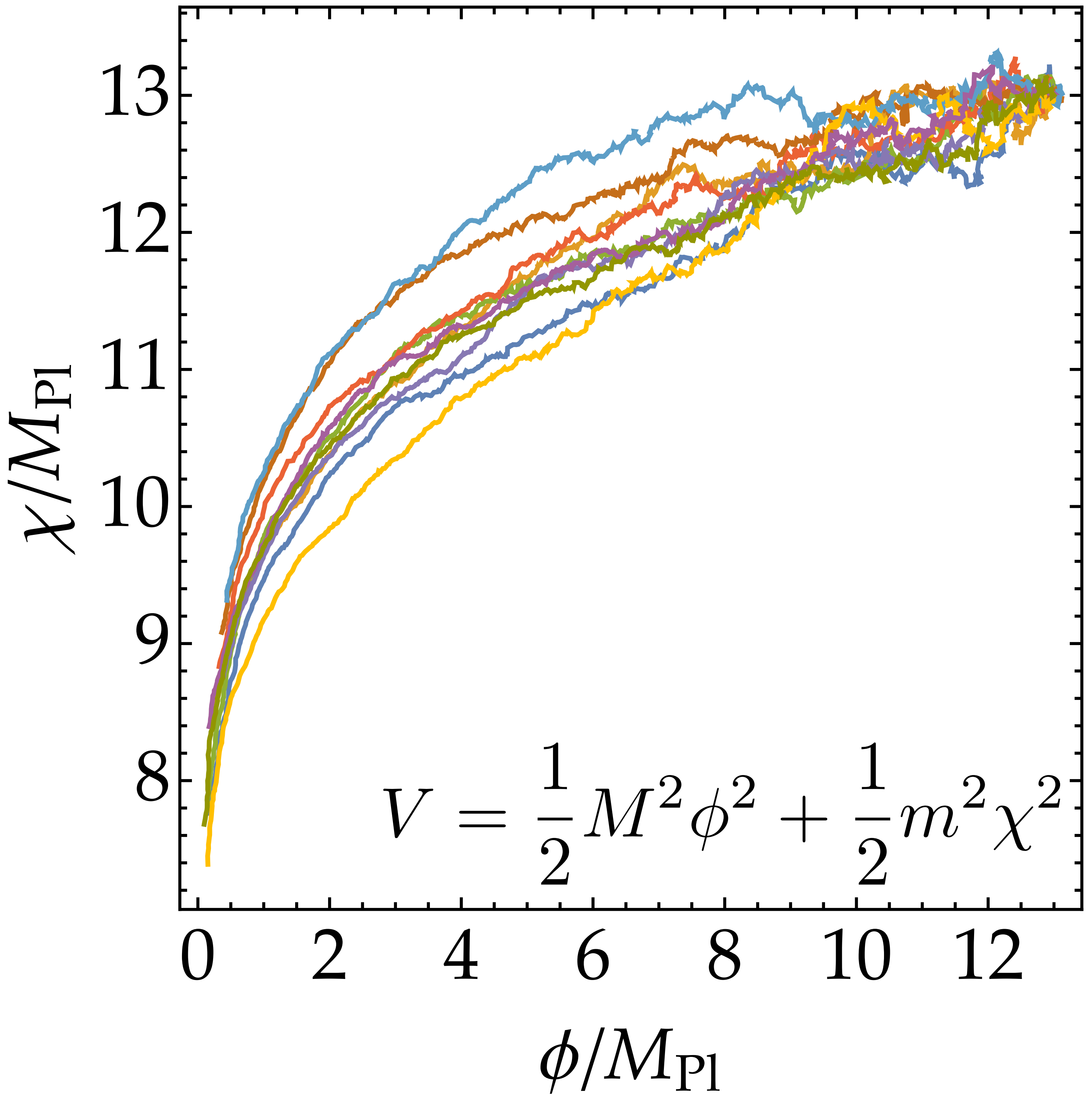

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Stochastic formalism Starobinsky 1986

- **How will be the Langevin EoM in the non-flat field space case?**
- **Can we derive them more rigorously?**

✱ rough expectation EoM for IR modes

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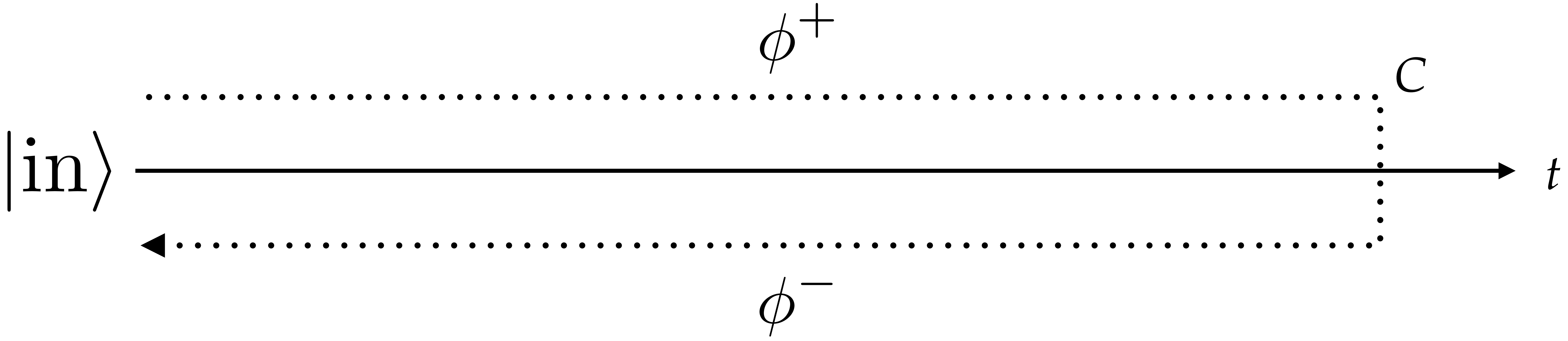
Langevin EoM by in-in formalism

Effective action Γ for $\langle \mathrm{in} | \mathcal{O}_{\mathrm{IR}} | \mathrm{in} \rangle$ is given by path integral on closed time path

$$Z[J] = \mathcal{N} \int_C \mathscr{D}\phi \, \mathrm{e}^{iS + i \int \mathrm{d}^4 x J \phi} = \mathcal{N} \int_{t=-\infty \to \infty} \mathscr{D}\phi^+ \mathscr{D}\phi^- \mathrm{e}^{iS^+ - iS^- + i \int \mathrm{d}^4 x (J^+ \phi^+ - J^- \phi^-)}$$



$$Z[J] = \mathcal{N} \mathrm{e}^{iW[J]}, \quad \to \quad \Gamma[\langle \phi \rangle] = W[J] - J \langle \phi \rangle$$

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- UV modes: quantum but perturbative (quadratic)

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- UV modes: quantum but perturbative (quadratic)
- IR modes: classical (Z is mainly given around $\langle \phi_{\mathrm{IR}} \rangle$) but non-perturbative
- correction terms for IR effective action can be interpreted as noise

Morikawa 1990

Langevin EoM by in-in formalism

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2R^2 - \frac{1}{2}g^{\mu\nu}G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J - V\right] = \int \mathrm{d}^4x\left[\pi_I\dot{\phi}^I - \mathscr{H}\right]$$

ADM in flat gauge

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + a^2\delta_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t)$$

N and β appear in S as multipliers → They can be integrated out in advance

Langevin EoM by in-in formalism

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ADM in flat gauge

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N and β appear in S as multipliers → They can be integrated out in advance

- $S\simeq S^{(0)}+S^{(1)}+S^{(2)}$

$$S^{(2)}=\frac{1}{2}\int\mathrm{d}^4x\,(\phi_{\mathrm{UV},Ia}\,,\,\pi_{\mathrm{UV},Ia})\,\Lambda^I{}_J{}^a{}_b\begin{pmatrix}\phi_{\mathrm{UV}}^{Jb}\\ \pi_{\mathrm{UV}}^{Jb}\end{pmatrix},\quad a,b=+\text{ or }-$$

→ c.f. UV EoM: $\Lambda^I{}_J{}^a{}_b\begin{pmatrix}\phi_{\mathrm{UV}}^{Jb}\\ \pi_{\mathrm{UV}}^{Jb}\end{pmatrix}=0$

Langevin EoM by in-in formalism

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- **IR-UV coupling**

$$S^{(1)}=\int \mathrm{d}^4x\left[\pi_{\mathrm{IR},Ia}\dot{\phi}^{Ia}_{\mathrm{UV}}+\pi_{\mathrm{UV},Ia}\dot{\phi}^{Ia}_{\mathrm{IR}}+\mathcal{O}_{\mathrm{IR}}\mathscr{O}^{(1)}_{\mathrm{UV}}\right]$$

Langevin EoM by in-in formalism

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IR $\perp$ UV

Langevin EoM by in-in formalism

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IR-UV coupling

IR $\perp$ UV

$$S^{(1)} = \int \mathrm{d}^4x \left[\underline{\pi_{\mathrm{IR},Ia} \dot{\phi}_{\mathrm{UV}}^{Ia}} + \pi_{\mathrm{UV},Ia} \dot{\phi}_{\mathrm{IR}}^{Ia} + \cancel{\mathcal{O}_{\mathrm{IR}} \mathcal{O}_{\mathrm{UV}}^{(1)}} \right]$$

$$\supset \dot{\theta}(k - \sigma a H) \pi_{\mathrm{IR}} \phi_k = -\delta(t - t_\sigma) \pi_{\mathrm{IR}} \phi_k, \quad k = \sigma a H \,@\, t_\sigma$$

Langevin EoM by in-in formalism

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IR-UV coupling

IR ⊥ UV

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$$\supset \dot{\theta}(k-\sigma aH)\pi_{\mathrm{IR}}\phi_k = -\delta(t-t_\sigma)\pi_{\mathrm{IR}}\phi_k, \quad k = \sigma aH \,@\, t_\sigma$$

$$\overset{??}{=} \int \mathrm{d}t \int \frac{\mathrm{d}^3k}{(2\pi)^3}\delta(t-t_\sigma)\left[-\pi^\Delta_{\mathrm{IR}}\bar{\phi}_{\mathrm{UV}} + \phi^\Delta_{\mathrm{IR}}\bar{\pi}_{\mathrm{UV}} - \bar{\phi}_{\mathrm{IR}}\pi^\Delta_{\mathrm{UV}} + \bar{\pi}_{\mathrm{UV}}\pi^\Delta_{\mathrm{IR}}\right]$$

Keldysh basis: $\begin{cases}\bar{\phi} = \frac{1}{2}(\phi^+ + \phi^-) \\ \phi^\Delta = \phi^+ - \phi- \end{cases}$

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IR-UV coupling

IR $\perp$ UV

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$$\overset{??}{=} \int \mathrm{d}t \int \frac{\mathrm{d}^3k}{(2\pi)^3} \delta(t - t_\sigma) \left[-\pi_{\mathrm{IR}}^\Delta \bar{\phi}_{\mathrm{UV}} + \phi_{\mathrm{IR}}^\Delta \bar{\pi}_{\mathrm{UV}} - \bar{\phi}_{\mathrm{IR}} \pi_{\mathrm{UV}}^\Delta + \bar{\pi}_{\mathrm{UV}} \pi_{\mathrm{IR}}^\Delta \right]$$

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Tokuda & Tanaka 2017
Junsei's Talk on Monday

dissipation and mass-renormalization
inconsistent with original theory
even in free test particle cases

Langevin EoM by in-in formalism

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\mathrm{Pl}}^2R^2-\frac{1}{2}g^{\mu\nu}G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J-V\right]=\int \mathrm{d}^4x\left[\pi_I\dot{\phi}^I-\mathscr{H}\right]$$

IR-UV coupling

IR $\perp$ UV

$$S^{(1)}=\int \mathrm{d}^4x\left[\pi_{\mathrm{IR},Ia}\dot{\phi}^{Ia}_{\mathrm{UV}}+\pi_{\mathrm{UV},Ia}\dot{\phi}^{Ia}_{\mathrm{IR}}+\mathcal{O}_{\mathrm{IR}}\mathcal{O}^{(1)}_{\mathrm{UV}}\right]$$

$$\supset\dot{\theta}(k-\sigma aH)\pi_{\mathrm{IR}}\phi_k=-\delta(t-t_\sigma)\pi_{\mathrm{IR}}\phi_k,\quad k=\sigma aH\,@\,t_\sigma$$

$$\overset{??}{=}\int\mathrm{d}t\int\frac{\mathrm{d}^3k}{(2\pi)^3}\delta(t-t_\sigma)\left[-\pi^{\Delta}_{\mathrm{IR}}\bar{\phi}_{\mathrm{UV}}+\phi^{\Delta}_{\mathrm{IR}}\bar{\pi}_{\mathrm{UV}}-\bar{\phi}_{\mathrm{IR}}\pi^{\Delta}_{\mathrm{UV}}+\bar{\pi}_{\mathrm{UV}}\pi^{\Delta}_{\mathrm{IR}}\right]$$

Keldysh basis: $\begin{cases}\bar{\phi}=\frac{1}{2}(\phi^++\phi^-)\\ \phi^{\Delta}=\phi^+-\phi-\end{cases}$

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Langevin EoM by in-in formalism

$$\Gamma_{\mathrm{IR}} = S^{(0)+} - S^{(0)-} + S_{\mathrm{IA}}$$

$$S_{\mathrm{IA}} = \frac{i}{2}\int \mathrm{d}^4x\mathrm{d}^4x' \begin{pmatrix}\pi_I^{\Delta}(x) & -\phi_I^{\Delta}(x)\end{pmatrix} \mathrm{Re}[\Pi^I{}_J(x,x')] \begin{pmatrix}\pi^{J\Delta}(x') \\ -\phi^{J\Delta}(x')\end{pmatrix}$$

$$\Pi^I{}_J(x,x') = \frac{\dot{k}_\sigma}{k_\sigma}\frac{\sin k_\sigma r}{k_\sigma r}\delta(t-t')\begin{pmatrix}\mathcal{P}_{\phi\phi}(k_\sigma) & \mathcal{P}_{\phi\pi}(k_\sigma) \\ \mathcal{P}_{\pi\phi}(k_\sigma) & \mathcal{P}_{\pi\pi}(k_\sigma)\end{pmatrix}, \quad k_\sigma = \sigma a H$$

Langevin EoM by in-in formalism

$$\Gamma_{\mathrm{IR}} = S^{(0)+} - S^{(0)-} + S_{\mathrm{IA}}$$

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* **Influence action S_{IA} is pure imaginary**

$$\mathrm{e}^{iS_{\mathrm{IA}}} = \mathrm{e}^{-\mathrm{Im}S_{\mathrm{IA}}} = \int \mathscr{D}\xi \underline{P[\xi]}\mathrm{e}^{i\int \mathrm{d}^4x(\pi_I^\Delta \xi_\phi^I - \phi_I^\Delta \xi_\pi^I)}$$

Gaussian weight

Langevin EoM by in-in formalism

$$S_{\mathrm{eff}} = S^{(0)+} - S^{(0)-} + \int \mathrm{d}^4x (\pi_I^\Delta \xi_\phi^I - \phi_I^\Delta \xi_\pi^I)$$

$$\left.\frac{\delta S}{\delta \phi^\Delta}\right|_{\phi^\Delta=0} = 0 \longrightarrow \begin{cases} \dot{\phi}^I = \dfrac{N_{\mathrm{IR}}}{a^3} G^{IJ}\bar{\pi}_J + \xi_\phi^I \\ D_t \bar{\pi}_I = -a^3 N_{\mathrm{IR}} V_I + \xi_{\pi I} - \Gamma^J_{IK}\bar{\pi}_J \xi_\phi^K \end{cases}$$

$$D_t\bar{\pi}_I = \dot{\bar{\pi}}_I - \Gamma^J_{IK}\dot{\phi}^K\bar{\pi}_J$$

- $\langle \xi^I(x)\xi_J(x')\rangle = \int \mathscr{D}\xi P[\xi]\xi^I(x)\xi_J(x') = \Pi^I{}_J(x,x')$

- S_{IA} does not include N_{IR}

$\longrightarrow$ Friedmann eq.: $\dfrac{3M_{\mathrm{Pl}}^2\mathcal{H}^2}{N_{\mathrm{IR}}^2} = \dfrac{1}{2a^6}G^{IJ}\bar{\pi}_I\bar{\pi}_J + V$

Conclusions

- Stochastic formalism in curved field space by the in-in formalism
- IR-UV coupling should be determined to recover the original propagator (Tanaka & Tokuda 2017)
- Friedmann constraint still holds for each local patch in the stochastic formalism
- Future work: analyze the geometrical destabilization w/ this stochastic formalism