Can we show that cosmological structures are of quantum mechanical origin?

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Outline

Based on - J Martin and **VV** 2015, *arXiv:1510.04038* (PRD)
- J Martin and **VV** 2016, *arXiv:1605.02944* (PRA)
- J Martin and **VV** 2016, *arXiv:1611.01785* (PRA)
- J Martin and **VV** 2017, *arXiv:1706.05001* (PRD)

- Do primordial density fluctuations contain quantum correlations?

- Can we violate Bell inequalities?

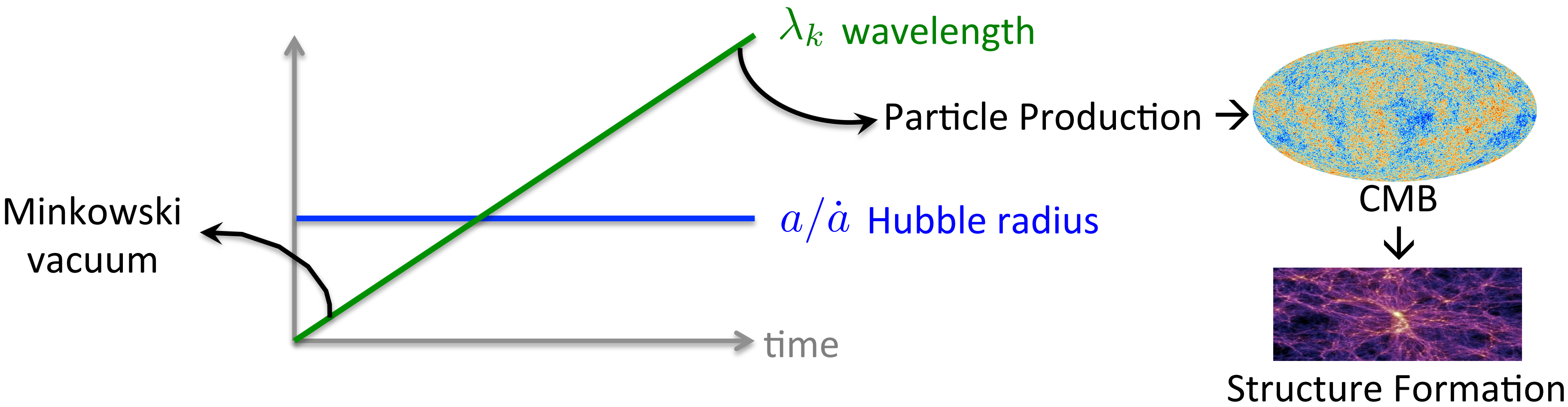
- Can we exclude a purely classical explanation?

Cosmological Perturbations in Inflation

- Inflation is a high energy phase of accelerated expansion in the early Universe

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\vec{x}^2 \quad \text{with} \quad \ddot{a} > 0$$

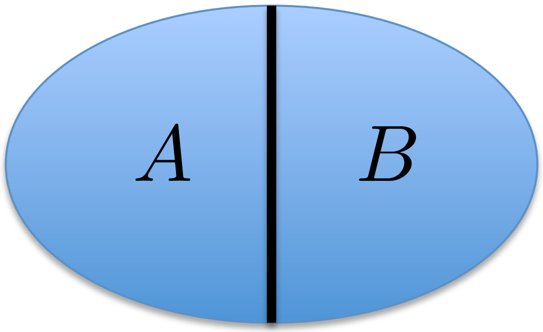
- Quantum vacuum fluctuations are stretched to cosmological scales



Quantum Mechanics on **Cosmological** Scales! *Can we test it?*

Quantum Discord

Henderson and Vedral 2001,
Ollivier and Zurek 2001
Adesso, Bromley, Cianciaruso 2016



Idea: Find two ways to calculate the mutual information between A and B that coincide for classical correlations but may differ in quantum systems

$$\mathcal{I} = S(A) + S(B) - S(A, B)$$

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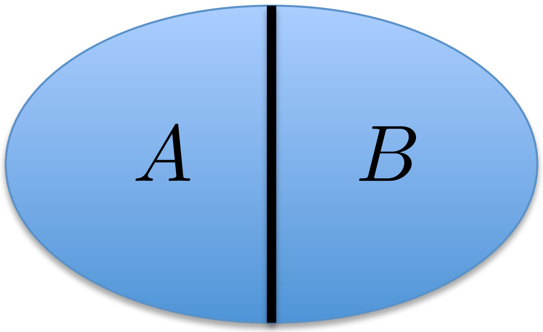

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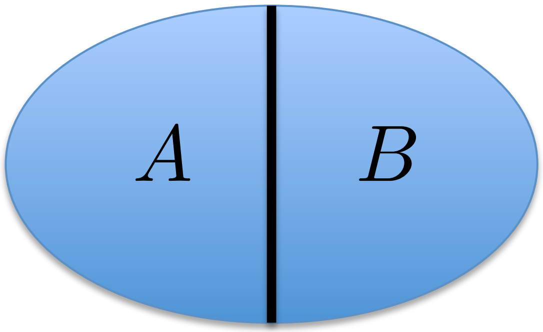

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$\hat{\Pi}_j$: complete set of projectors defined on $\mathcal{E}_B$

$\hat{\rho} \to \hat{\rho}\hat{\Pi}_j/p_j$ with probability $p_j = \mathrm{Tr}\left(\hat{\rho}\hat{\Pi}_j\right)$ and $\rho_{A;\hat{\Pi}_j} = \mathrm{Tr}_B\left(\rho\hat{\Pi}_j/p_j\right)$

$$S(A|B) = \sum_j p_j S\left(\rho_{A;\hat{\Pi}_j}\right)$$

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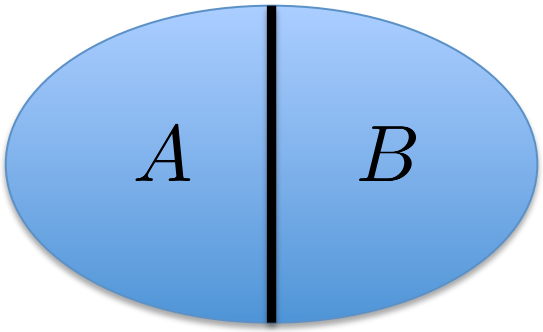

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$$\mathcal{I} = S(A) + S(B) - S(A,B)$$

$$\mathcal{J} = S(A) - S(A|B)$$ with respect to measurements $\hat{\Pi}_j$

$$\delta(A,B) = \min_{\{\hat{\Pi}_j\}} \left(\mathcal{I} - \mathcal{J}\right)$$

Quantum Discord of Cosmic Inflation

- One scalar degree of freedom: $v \propto \zeta$ (curvature perturbation) $\propto \delta T/T$ (CMB T° fluctuation)

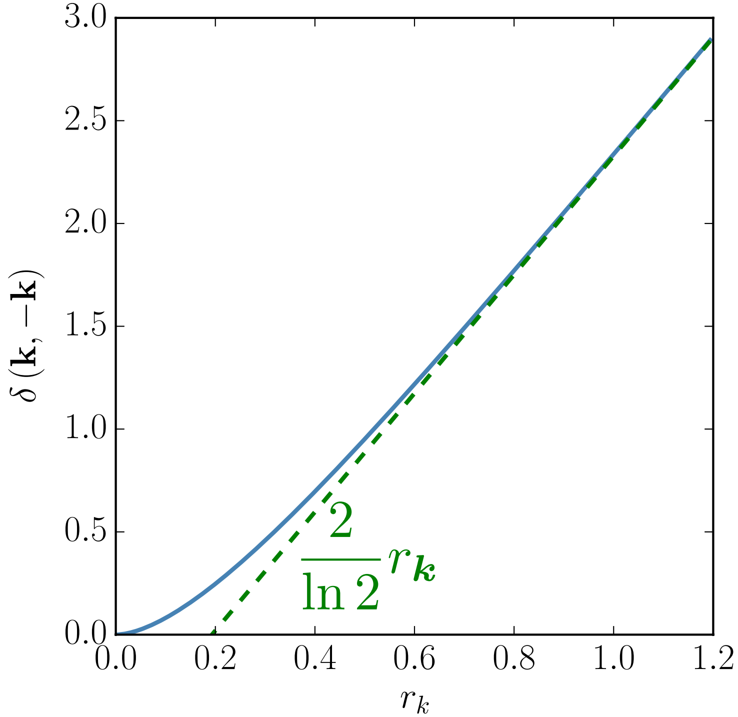
- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k}\sum_{n=0}^{\infty} e^{2in\varphi_k}(-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

  Two-mode squeezed state (Gaussian state)

- $$\delta(\boldsymbol{k},-\boldsymbol{k}) = \cosh^2 r_k \log_2\left(\cosh^2 r_k\right) - \sinh^2 r_k \log_2\left(\sinh^2 r_k\right)$$
  ~ 150 at the end of inflation

Large degree of quantum correlations!

Can we design a Bell-type experiment?



Bell Experiments in the CMB

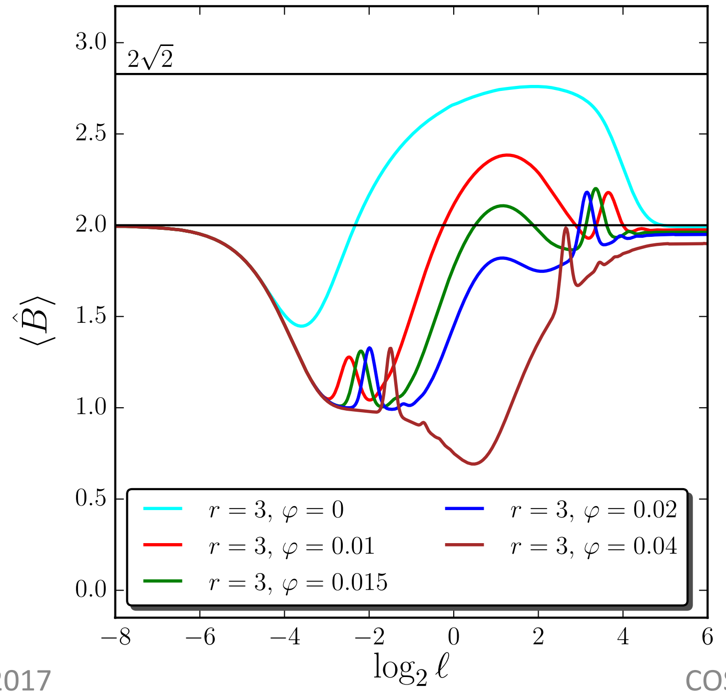
- Spin operators for continuous variables *(Larsson 2004)*

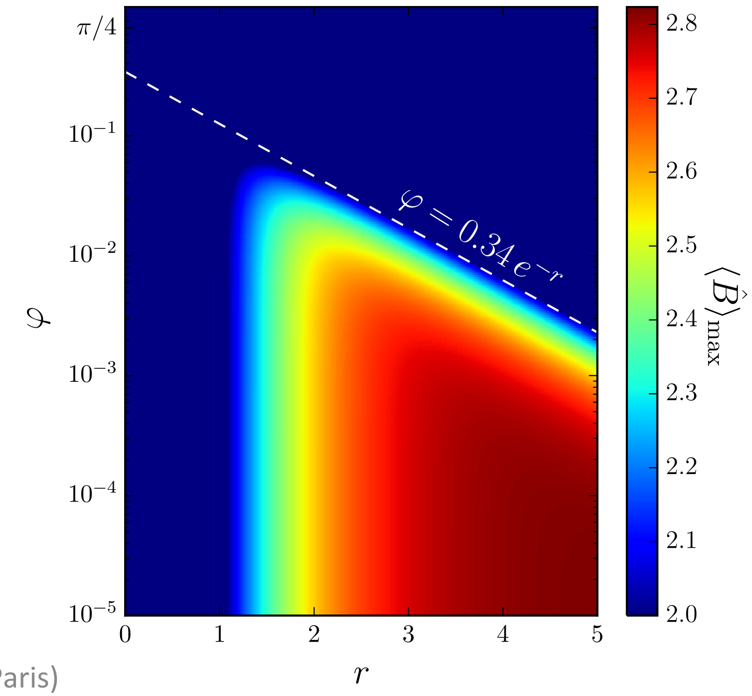
$$\hat{S}_z(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{n\ell}^{(n+1)\ell} \mathrm{d}Q |Q\rangle\langle Q|$$

$$\hat{S}_x(\ell) = \hat{S}_+(\ell) + \hat{S}_+^\dagger(\ell)$$

with $\hat{S}_+(\ell) = \sum_{n=-\infty}^{\infty} (-1)^n \int_{2n\ell}^{(2n+1)\ell} \mathrm{d}Q |Q\rangle\langle Q + \ell|$

$$\hat{S}_y(\ell) = -i\left[\hat{S}_+(\ell) - \hat{S}_+^\dagger(\ell)\right]$$





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- How do we measure the spin operators?

$\delta T/T \to \zeta_k \to v_k$, find n such that $n\ell \leq v_k < (n+1)\ell$ and $S_z = (-1)^n$

position measurement only

$\hat{S}_x$ and $\hat{S}_y$ require to measure the conjugated momentum $\zeta_k' \sim e^{-r_k}\zeta_k$, which may be concealed from us for ever...

See also Campo and Parentani 2005
Maldacena 2014

Can we detect quantum correlations using position measurements only?

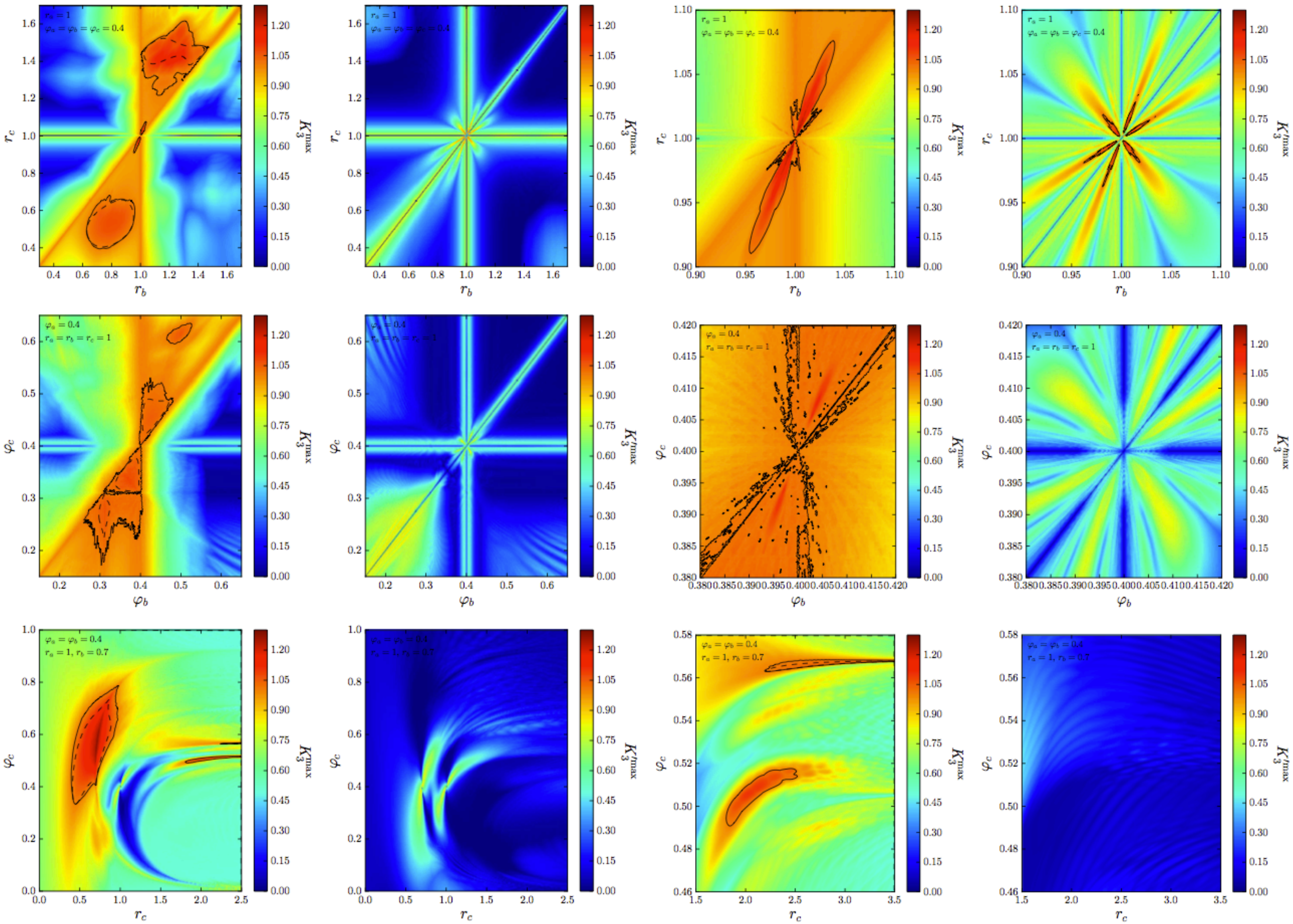
Legget-Garg Experiments in the CMB

- Two-time correlators $C_{ab} = \left\langle \hat{S}_z(t_a, \ell)\hat{S}_z(t_b, \ell) \right\rangle$

- Legget-Garg three-strings: $K_3 = C_{ab} + C_{bc} - C_{ac}, \quad K_3' = -C_{ab} - C_{bc} - C_{ac}$

Classically $-3 \leq K_3, K_3' \leq 1$ "temporal Bell inequalities"

Legget-Garg Experiments in the CMB



Legget-Garg Experiments in the CMB

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Classically $-3 \leq K_3, K_3' \leq 1$ "temporal Bell inequalities"

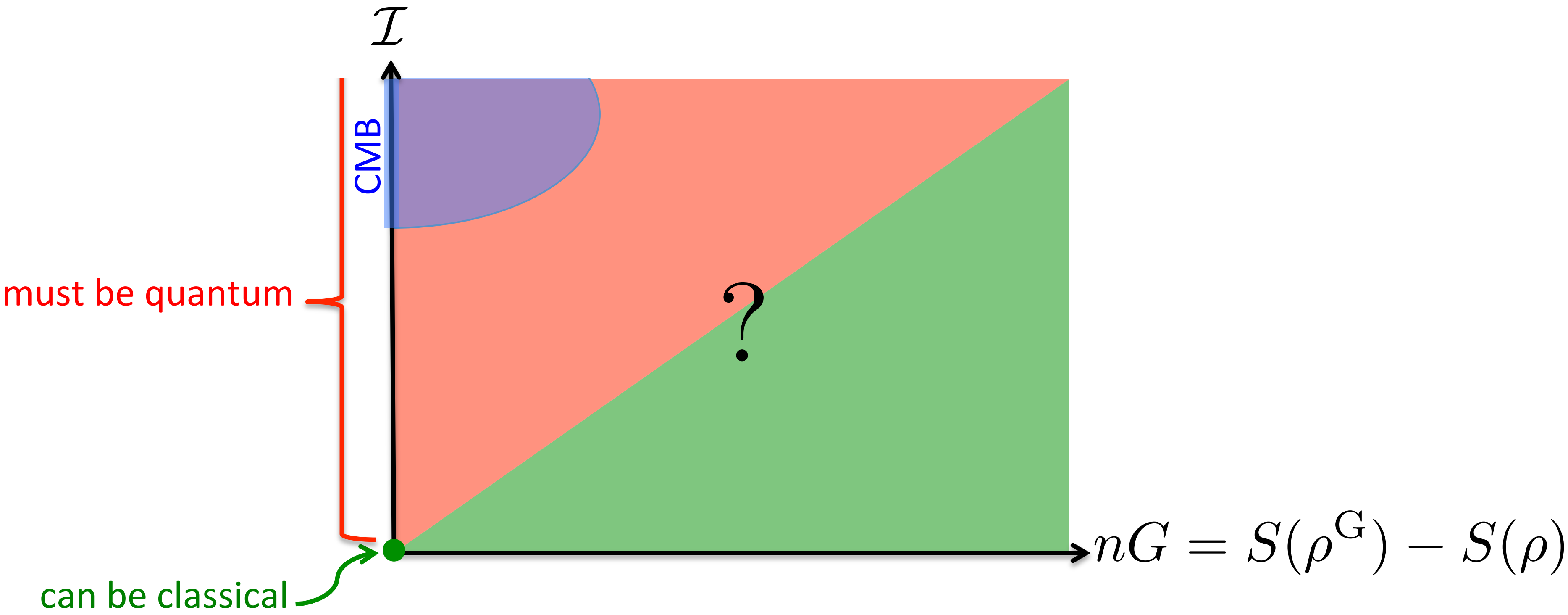
- But requires to measure ζ at three different times…

Can we detect quantum correlations
using single-time, position measurements only?

Back to Discord: Classical States

- Classical states ≡ non-discordant states (δ=0)   *Can we exclude non-discordant states?*
- Theorem: the only classical Gaussian states are product states
  (Adesso and Datta 2010; Rahimi-Keshari, Caves, Ralph 2013; Mista, McNulty, Adesso 2014)

$$\delta = 0, nG = 0 \Rightarrow \mathcal{I} = 0$$



Conclusions

- **Cosmological perturbations** are placed in a two-mode highly **squeezed state** in the very early Universe

- Such a state has a large **quantum discord**, denoting the presence of **large quantum correlations** between particles created with opposite wave momenta

- In principle, **Bell experiments** can therefore be constructed that would prove that CMB anisotropies are of quantum mechanical origin

- In practice, these experiments require to measure exponentially small quantities ($\propto$ **decaying mode**), at least in the standard setup

- **Legget-Garg** inequalities evade this issue but require to measure perturbations at **different times**

- The CMB cannot have been placed in a **classical Gaussian state**. Current constraints on non-Gaussianities may be already sufficient to exclude non-discordant states!