

Relativistic effects in large-scale structure

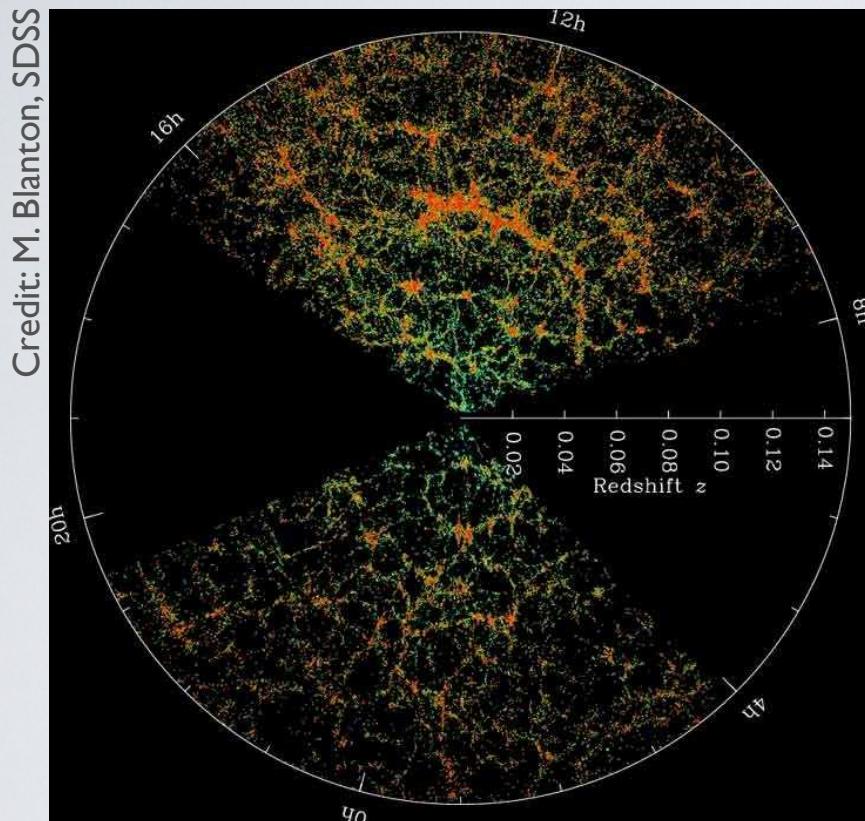
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COSMO
August 2017

Galaxy survey

The **distribution** of galaxies is sensitive to:



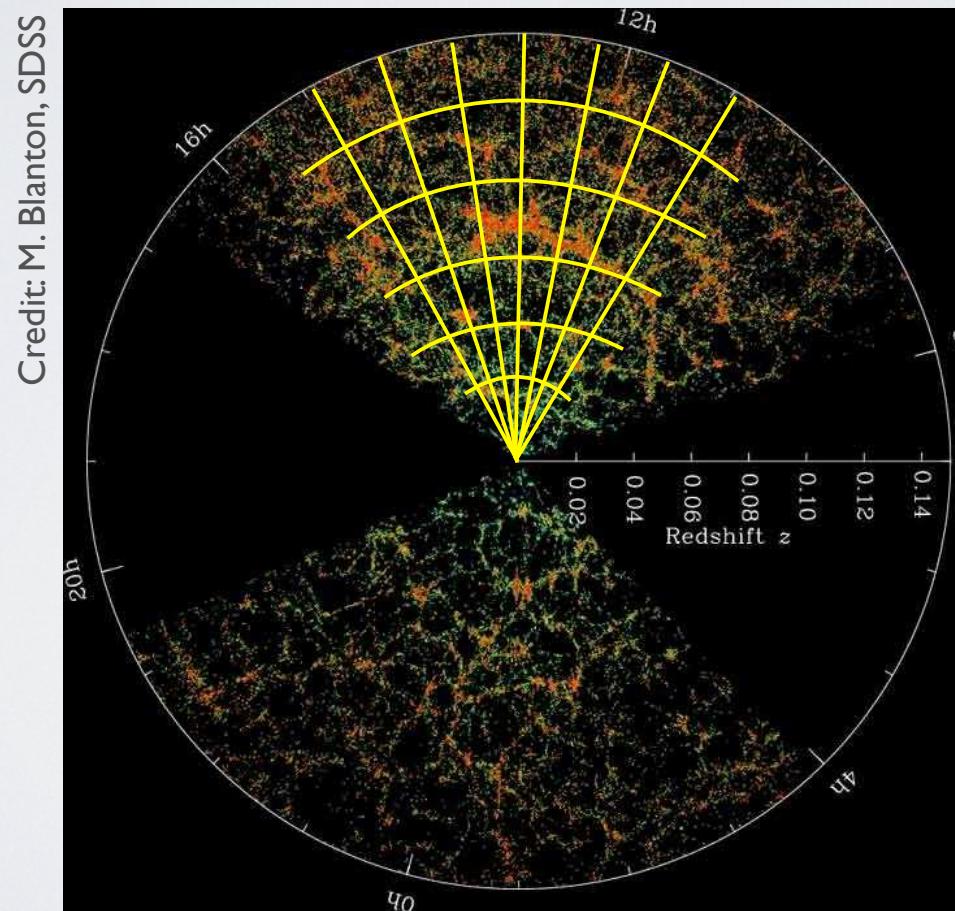
- ◆ the initial conditions
- ◆ the theory of gravity
- ◆ the content of the universe

→ The large-scale structure contains valuable **information**

To interpret properly this information, we need to understand **what** we are **measuring**.

Galaxy survey

- ◆ We count the number of **galaxies** per **pixel**: $\Delta = \frac{N - \bar{N}}{\bar{N}}$
- ◆ How is Δ related to: the initial conditions, the theory of gravity and dark energy?

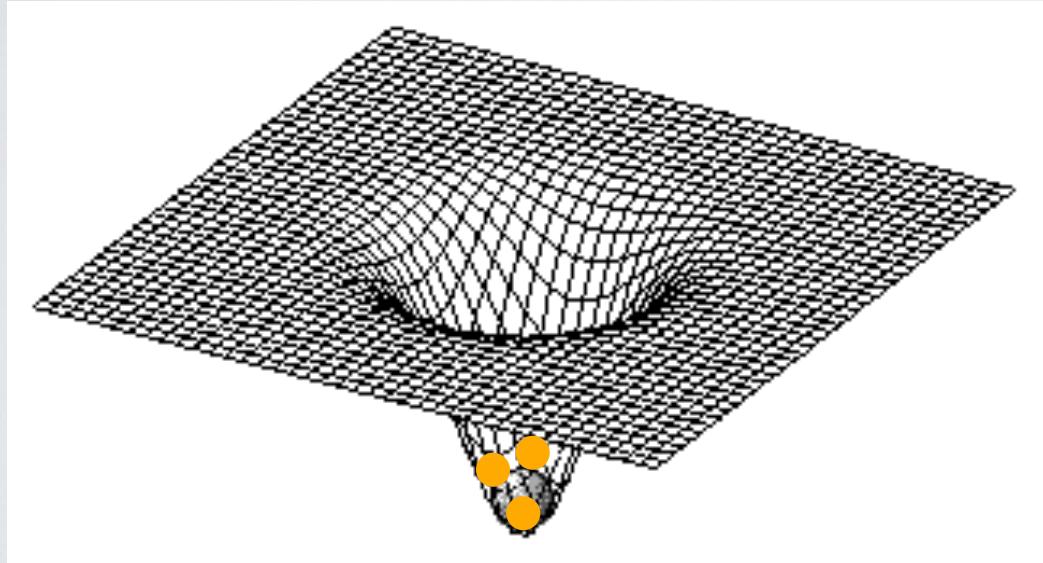


Galaxy distribution

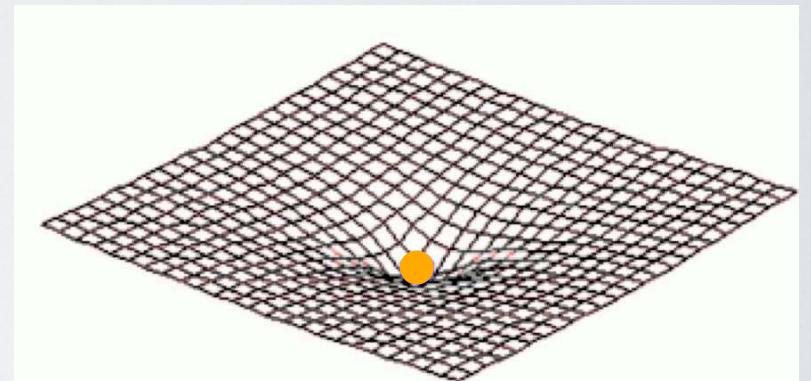
Simple picture:

- ◆ **dark matter** is not homogeneously distributed
- ◆ it creates **gravitational potential** wells
- ◆ **baryons** fall into them and form galaxies

More dark matter



Less dark matter



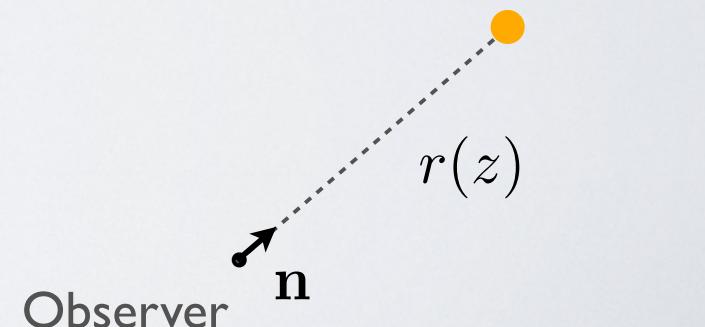
$$\Delta = \frac{\delta\rho}{\rho} \equiv \delta$$

Complications

- ◆ **Bias:** the distribution of galaxies does not trace directly the distribution of dark matter $\Delta = b \cdot \delta$
- ◆ We never observe directly the **position** of galaxies, we observe the **redshift** z and the **direction** of incoming photons **n**.

In a **homogeneous** universe:

- we calculate the distance $r(z)$
- light propagates on straight lines

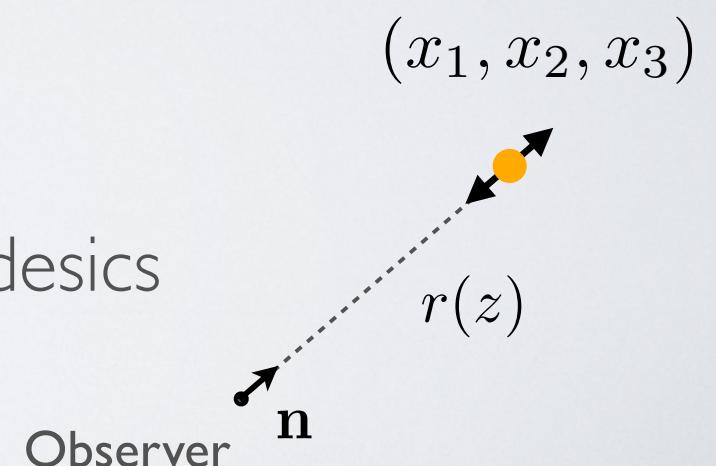


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In a **inhomogeneous** universe:

- the redshift is perturbed $r(z)$
- light propagates on perturbed geodesics



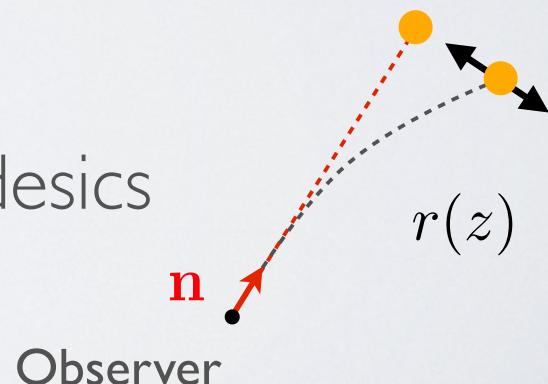
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In a **inhomogeneous** universe:

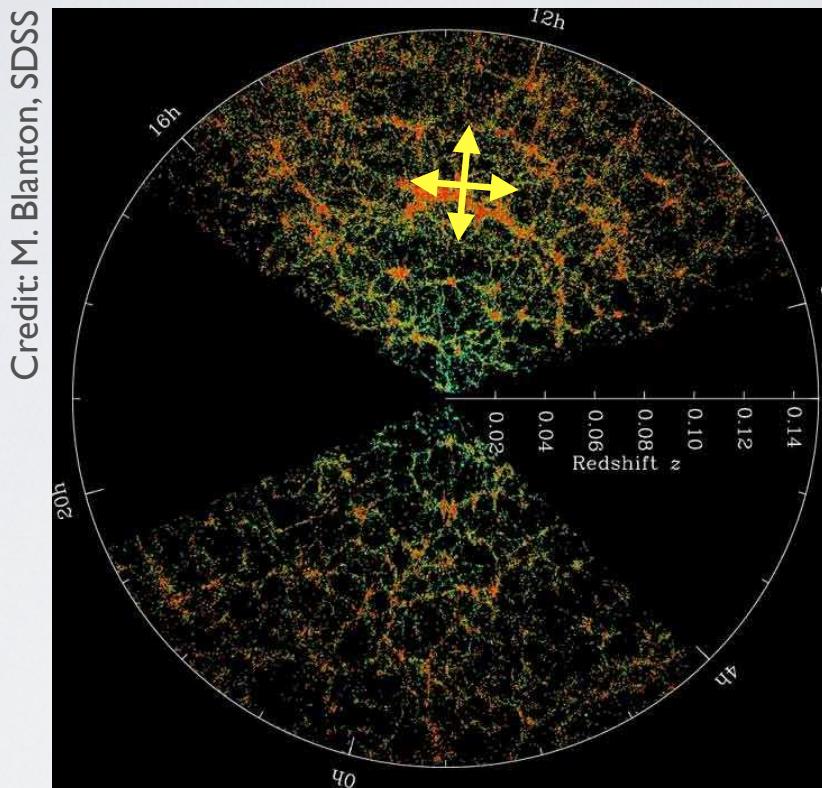
$$(x_1, x_2, x_3)$$

- the redshift is perturbed $r(z)$
- light propagates on perturbed geodesics



Galaxy distribution

The **structures** seen on a galaxy map do **not reflect** directly the underlying dark matter structures. The observed **position** of galaxies are **shifted** radially and transversally.



To extract **information** from a galaxy map, we need to understand exactly which **distortions** there are.

Outline

- ◆ Expression for Δ encoding all distortions at linear order

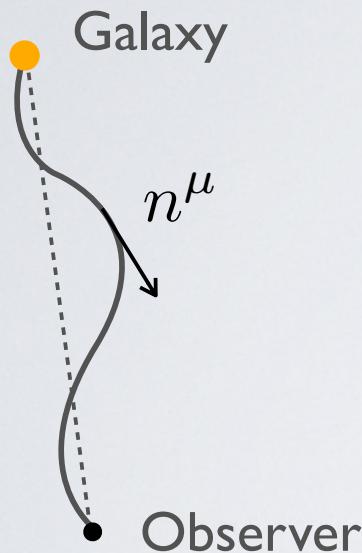
$$\Delta = \text{density} + \text{redshift distortions}$$
$$+ \text{lensing} + \text{relativistic effects}$$

- ◆ Impact of the different terms on the **correlation** function
 - The distortions **change** the **standard** multipoles
 - The distortions generate **new** multipoles

Calculation of the distortions

Perturbed Friedmann universe:

$$ds^2 = -a^2 \left[(1 + 2\Psi) d\eta^2 + (1 - 2\Phi) \delta_{ij} dx^i dx^j \right]$$



We calculate the **propagation of photons**, i.e. the null geodesics and infer:

- ◆ the change in **energy**
- ◆ the change in **direction**

What we really observe

$$\begin{aligned}\Delta(z, \mathbf{n}) = & b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n}) \\ & - \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega(\Phi + \Psi) \\ & + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi \\ & + \frac{2}{r} \int_0^r dr' (\Phi + \Psi) + 3\mathcal{H} \nabla^{-2} (\nabla \mathbf{V}) + \Psi - 2\Phi \\ & + \frac{1}{\mathcal{H}} \dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) \left[\Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right]\end{aligned}$$

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

What we really observe

$$\begin{aligned}\Delta(z, \mathbf{n}) = b \cdot \delta & - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n}) \quad \text{Velocities} \\ & - \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega(\Phi + \Psi) \\ & + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi \\ & + \frac{2}{r} \int_0^r dr' (\Phi + \Psi) + 3\mathcal{H} \nabla^{-2} (\nabla \mathbf{V}) + \Psi - 2\Phi \\ & + \frac{1}{\mathcal{H}} \dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) \left[\Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right]\end{aligned}$$

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What we really observe

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$$+ \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi$$

$$+ \frac{2}{r} \int_0^r dr' (\Phi + \Psi) + 3\mathcal{H} \nabla^{-2} (\nabla \mathbf{V}) + \Psi - 2\Phi \quad \text{Potentials}$$

$$+ \frac{1}{\mathcal{H}} \dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) \left[\Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right]$$

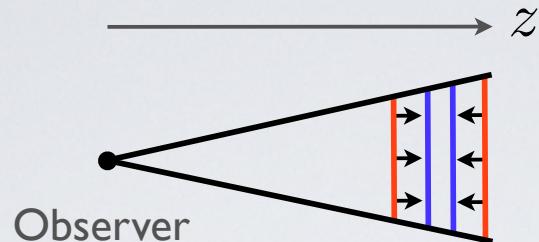
Yoo et al (2010)
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Distortions

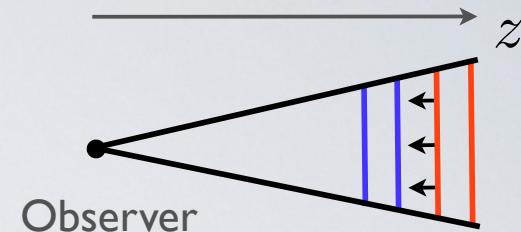
◆ Velocities

Kaiser (1987), Lilje & Efstathiou (1989), Hamilton (1992)

Change in the bin size:
Redshift distortions



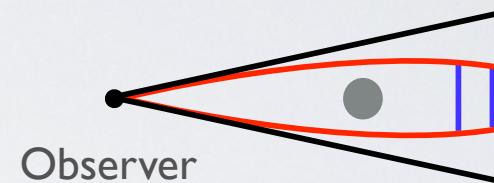
Change in the bin position:
Doppler effect



◆ Lensing

Gunn (1967), Schneider (1989), Broadhurst, Taylor & Peacock (1995)

Change in the solid angle



◆ Potentials

Yoo et al (2010)
CB and Durrer (2011)
Challinor and Lewis (2011)

Local terms:
e.g. gravitational redshift



Integrated terms: e.g.
Shapiro time-delay and
Integrated Sachs-Wolfe



What we really observe

$$\Delta(z, \mathbf{n}) = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

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What we really observe

$$\begin{aligned}
 \Delta(z, \mathbf{n}) = & \boxed{b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})} \quad \text{current standard analysis} \\
 & - \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega(\Phi + \Psi) \\
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What we really observe

$$\Delta(z, \mathbf{n}) = b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n})$$

current standard analysis

$$- \int_0^r dr' \frac{r-r'}{rr'} \Delta_\Omega (\Phi + \Psi)$$

lensing: important at high redshift

$$+ \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi$$

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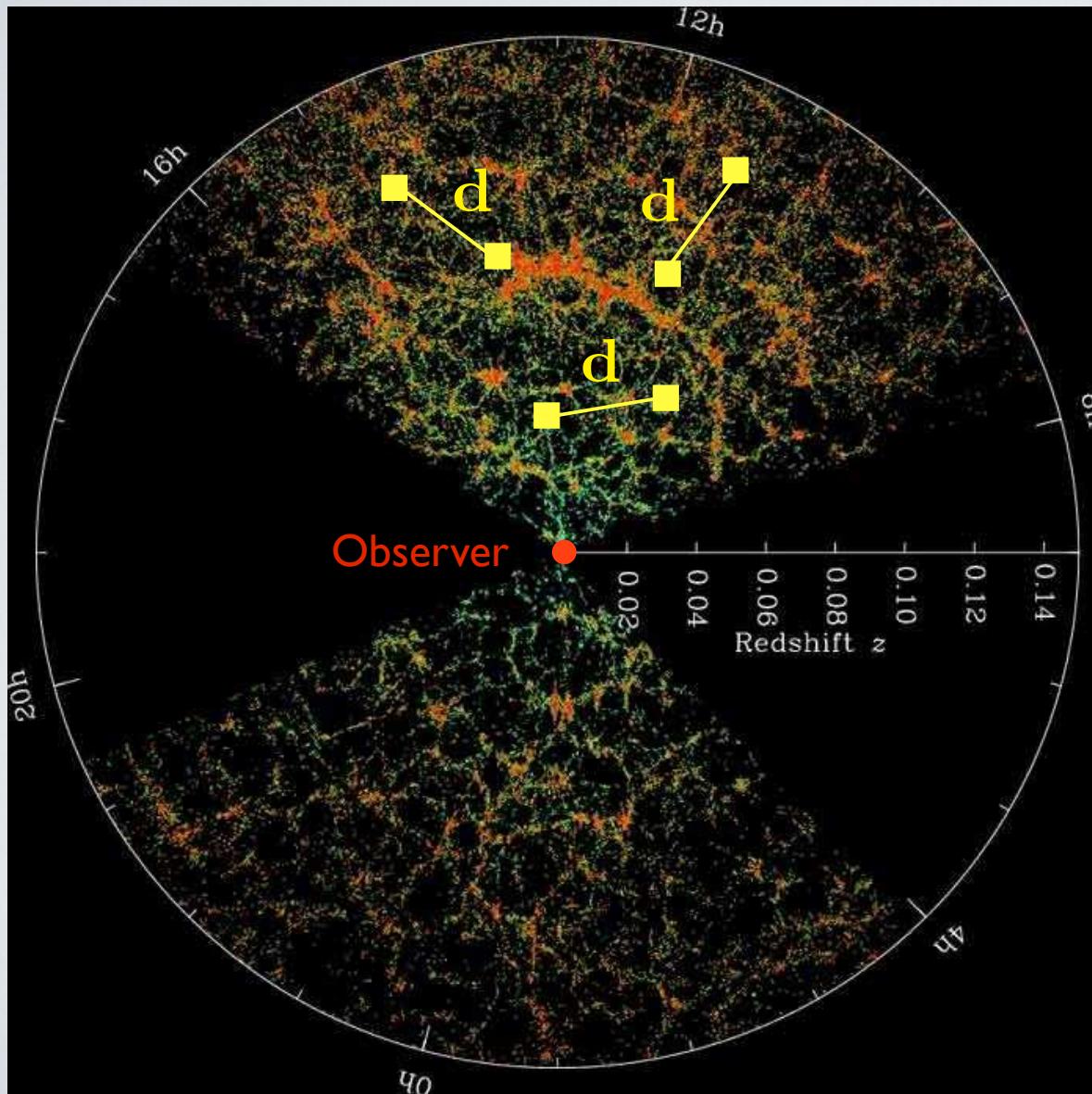
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relativistic effects:
important at large separation

Correlation function

Credit: M. Blanton, SDSS



$$\xi = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}') \rangle$$

The dark matter fluctuations generate **isotropic** correlations

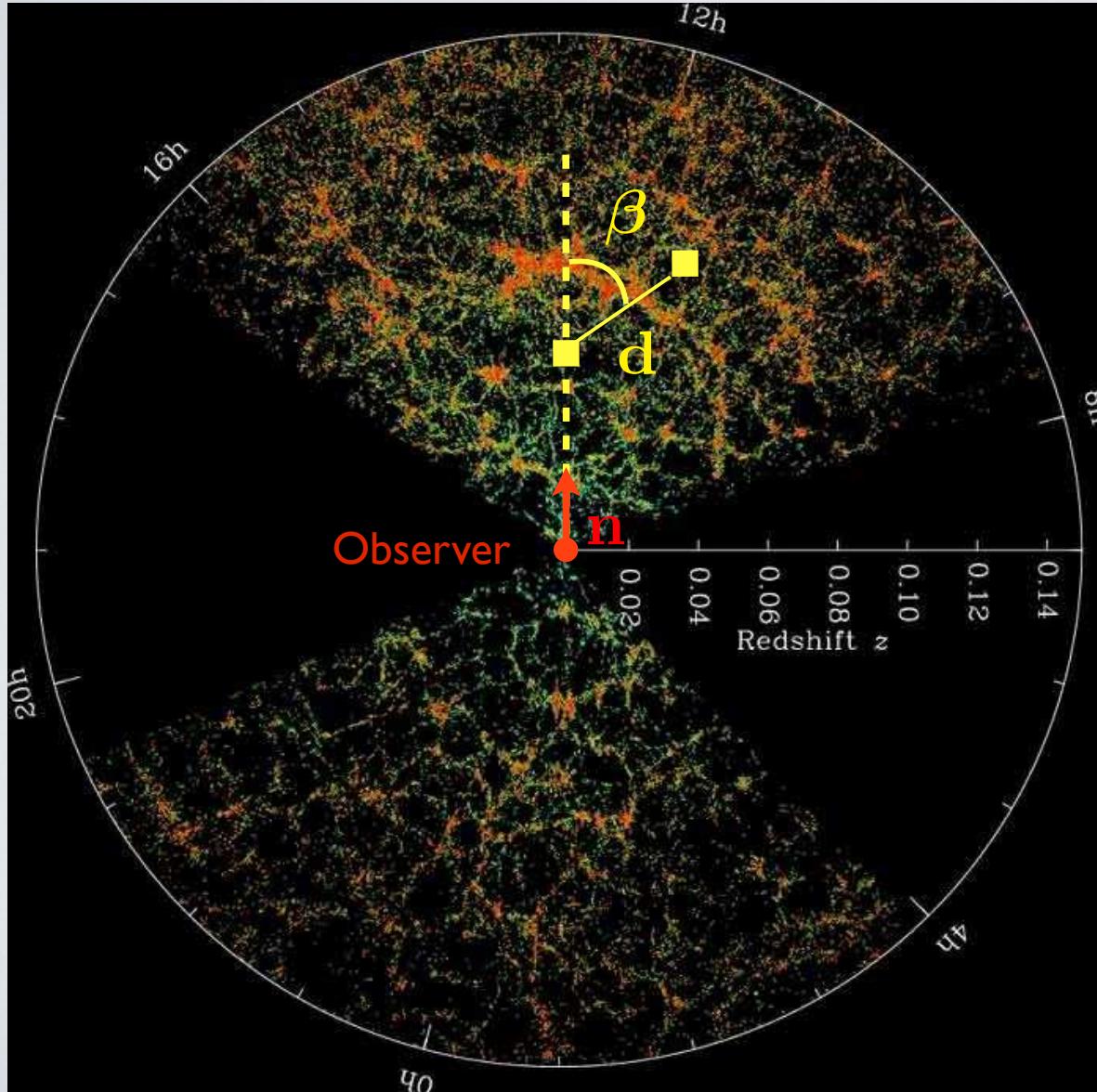
$$\Delta = b \cdot \delta$$

$$\xi(d) = C_0(d)$$

Correlation function

The distortions **break** the **isotropy** of the correlation function

Credit: M. Blanton, SDSS



$$\xi = \langle \Delta(\mathbf{x})\Delta(\mathbf{x}') \rangle$$

Redshift distortions
generate a quadrupole
and hexadecapole

Kaiser (1987)
Hamilton (1992)

$$\xi = C_0(d) + C_2(d)P_2(\cos \beta)$$

$$+ C_4(d)P_4(\cos \beta)$$

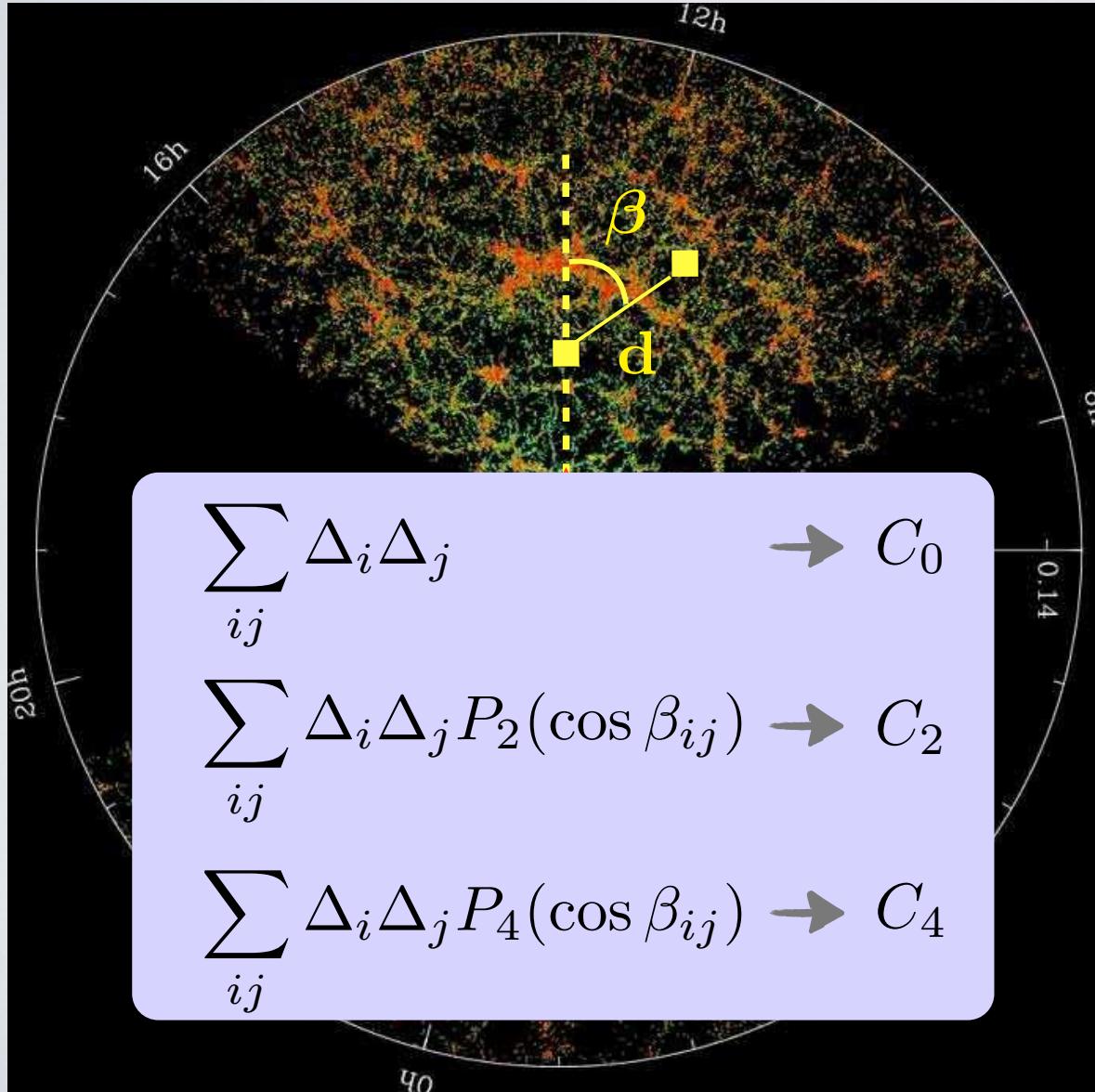


Legendre polynomials

Correlation function

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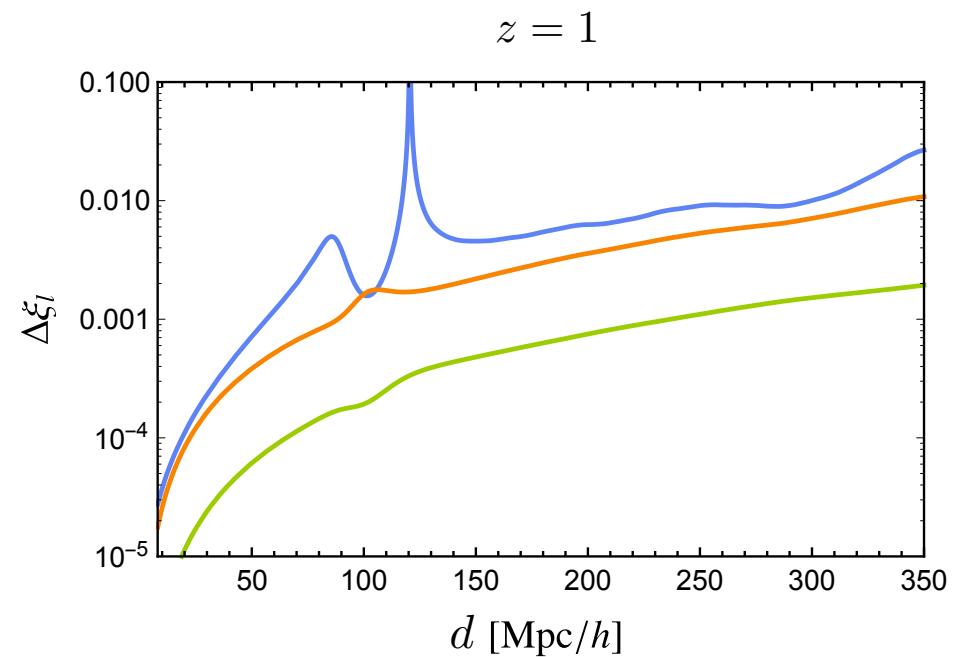
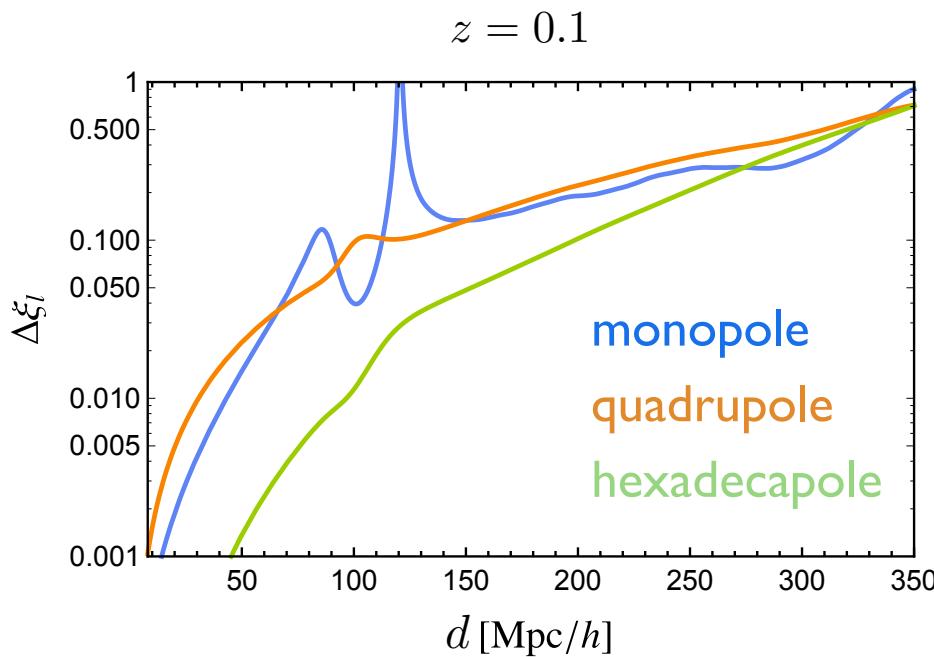
↓
Legendre polynomials

Corrections to the standard multipoles

Tansella, CB, Durrer, Gosh and Sellentin (2017)

How do the **distortions** change the **amplitude** of the monopole, quadrupole and hexadecapole?

Fractional difference from **local relativistic** effects $\Delta \xi_\ell = \frac{\xi_\ell^{\text{rel}}}{\xi_\ell^{\text{st}}}$



Dominant distortion studied in Papai and Szapudi (2008)

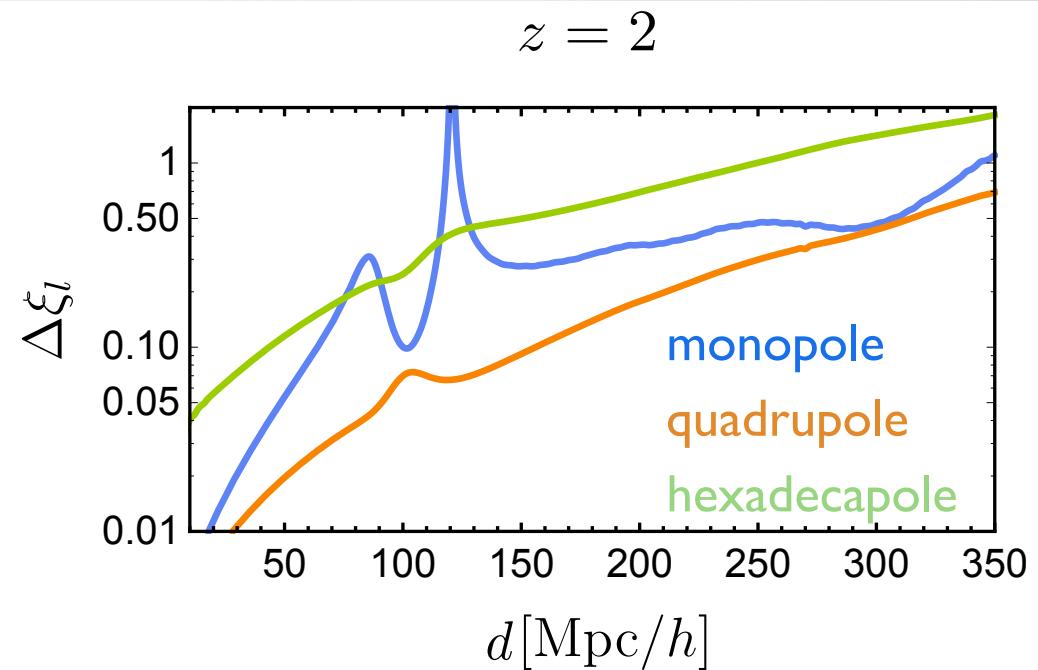
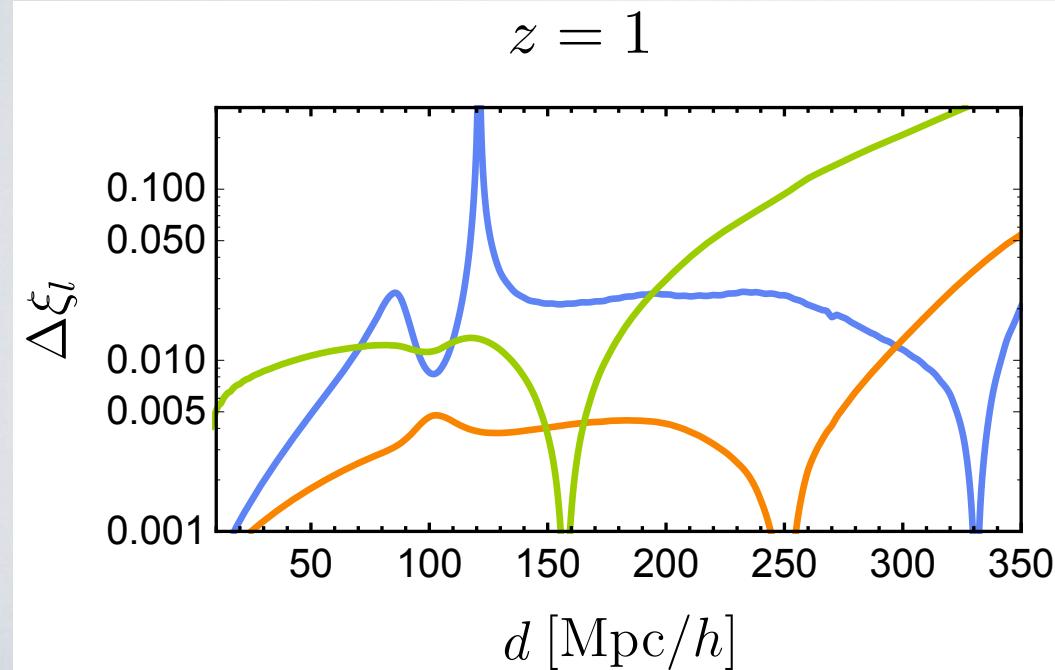
Raccanelli, Samushia and Percival (2010)

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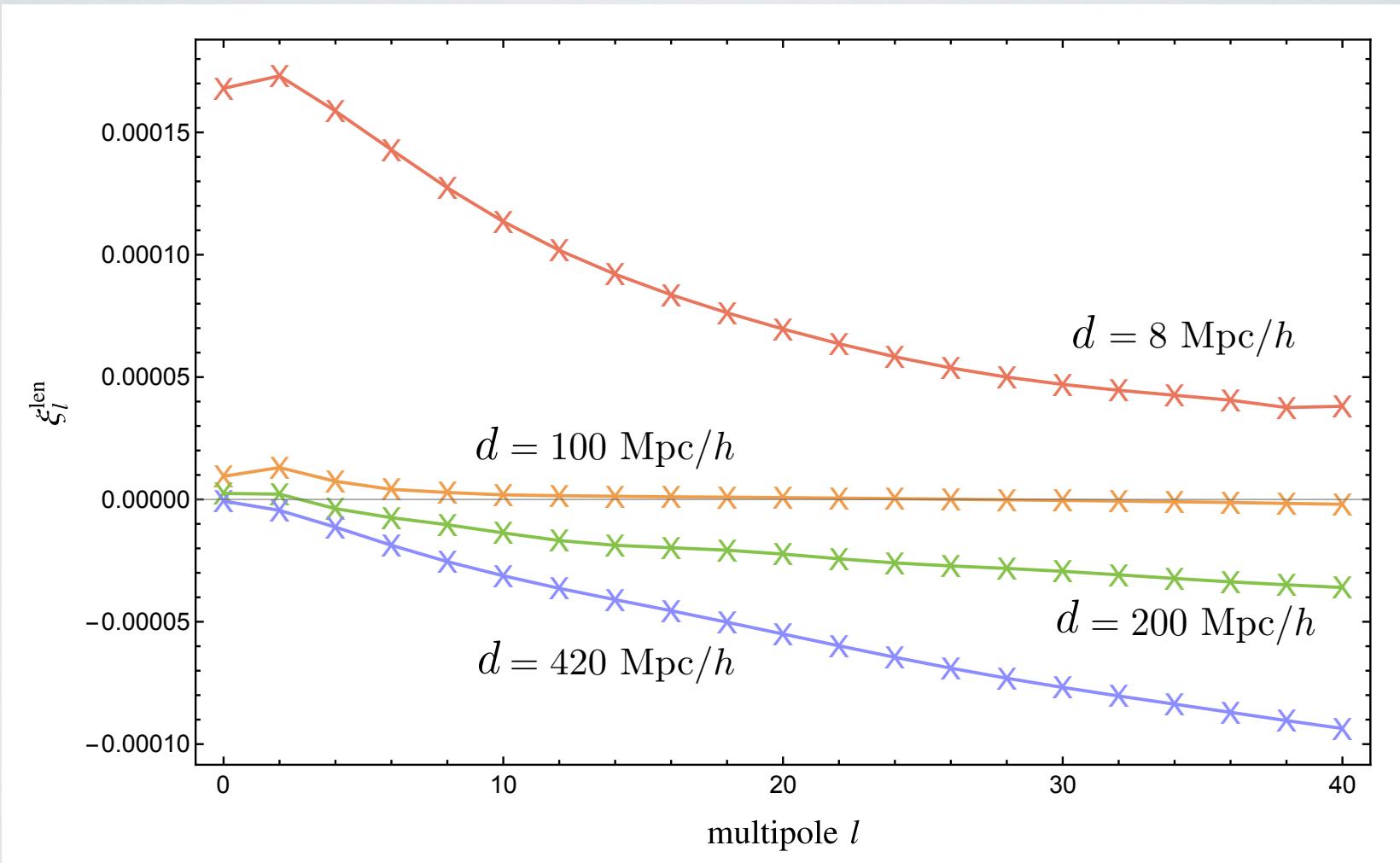
Fractional difference from **gravitational lensing** $\Delta \xi_\ell = \frac{\xi_\ell^{\text{lens}}}{\xi_\ell^{\text{st}}}$



The distortions change the multipole expansion

Tansella, CB, Durrer, Gosh and Sellentin (2017)

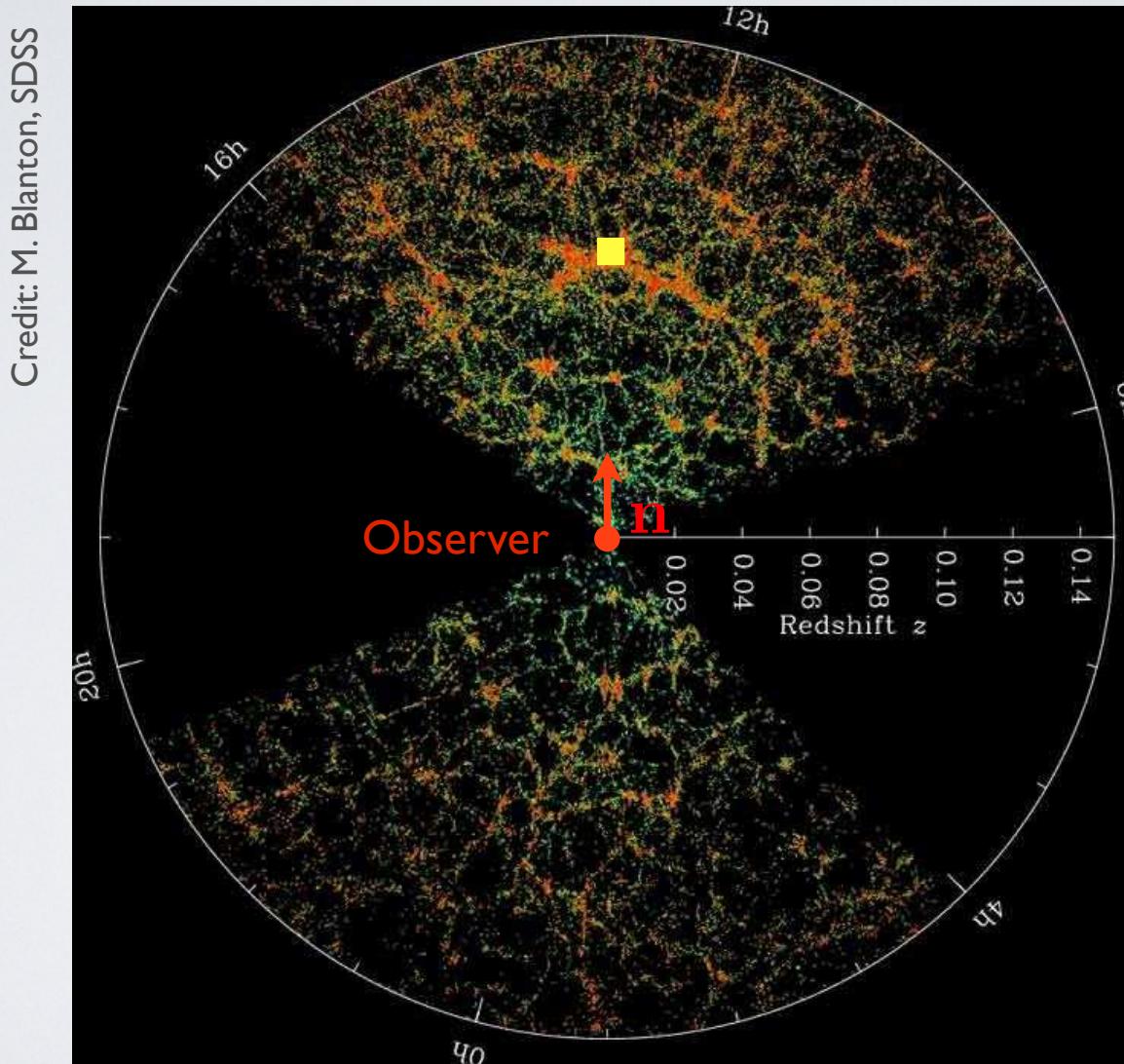
Lensing generates **higher** multipoles



The distortions change the multipole expansion

CB, Hui & Gaztanaga (2014)

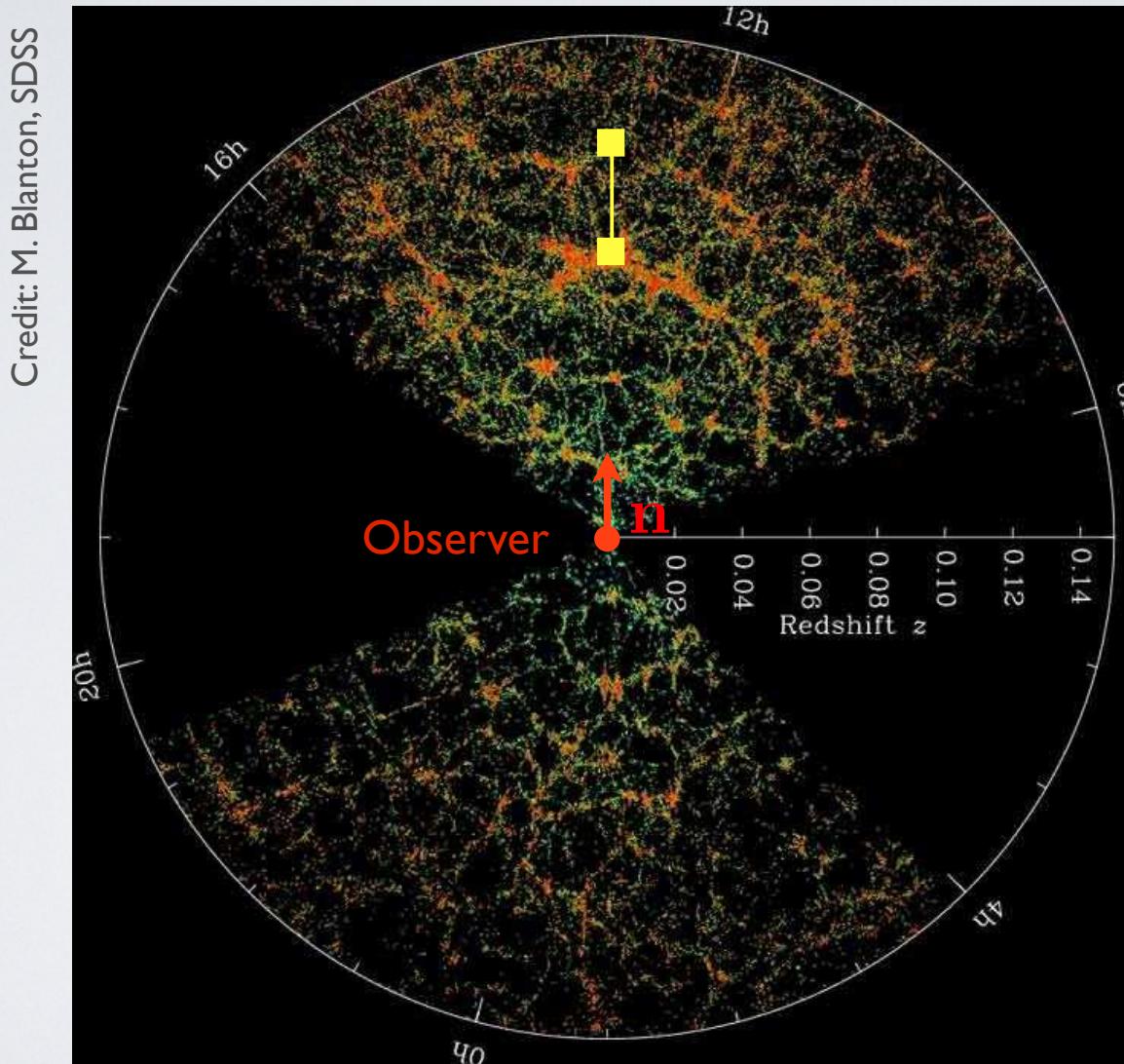
Relativistic effects generate **odd** multipoles



The distortions change the multipole expansion

CB, Hui & Gaztanaga (2014)

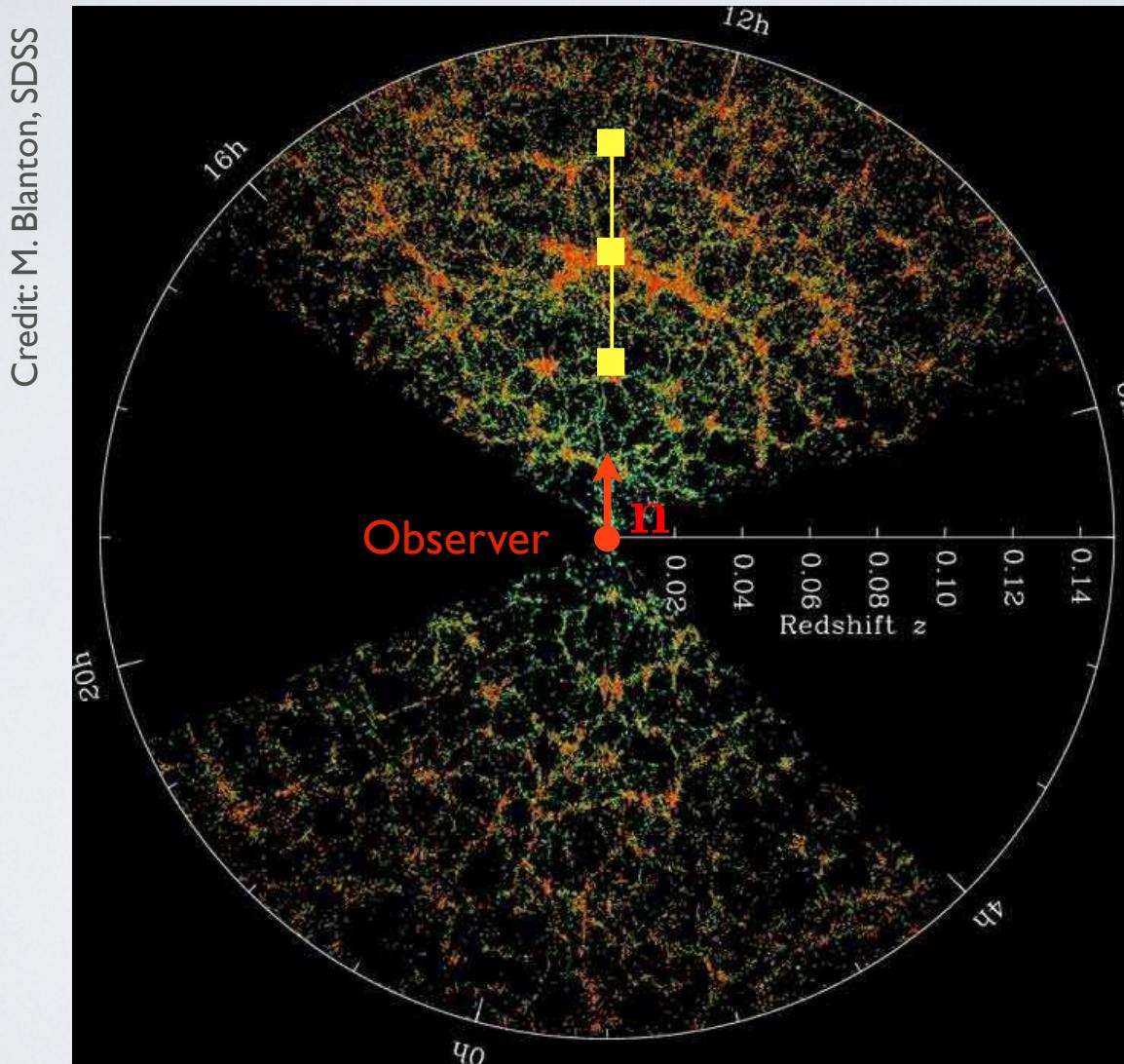
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The distortions change the multipole expansion

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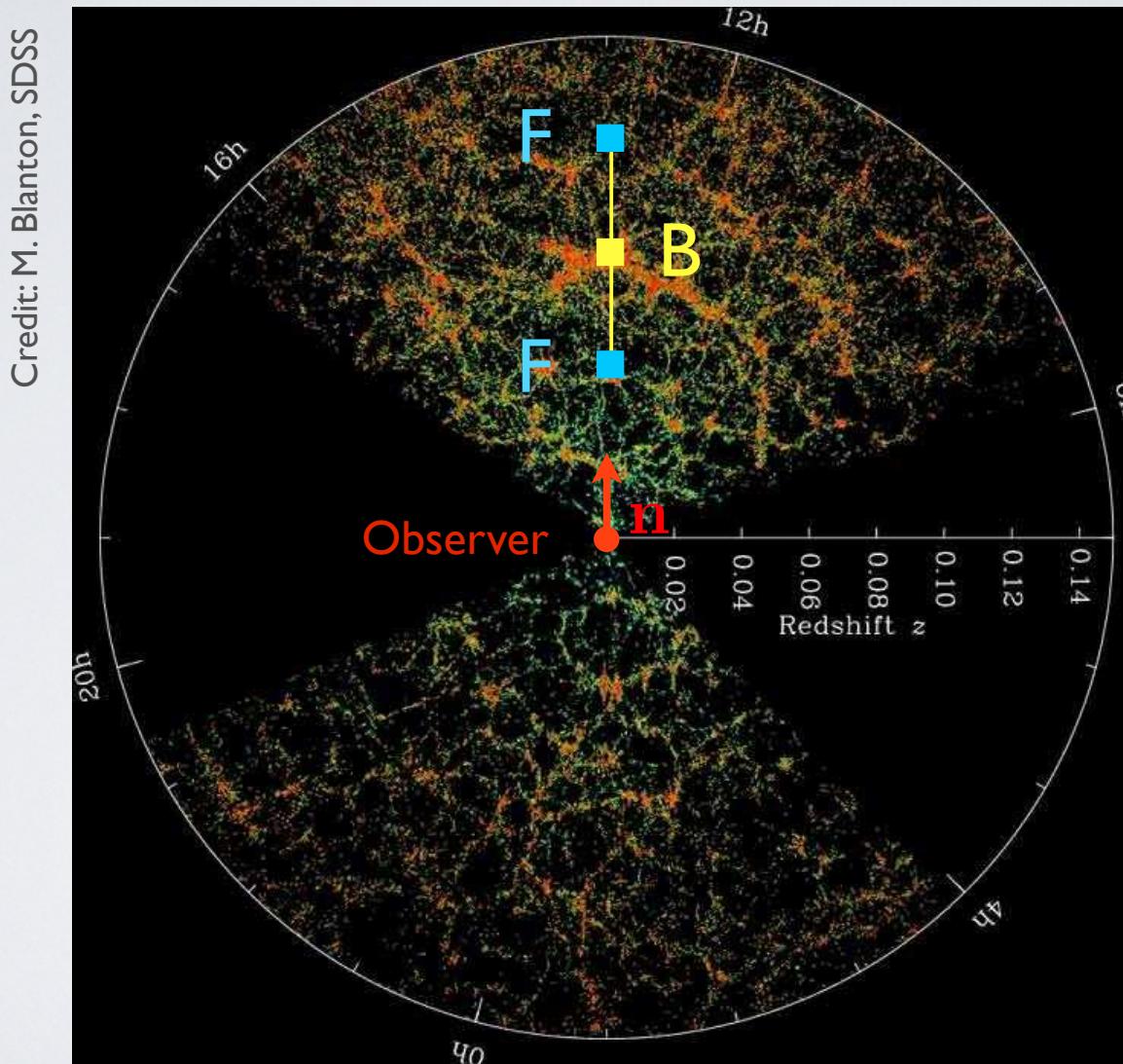
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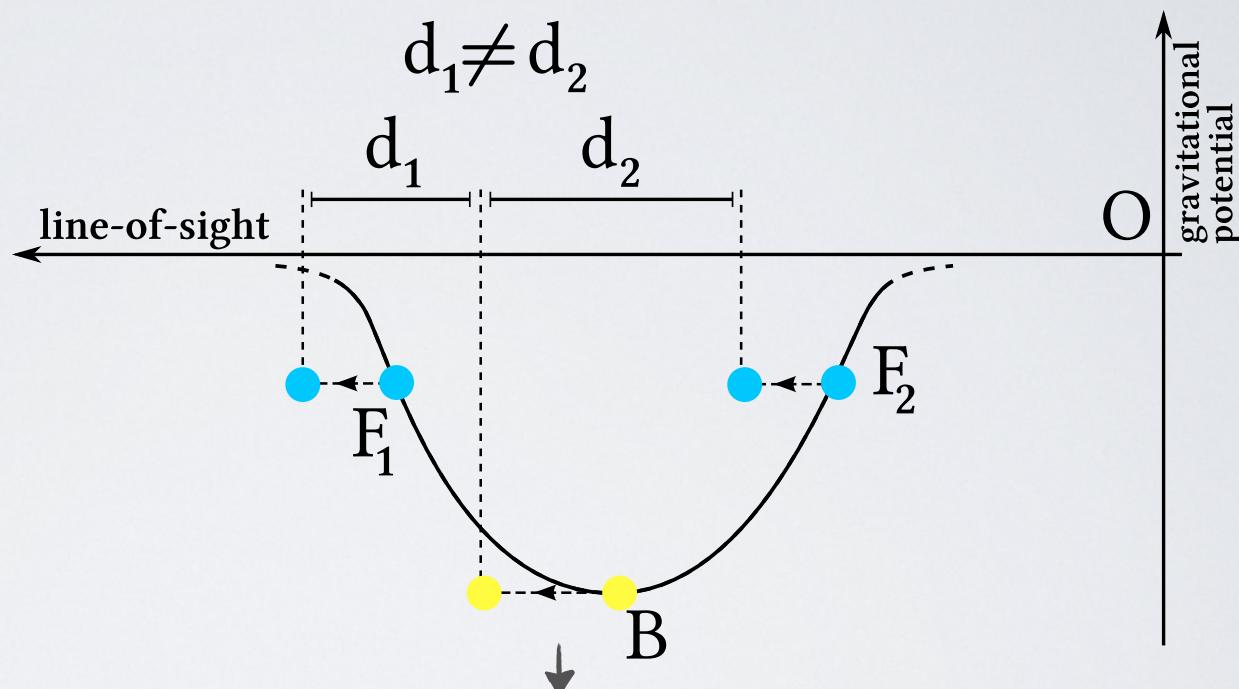
Relativistic effects generate **odd** multipoles



Breaking of symmetry from gravitational redshift

CB, Hui & Gaztanaga (2014)

$$\Delta = \frac{1}{\mathcal{H}} \partial_r \Psi$$



shift in position due to gravitational redshift

The number count at linear order

$$\begin{aligned}\Delta(z, \mathbf{n}) = & b \cdot \delta - \frac{1}{\mathcal{H}} \partial_r (\mathbf{V} \cdot \mathbf{n}) \\ & - \int_0^r dr' \frac{r - r'}{rr'} \Delta_\Omega(\Phi + \Psi) \\ & + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \partial_r \Psi \\ & + \frac{2}{r} \int_0^r dr' (\Phi + \Psi) + 3\mathcal{H} \nabla^{-2} (\nabla \mathbf{V}) + \Psi - 2\Phi \\ & + \frac{1}{\mathcal{H}} \dot{\Phi} + \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r\mathcal{H}} \right) \left[\Psi + \int_0^r dr' (\dot{\Phi} + \dot{\Psi}) \right]\end{aligned}$$

The number count at linear order

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Dipole in the correlation function

$$- \int_0^r dr' \frac{r - r'}{rr'} \Delta \rightarrow \sum_{ij} \Delta_i \Delta_j \cos \beta_{ij}$$

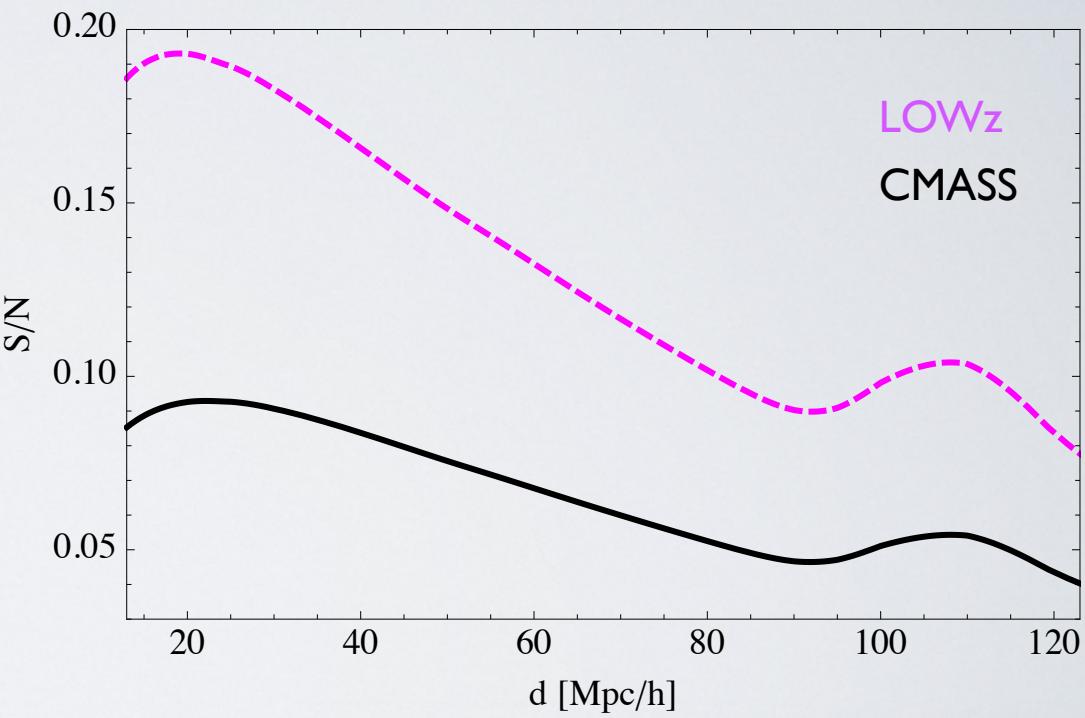
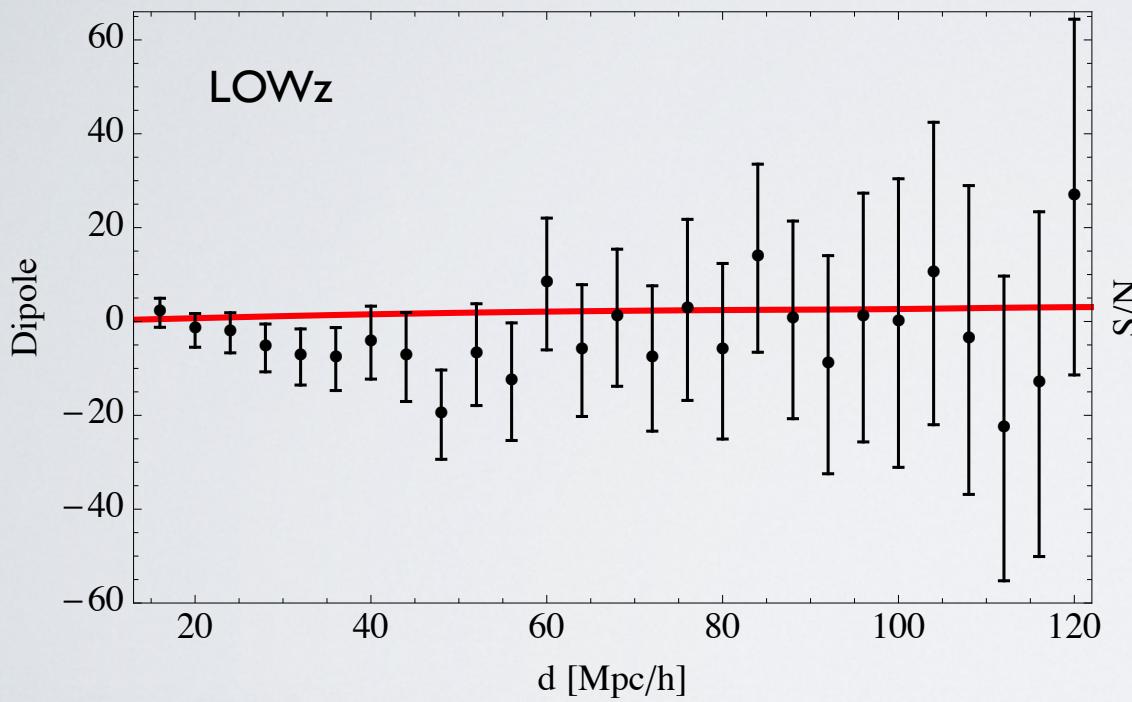
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Measuring the dipole in BOSS

We split the LOWz and CMASS samples into 2 populations and measure the dipole.

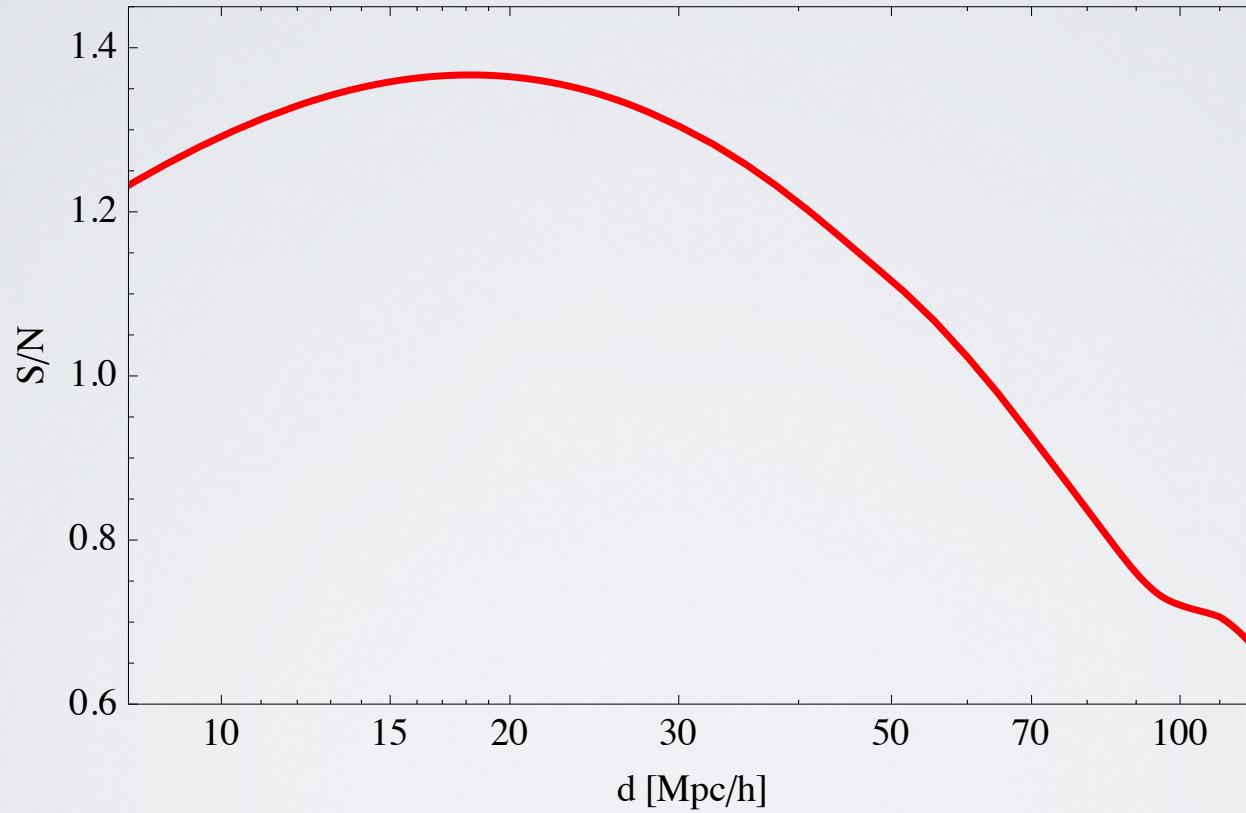


Improvements

- ◆ Measure the dipole at **lower** redshift to increase the signal.
- ◆ Use a sample with **diverse** populations to increase the bias difference.
- ◆ Divide the sample into **more** than 2 **populations** to gain in statistics.
- ◆ Use an **optimal** estimator: weight each pair by the bias difference.

Forecasts

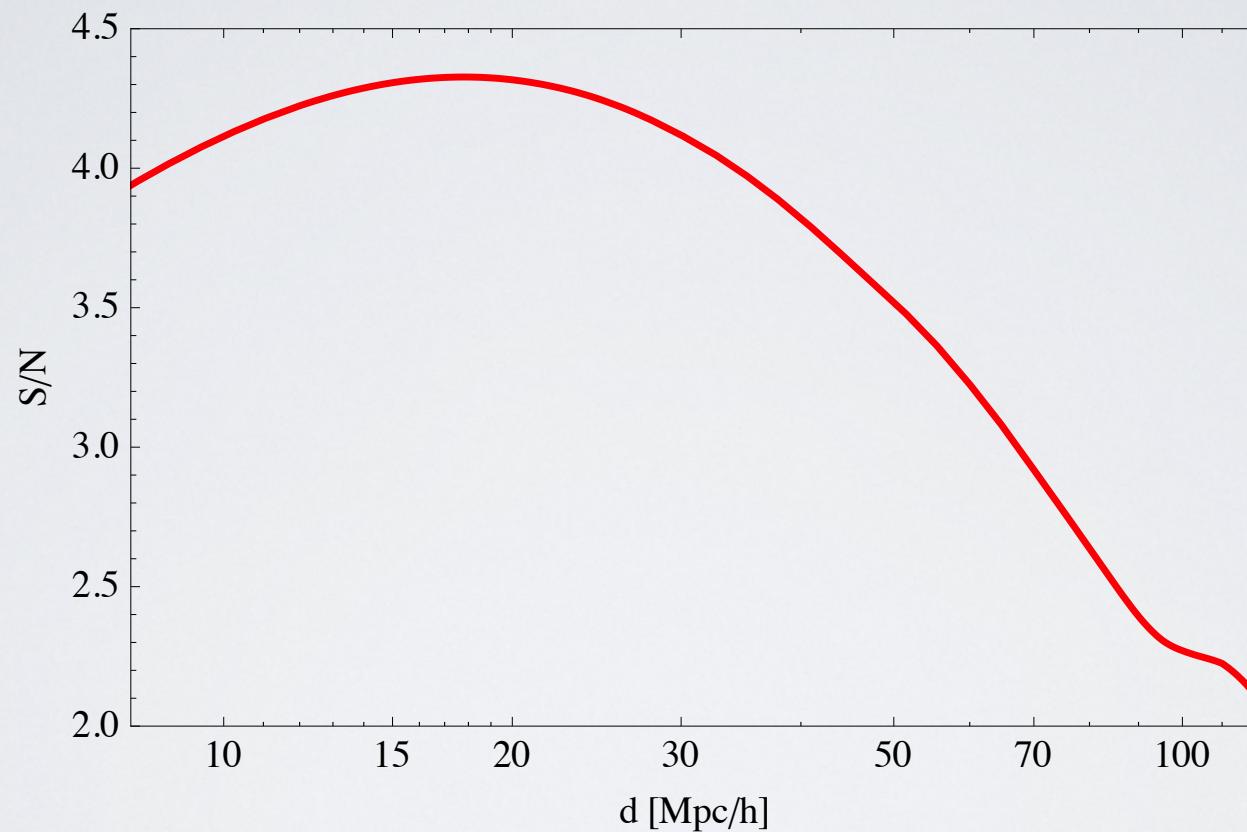
Main sample of SDSS: 465'000 galaxies in total,
6 populations with bias from 0.96 to 2.16 Percival et al (2007)



Cumulative signal-to-noise of 2.4

Forecasts

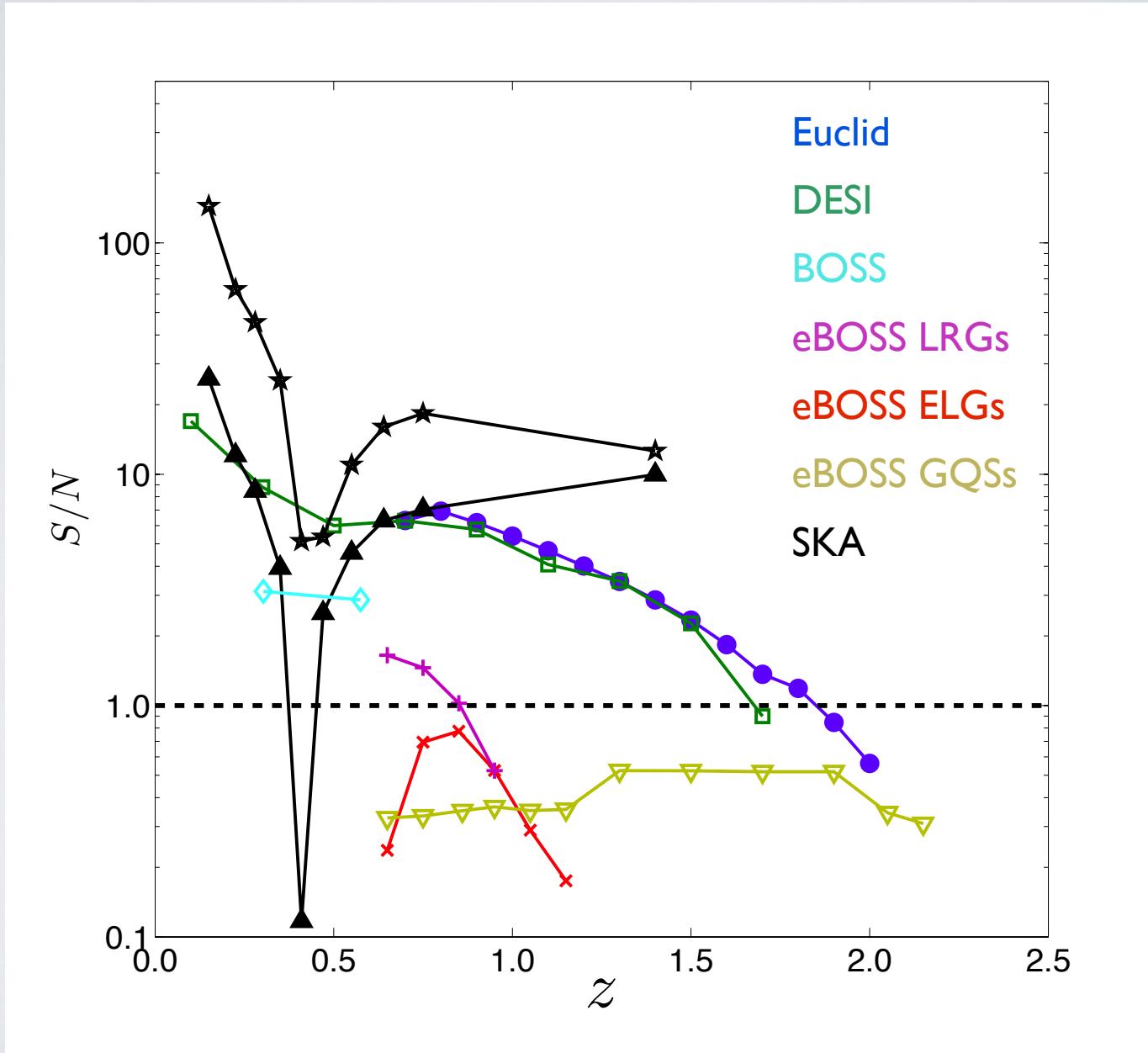
DESI Bright Sample: 10 million galaxies,
6 populations with bias from 0.96 to 2.16



Cumulative signal-to-noise of 7.4

Forecasts with 21cm intensity mapping from SKA

Hall and CB (2016)



Interest

The dipole is sensitive to the **gravitational potential**.

$$\Delta_{\text{rel}} = \frac{1}{\mathcal{H}} \partial_r \Psi + \left(1 - \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - \frac{2}{r\mathcal{H}} \right) \mathbf{V} \cdot \mathbf{n} + \frac{1}{\mathcal{H}} \dot{\mathbf{V}} \cdot \mathbf{n}$$

Combining the dipole with the quadrupole, we can test **Euler equation**. CB and Fleury, in preparation

$$\dot{\mathbf{V}} \cdot \mathbf{n} + \mathcal{H} \mathbf{V} \cdot \mathbf{n} + \partial_r \Psi = 0$$

Conclusion

- ◆ The **fluctuations** in the number of **galaxies** is affected by many effects besides the matter density fluctuations.
- ◆ These effects have a different **signature** in the **correlation** function:
 - density → monopole
 - redshift distortions → quadrupole and hexadecapole
 - lensing → higher multipoles
 - relativistic effects → dipole
- ◆ The dipole should be **detectable** in the near future and allow for new **tests** of **gravity**.