Update on Bouncing and Emergent Cosmologies

Robert Brandenberger
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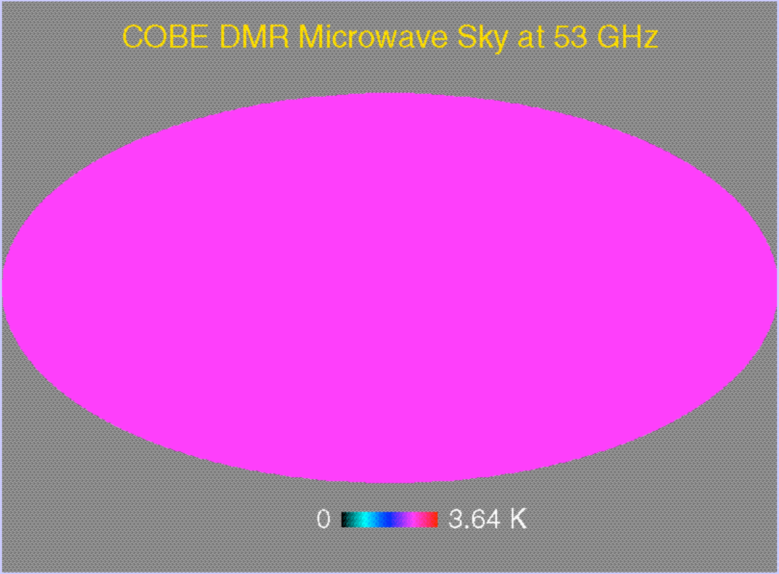
COSMO-17, Paris, August 28, 2017

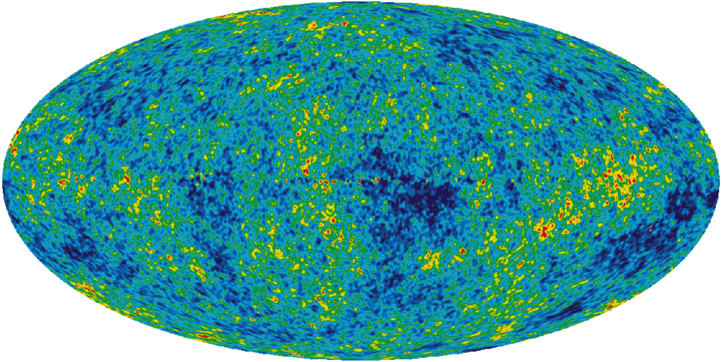
Outline

Plan

Isotropic CMB Background

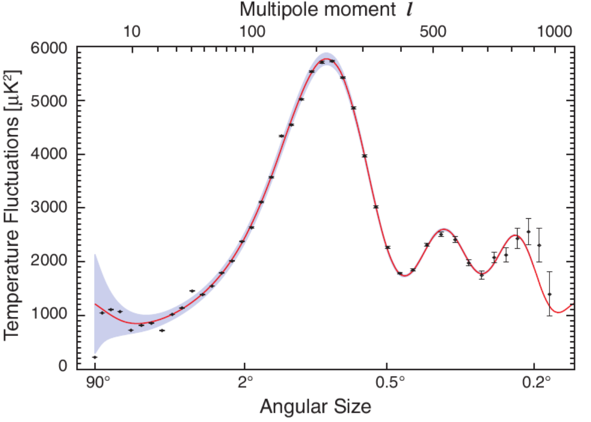


Map of the Cosmic Microwave Background (CMB)



Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Early Work

1970 p

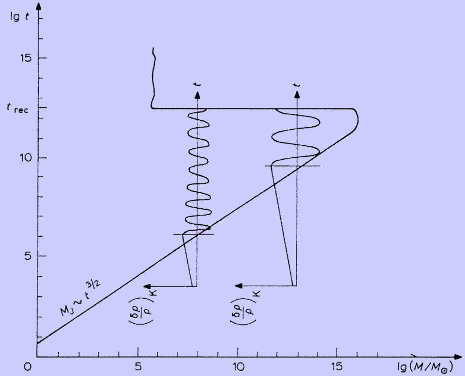


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Key Realization

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.

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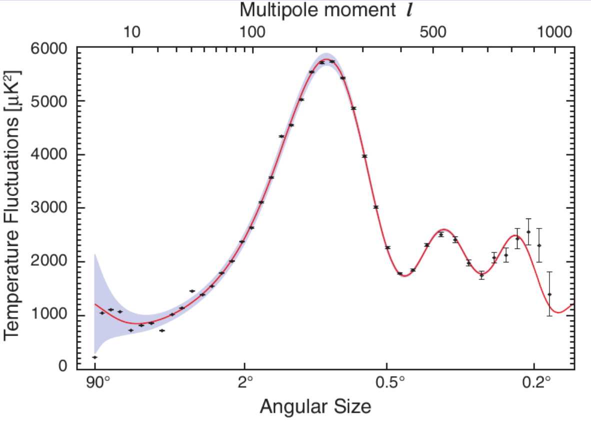

Credit: NASA/WMAP Science Team

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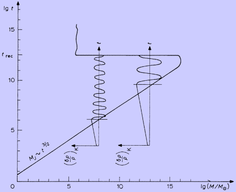

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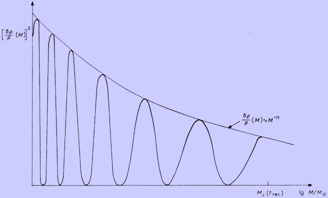

Fig. 1b. The dependence of the square of the amplitude of density perturbations of matter on scale. The fine line designates the usually assumed dependence $(\delta\varrho/\varrho)_M \sim M^{-n}$. It is apparent that fluctuations of relic radiation should depend on scale in a similar manner.

R. Sunyaev & Ya. Zeldovich, Astrophysics and Space Science 7, 3–19 (1970)

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

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- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.

Key Challenge

How does one obtain such a spectrum?

- Inflationary Cosmology is the first scenario based on causal physics which yields such a spectrum.
- But it is not the only one.

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Plan

Hubble Radius vs. Horizon

- Horizon: Forward light cone of a point on the initial Cauchy surface.
- Horizon: region of causal contact.
- Hubble radius: $l_H(t) = H^{-1}(t)$ inverse expansion rate.
- Hubble radius: local concept, relevant for dynamics of cosmological fluctuations.
- In Standard Big Bang Cosmology: Hubble radius = horizon.
- In any theory which can provide a mechanism for the origin of structure: Hubble radius $\neq$ horizon.

Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the "horizon problem" of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Squeezing of fluctuations on super-Hubble scales in order to obtain the acoustic oscillations in the CMB angular power spectrum.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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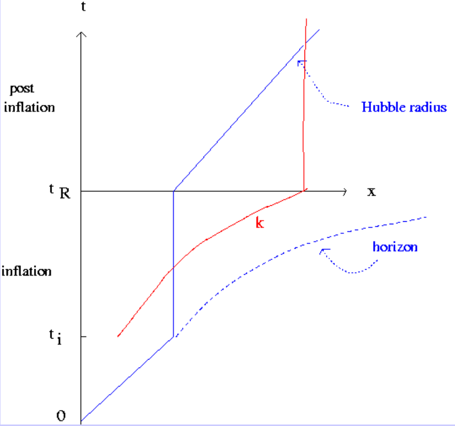
Inflation as a Solution

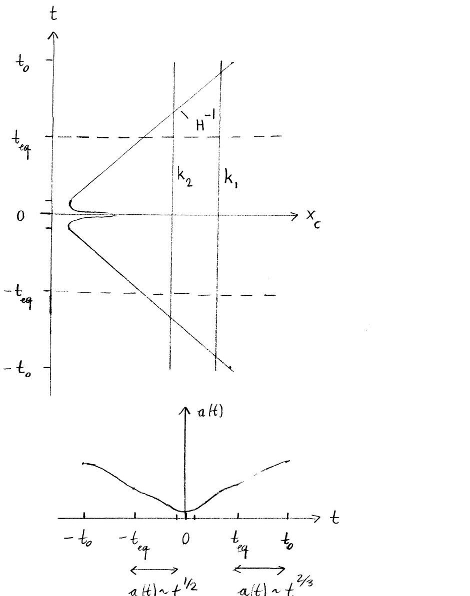


Addressing the Criteria

- Exponential increase in horizon relative to Hubbe radius.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Time translation symmetry $\rightarrow$ scale-invariant spectrum (Press, 1980).

Bouncing Cosmologies as a Solution

M. Gasperini and G. Veneziano (1992); J. Khoury et al (2001); F. Finelli and R.B., (2002), D. Wands(1999)



Addressing the Criteria

- Horizon infinite, Hubble radius decreasing.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- **Curvature fluctuations** starting from the vacuum acquire a scale-invariant spectrum on scales which exit the Hubble radius during matter domination ("matter bounce scenario").
- **Entropy fluctuations** starting from the vacuum induce a scale-invariant spectrum in the Ekpyrotic bounce scenario.

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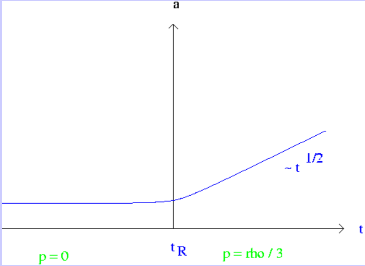
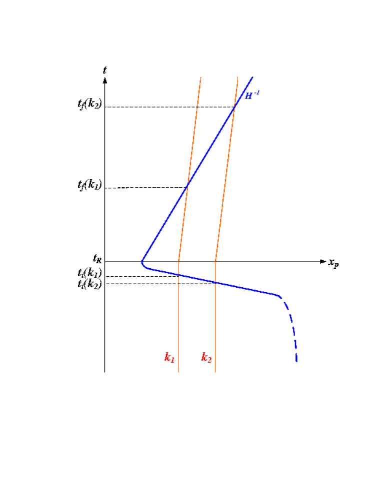
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Addressing the Criteria

- Horizon given by the duration of the quasi-static phase, Hubble radius decreass suddenly at the phase transition $\rightarrow$ horizon $\gg$ Hubble radius at the beginning of the Standard Big Bang phase.
- Fluctuations originate on sub-Hubble scales.
- Long period of super-Hubble evolution.
- Curvature fluctuations starting from thermal matter inhomogeneities acquire a scale-invariant spectrum if the thermodynamics obeys holographic scaling.

Plan

Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$\begin{aligned} ds^2 &= a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \\ \varphi &= \varphi_0 + \delta\varphi \end{aligned}$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$\begin{aligned} S^{(2)} &= \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right) \\ v &= a\left(\delta\varphi + \frac{z}{a}\Phi\right) \\ z &= a\frac{\varphi_0'}{\mathcal{H}} \end{aligned}$$

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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1}$$

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More on Perturbations I

In the case of adiabatic fluctuations, there is only one degree of freedom for the scalar metric inhomogeneities. It is

$$\zeta = z^{-1} v$$

Its physical meaning: curvature perturbation in comoving gauge.

- In an expanding background, ζ is conserved on super-Hubble scales.
- In a contracting background, ζ grows on super-Hubble scales.

More on Perturbations II

- In the case of entropy fluctuations there are more than one degrees of freedom for the scalar metric inhomogeneities. Example: extra scalar field.
- Entropy fluctuations seed an adiabatic mode even on super-Hubble scales.

$$\dot{\zeta} = \frac{\dot{p}}{p + \rho} \delta S$$

- Example: topological defect formation in a phase transition.
- Example: Axion perturbations when axions acquire a mass at the QCD scale (M. Axenides, R.B. and M. Turner, 1983).

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Gravitational Waves

$$ds^2 = a^2\left[(1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right]$$

- $h_{ij}(\mathbf{x}, t)$ transverse and traceless
- Two polarization states

$$h_{ij}(\mathbf{x}, t) = \sum_{a=1}^{2} h_a(\mathbf{x}, t)\epsilon_{ij}^a$$

- At linear level each polarization mode evolves independently.

Gravitational Waves II

Canonical variable for gravitational waves:

$$u(\mathbf{x}, t) \, = \, a(t) h(\mathbf{x}, t)$$

Equation of motion for gravitational waves:

$$u_k^{''} + \left(k^2 - \frac{a^{''}}{a}\right) u_k \, = \, 0 \, .$$

Squeezing on super-Hubble scales, oscillations on sub-Hubble scales.

Consequences for Tensor to Scalar Ratio r

R.B., arXiv:1104.3581

- If EoS of matter is time independent, then $z \propto a$ and $u \propto v$.
- Thus, generically models with dominant adiabatic fluctuations lead to a large value of r. A large value of r is not a smoking gun for inflation.
- During a phase transition EoS changes and u evolves differently than v
- $\rightarrow$ Suppression of r.
- This happens during the inflationary reheating transition.
- Simple inflation models typically predict very small value of r.

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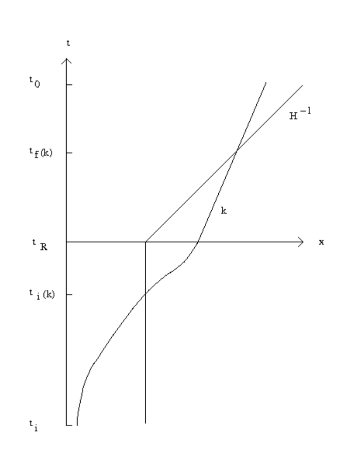
Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Origin of Scale-Invariance in Inflation

- **Initial vacuum spectrum** of ζ ($\zeta \sim v$): (Chibisov and Mukhanov, 1981).

$$P_\zeta(k) \equiv k^3 |\zeta(k)|^2 \sim k^2$$

- $v \sim z \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_\zeta(k,t) \equiv P_\zeta(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \sim k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_\zeta(k,t) \sim \mathrm{const}$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \left(\frac{a(t)}{a(t_i(k))}\right)^2 \simeq k^2 a(t_i(k))^{-2}$$

- Hubble radius crossing: $ak^{-1} = H^{-1}$
- $\rightarrow P_h(k,t) \simeq H^2$

Note: If NEC holds, then $\dot{H} < 0 \;\; \rightarrow$ red spectrum, $n_t < 0$

Scale-Invariance of Gravitational Waves in Inflation

- **Initial vacuum spectrum** of u (Starobinsky, 1978):

$$P_h(k) \equiv k^3 |h(k)|^2 \sim k^2$$

- $u \sim a$ on super-Hubble scales
- At late times on super-Hubble scales

$$P_h(k,t) \equiv a^{-2}(t) P_u(k, t_i(k)) \Big(\frac{a(t)}{a(t_i(k))}\Big)^2 \simeq k^2 a(t_i(k))^{-2}$$

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Matter Bounce: Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) \, = \, k^3 |\zeta(k)|^2 \, \sim \, k^2$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \, = \, c_1 \eta^2 + c_2 \eta^{-1} \, ,$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

$$\begin{aligned} P_\zeta(k, \eta) \, & \sim \, k^3 |v_k(\eta)|^2 a^{-2}(\eta) \\ & \sim \, k^3 |v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \, \sim \, k^{3-1-2} \\ & \sim \, \mathrm{const} \, , \end{aligned}$$

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gao et al, 2009).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Plan

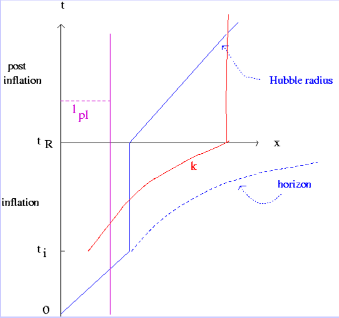
Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field φ (the "inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Origin of Inflation?

- To obtain inflationary dynamics free of initial condition fine tuning we require super-Planckian field values.
- $\rightarrow$ requires embedding of inflation into a quantum gravitational theory.
- But: No-go theorems on obtaining de Sitter space in string theory.

Trans-Planckian Problem

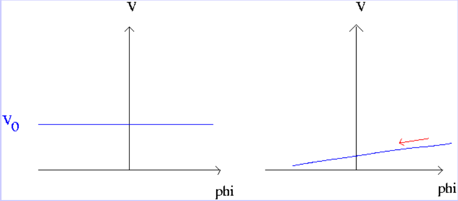


- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- $\rightarrow$ new physics MUST enter into the calculation of the fluctuations.

Singularity Problem

- Standard cosmology: Penrose-Hawking theorems $\rightarrow$ initial singularity $\rightarrow$ incompleteness of the theory.
- Inflationary cosmology: In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] $\rightarrow$ incompleteness of the theory.

Cosmological Constant Problem



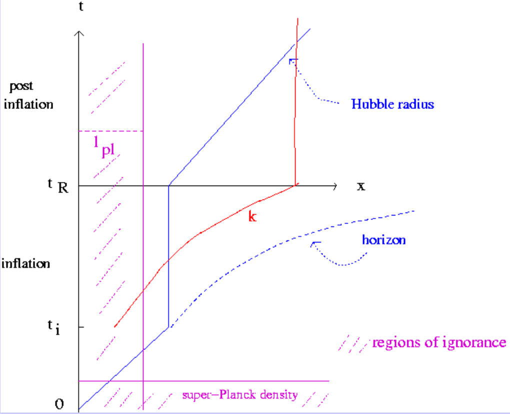
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120}$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Anisotropy Problem of the Contracting Phase

Y. Cai, R.B. and P. Peter, arXiv:1301.4703

Problem: The energy density in anisotropies increases faster than the energy density in matter and radiation in the contracting phase.

$$ds^2 = dt^2 - a^2(t)\sum_i e^{2\theta_i(t)}\sigma_i^2$$

$$H^2 = \frac{\rho}{3m_{pl}^2} + \frac{1}{6}\sum_i \dot{\theta}_i^2$$

$$\ddot{\theta}_i + 3H\dot{\theta}_i = 0$$

$$\rightarrow \quad \rho_{anis} \sim a^{-6}$$

Note: This is not a problem for Ekpyrotic Cosmology.

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Black Hole Formation in the Contracting Phase

J. Quintin and R.B., arXiv:1609.02556

Worry: Cosmological fluctuations become nonlinear on sub-Hubble scales and form black holes.

Starting point: scalar cosmological perturbations in longitudinal gauge:

$$\mathrm{d}s^2 = a(\eta)^2 \left\{ [1 + 2\Phi(\eta, \mathbf{x})]\, \mathrm{d}\eta^2 - [1 - 2\Phi(\eta, \mathbf{x})]\, \delta_{ij} \mathrm{d}x^i \mathrm{d}x^j \right\} \, .$$

Equation of motion:

$$\Phi_k'' - \frac{6(1 + c_\mathrm{s}^2)}{1 + 3w} \frac{1}{(-\eta)} \Phi_k' + \left(c_\mathrm{s}^2 k^2 + \frac{12(c_\mathrm{s}^2 - w)}{(1 + 3w)^2} \frac{1}{(-\eta)^2} \right) \Phi_k = 0 \, .$$

Black Hole Formation (ctd.)

Resulting fractional density contrast:

$$\delta_k \equiv \frac{\delta\rho_k^{(\mathrm{gi})}}{\rho^{(0)}} = -\frac{2}{3}\left(\frac{k^2}{\mathcal{H}^2}\Phi_k + \frac{3}{\mathcal{H}}\Phi_k' + 3\Phi_k\right) .$$

Criterium for direct black hole formation.

$$\int_{R \leq R_s} \mathrm{d}\delta M \geq M_s .$$

Result: for Bunch-Davies vacuum initial conditions early in the contracting phase the first scale to form black holes is the Hubble scale.

Black Hole Formation (ctd.)

The condition that black holes form becomes

$$|H| \sim c_s^{12/5} w^{3/5} \left(\frac{M_{\mathrm{Pl}}}{H_{\mathrm{ini}}}\right)^{1/5} M_{\mathrm{Pl}}$$

- For $c_s \ll 1$ we have $H \ll M_{pl}$.
- For a radiation dominated phase at late stages of contraction no black holes form from the direct channel if $|H_{max}| < M_{pl}$.

Initial Condition Problem of the Contracting Phase

- **Q: Attractor Nature of the Background**
- A: o.k. for Ekpyrotic contraction, not o.k. for matter bounce.
- **Q: What initial conditions for fluctuations?**
- Usual answer: vacuum - but why?
- Note: For inflation the use of vacuum initial conditions for fluctuations can be justified.

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Phenomenological Challenges

J. Quintin et al, arXiv:1508.04141

- $r \sim 1$ at the end of the contracting phase.
- $f_{nl} \sim 1$ at the end of the contracting phase (Y. Cai, W. Xue, R.B. and X. Zhang, arXiv:0903.0631).
- Need mechanism to boost the scalar spectrum during the bounce phase.
- This mechanism will generically boost f_{nl}.

$\rightarrow$ no go "theorem" in simple single field matter bounce models.

Obtaining a Bounce

- **New matter which violates the Null Energy Condition.**
- Challenges: Instabilities.
- **Modifications of Gravity.**
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- **Quantum Resolution.**

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Some Examples

Modified Matter

- Ghost condensate [C. Lin, L. Perreault Levasseur and R.B., arXiv:1007.2654 [hep-th]]
- Galileon matter [A. Ijjas and P. Steinhardt, 2016]

Modified Gravity

- Horava-Lifshitz gravity [R.B., arXiv:0904.2835 [hep-th]]

Quantum Resolution

- Loop quantum cosmology [A. Ashtekar, M. Bojowald, A. Barrau, I. Agullo]
- Perfect bounce [S. Gielen and N. Turok]

Challenges for Emergent Cosmologies

- What is the **dynamics** which yields a quasi-static phase?
- **Stability** of the emergent phase?

Plan

Scenario 1: Temporal Duality

R.B., C. Kounnas, H. Partouche, S. Patil and N. Toumbas, arXiv:1312.2524 [hep-th]

Starting point: Type II superstring theory in the presence of non-trivial gravito-magnetic fluxes (Euclidean background)

Temperature duality:

$$Z(T) = Z(T_c^2/T).$$

T_c: Self-dual temperature (equals the Hagedorn temperature modulo coupling constants)

Physical temperature

$$\begin{aligned} T_p &= T \quad T \ll T_c \\ T_p &= \frac{T_c^2}{T} \quad T \gg T_c \end{aligned}$$

S-Brane

- For $T \ll T_c$ and $T \gg T_c$ the dynamics of the low energy modes of string theory is given by **dilaton gravity**
- Begin in a contracting phase with $T \gg T_c$ and T decreasing (i.e. T_p increasing).
- When $T = T_c$ a set of string states becomes massless (enhanced symmetry states)
- These states must be included in the action for the low energy modes.
- S-Brane: term in the action present only at $T = T_c$
- S-brane has $\rho < 0$ and $p = |\rho| > 0$ → S-brane is matter violating the NEC and can mediate a transition from contraction to expansion.
- → S-Brane bounce.

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} - \nabla_\mu\phi\nabla^\mu\phi\right] + \int d^4x\sqrt{-g}\, n^*\sigma_r T_E^4$$
$$-\kappa\int d\tau d^3\xi\sqrt{h}e^{\phi}\delta(\tau)\,.$$

Evolution of Fluctuations through the Bounce

- Consider initially scale-invariant cosmological fluctuations in the contracting phase on super-Hubble scales.
- **Matching conditions** across the S-brane: continuity of the induced metric and extrinsic curvature.
- Note: matching surface uniquely determined!
- **Result**: the spectrum of cosmological perturbations after the bounce on super-Hubble scales is scale-invariant.

Scenario 2: AdS/CFT Cosmology

R.B., Y. Cai, S. Das, E. Ferreira, I. Morrison and Y. Wang arXiv:1601.00231 [hep-th]

- Consider time dependent deformation of AdS via a time-dependent string coupling constant.
- Corresponding to a contracting universe for $t < t_b$ and an expanding universe for $t > t_b$.
- Curvature singularity at $t = t_b$.
- Gravitational coupling weak for $t < t_i$ and $t > t_f$.

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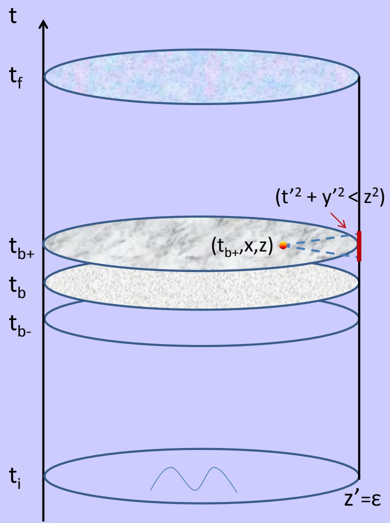
t
t_f
$(t'^2 + y'^2 < z^2)$
t_{b+}
(t_{b+},x,z)
t_b
t_{b-}
t_i
$z'=\varepsilon$

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Scenario 2: AdS/CFT Cosmology

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- Begin with a homogeneous and isotropic bulk solution: contracting universe.
- Map the bulk onto the boundary via AdS/CFT at $t = t_i$
- Evolve the system on the boundary for $t_i < t < f_f$.
- The conformal field theory can be continued to $t > t_b$ without encountering a singularity (S. Das et al.).
- Bulk cosmology can be reconstructed for $t > t_b$ via boundary-to-bulk propagators.
- $\rightarrow$ successful singularity resolution.

Scenario 2: AdS/CFT Cosmology

R.B., Y. Cai, S. Das, E. Ferreira, I. Morrison and Y. Wang arXiv:1601.00231 [hep-th]

Our work: include initial cosmological perturbations in the bulk in the contracting phase.

- Nonanalyticity in one of the two solutions of the fluctuation equations at t_b
- $\rightarrow$ need to add a cutoff.
- Need to use matching conditions at the level of the CFT (less ambiguities than when using matching conditions in the bulk!).
- Result: **Spectral index of the fluctuations unchanged across the bounce.**

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Principles of String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

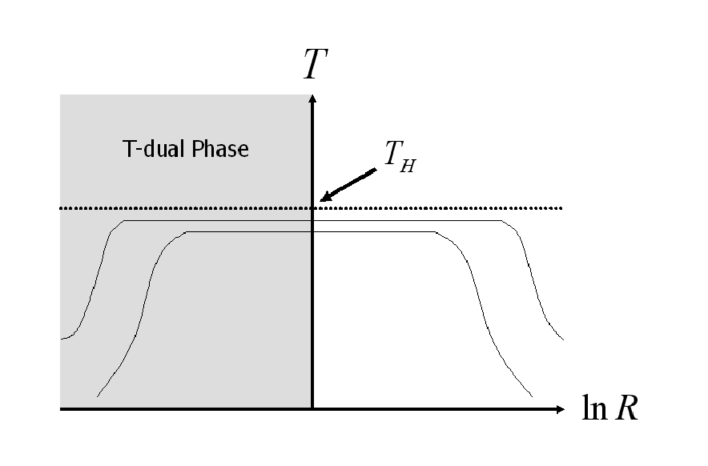
T-Duality

- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \to 1/R \quad (n, m) \to (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\to$ existence of D-branes

Adiabatic Considerations

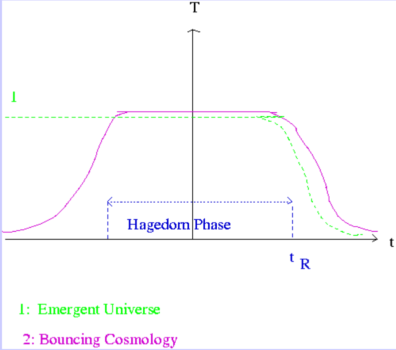
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Dynamics

Assume some action gives us $R(t)$



String Gas Bounce

Two possibilities:

- Thermal Bounce
- Emergent Scenario

In both cases, a **long Hagedorn phase** will allow thermalization of the string gas on large scales.

$\rightarrow$ thermal initial conditions for fluctuations

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Doubled Space in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

Candidate for dynamics in the Hagedorn phase: Double Field Theory [C. Hull and B. Zwiebach, 2009]
Idea: For each dimension of the underlying topological space there are two position operators [R.B. and C. Vafa]:

- x: dual to the momentum modes
- $\tilde{x}$: dual to the winding modes

We measure **physical length** in terms of the **light** degrees of freedom.

$$
\begin{aligned}
l(R) &= R \quad \text{for} \quad R \gg 1\,, \\
l(R) &= \frac{1}{R} \quad \text{for} \quad R \ll 1\,.
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Double Field Theory Approach

Idea Describe the low-energy degrees of freedom with an action in doubled space in which the T-duality symmetry is manifest.

$$S = \int dx d\tilde{x} e^{-2d} \mathcal{R},$$

$$\begin{aligned}
\mathcal{R} &= \frac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_N\mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{MN}\partial_M\mathcal{H}^{KL}\partial_K\mathcal{H}_{NL} \\
&+ 4\mathcal{H}^{MN}\partial_M\partial_N d - \partial_M\partial_N\mathcal{H}^{MN} - 4\mathcal{H}^{MN}\partial_M d\partial_N d \\
&+ 4\partial_M\mathcal{H}^{MN}\partial_N d + \frac{1}{2}\eta^{MN}\eta^{KL}\partial_M\mathcal{E}^A{}_K\partial_N\mathcal{E}^B{}_L\mathcal{H}_{AB} .
\end{aligned}$$

$$
\begin{aligned}
\mathcal{H}_{MN} &= \begin{bmatrix} g^{ij} & -g^{ik}b_{kj} \\ b_{ik}g^{kj} & g_{ij} - b_{ik}g^{kl}b_{lj} \end{bmatrix} . \\
X^M &= (\tilde{x}_i, x^i), \\
\eta^{MN} &= \begin{bmatrix} 0 & \delta_i{}^j \\ \delta^i{}_j & 0 \end{bmatrix} .
\end{aligned}
$$

Singularity Resolution in SGC

R.B., R. Costa, G. Franzmann, S. Patil and A. Weltman, in prep.

- Consider test particles in a DFT background.
- Derive geodesic equation of motion
- Consider a cosmological background with $b = 0$ and fixed dilaton.
- Find that the geodesics can be extended to infinite proper time in both time directions.
- $\rightarrow$ geodesic completeness in terms of physical time:

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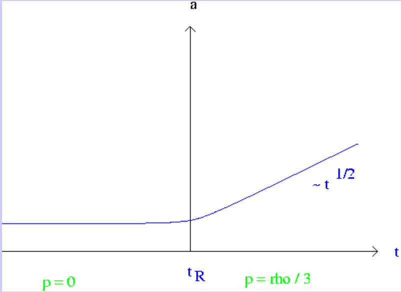
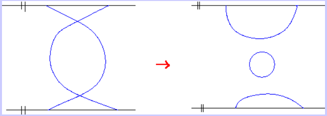
Emergent Dynamics

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

(see also numerical work by M. Sakellariadou)

Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow \quad V_{eff}(R)$ has a minimum at a finite value of $R, \quad \rightarrow \quad R_{min}$
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- $\rightarrow$ harmonic oscillator potential for θ
- $\rightarrow$ shape moduli stabilized

Dilaton stabilization in SGC

- The only remaining modulus is the dilaton.
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum.
- $\rightarrow$ diltaton is stabilized.
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008].
- Gaugino condensation induces (high scale) supersymmetry breaking [S. Mishra, W. Xue, R.B. and U. Yajnik, 2012].

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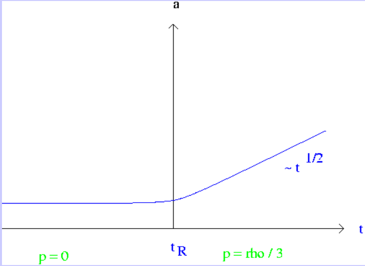
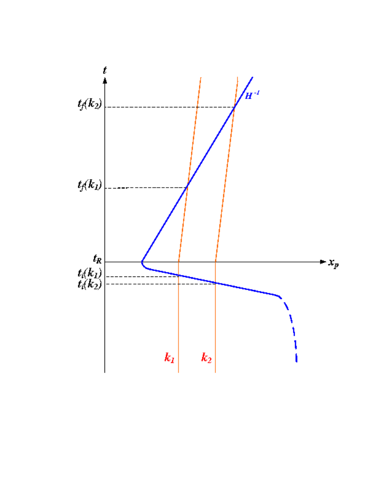
Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Structure Formation in String Gas Cosmology

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \, .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2 / \ell_s^3}{T(1 - T/T_H)} \, .$$

Power spectrum of cosmological fluctuations

$$
\begin{aligned}
P_\Phi(k) &= 8G^2k^{-1} < |\delta\rho(k)|^2 > \\
&= 8G^2k^2 < (\delta M)^2 >_R \\
&= 8G^2k^{-4} < (\delta\rho)^2 >_R \\
&= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H}
\end{aligned}
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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Running of the Spectrum

R.B., G. Franzmann and Q. Liang, arXiv:1708.06793

Using a simple parametrization of the transition between the Hagedorn phase and the radiation phase we find:

$$\alpha_s \sim -(1 - n_s) \, .$$

This is same sign but parametrically larger in amplitude than the running in simple inflationary models:

$$\alpha_s \sim -(n_s - 1)^2$$

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

$$
\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > \\
&= 16\pi^2 G^2 k^{-4} < |T_{ij}(R)|^2 > \\
&\sim 16\pi^2 G^2 \frac{T}{\ell_s^3}(1 - T/T_H)
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Key ingredient for string thermodynamics

$$
< |T_{ij}(R)|^2 > \sim \frac{T}{l_s^3 R^4}(1 - T/T_H)
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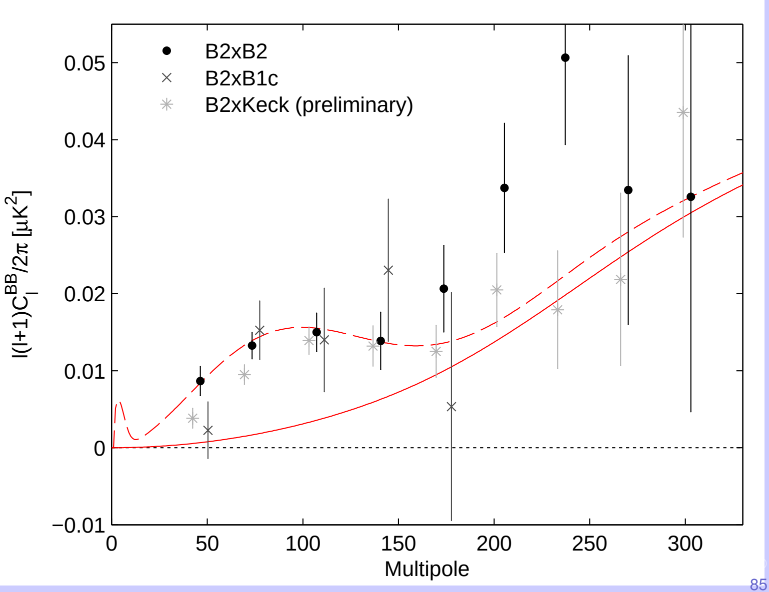
BICEP-2 Results



Requirements

- Emergent phase in thermal equilibrium
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Plan

Conclusions

- Alternatives to cosmological inflation exist.
- Two classes of alternatives are bouncing and emergent cosmologies.
- Generically, a large value of r results if the primordial fluctuations are in the adiabatic mode $\rightarrow$: a detection of $r \neq 0$ is not a "smoking gun" signal of inflation.
- Simple bouncing cosmologies described using **effective field theory** suffer from an anisotropy problem except for models with an Ekpyrotic phase of contraction.

Conclusions II

- Superstring cosmology $\rightarrow$ need to look beyond point particle effective field theory and beyond inflation.
- String Gas Cosmology: Model of cosmology of the very early universe based on new degrees of freedom and new symmetries of superstring theory.
- Thermal string fluctuations lead to an almost scale-invariant spectrum of cosmological fluctuations with **small red tilt** and a **negative running**.
- Key prediction: blue tilt of the tensor modes.
- String Theory testable through cosmological observations.