

Dark Energy

Filippo Vernizzi - IPhT, CEA Saclay

COSMO 2017, Paris - August 28, 2017

Dark Energy and Modified Gravity

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Effective Theory of Dark Energy and Modified Gravity

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Acceleration

In 1998, the Universe started accelerating... Confirmed by many independent datasets

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- Luminosity distance/redshift relation SNIa:

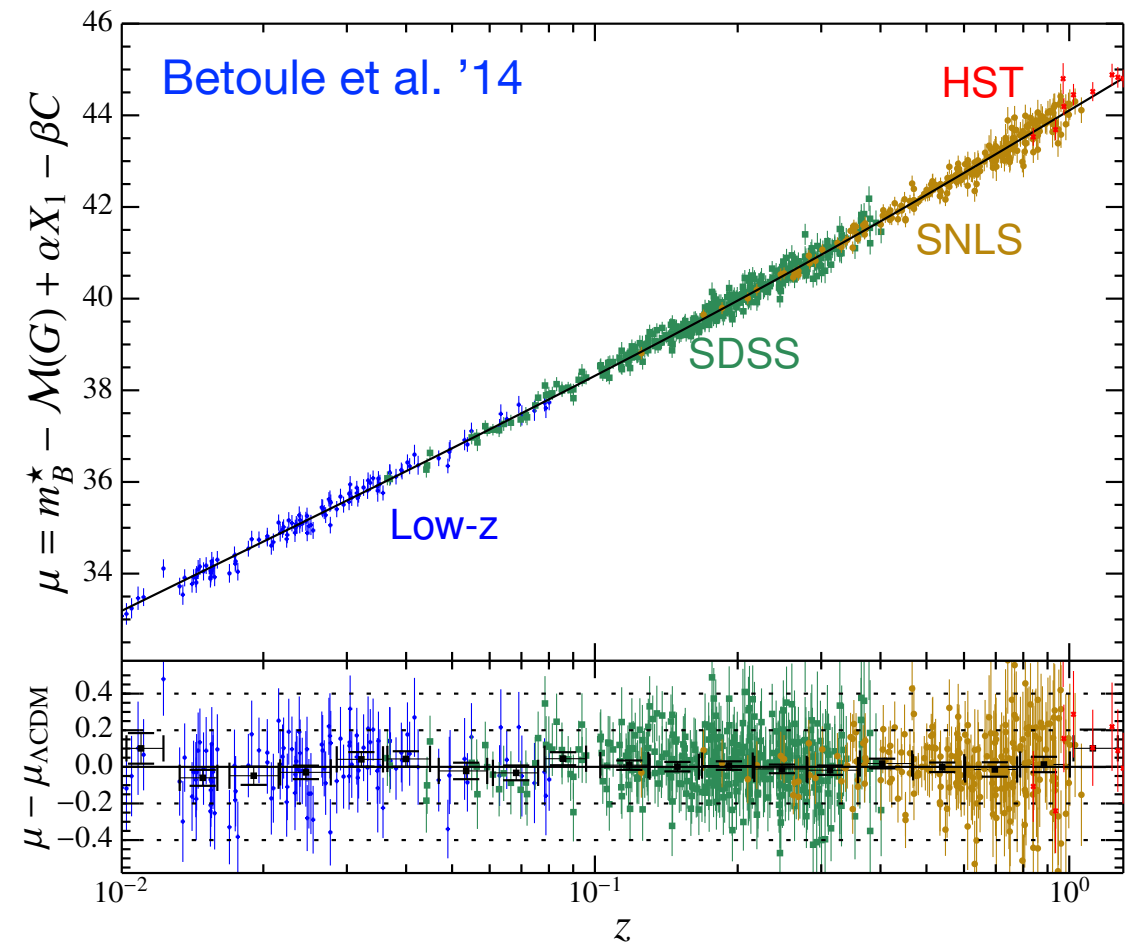
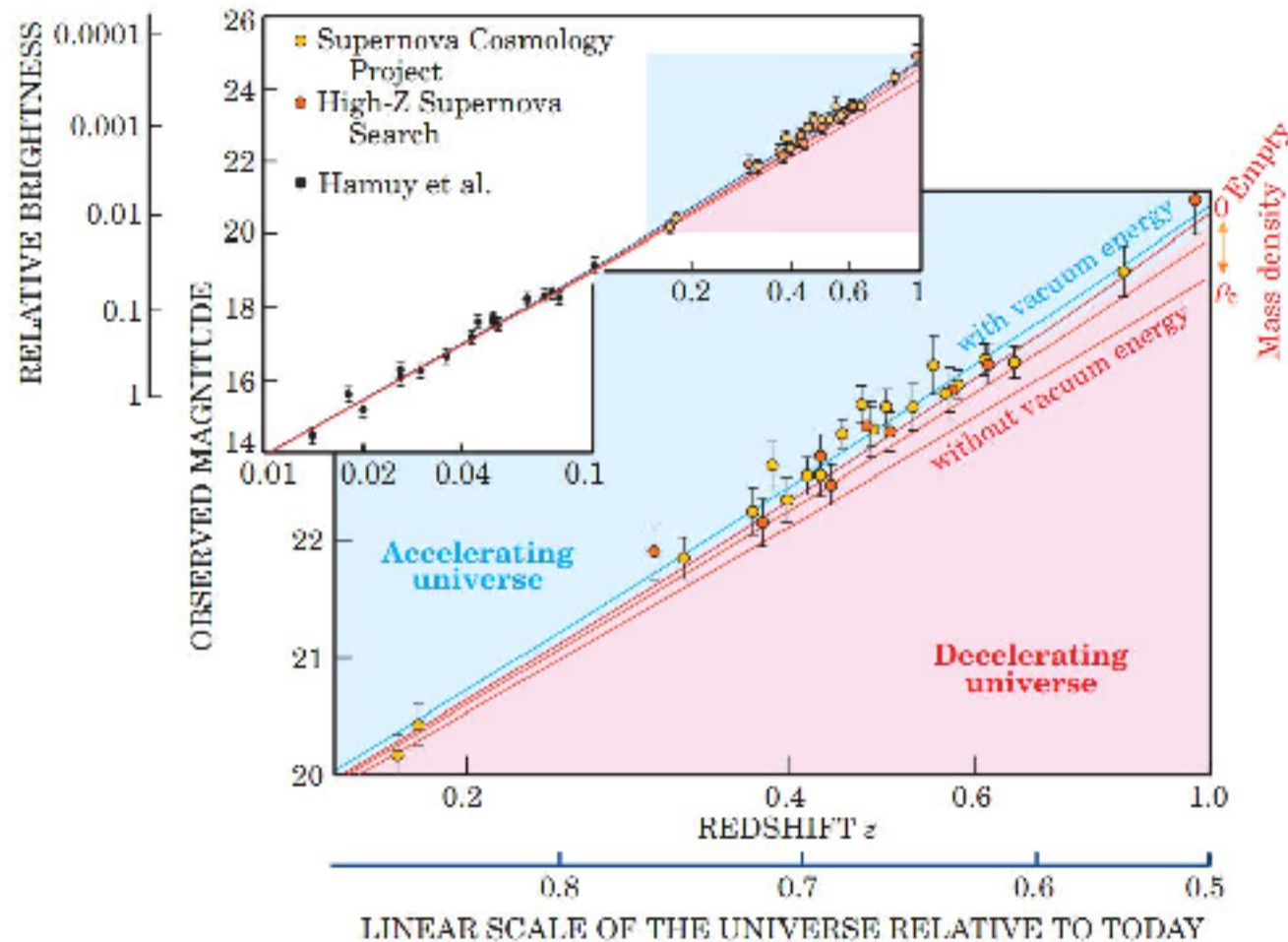


Fig. 8. *Top:* Hubble diagram of the combined sample. The distance modulus redshift relation of the best-fit Λ CDM cosmology for a fixed $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is shown as the black line. *Bottom:* Residuals from the best-fit Λ CDM cosmology as a function of redshift. The weighted average of the residuals in logarithmic redshift bins of width $\Delta z/z \sim 0.24$ are shown as black dots.

Acceleration

In 1998, the Universe started accelerating... Confirmed by many independent datasets

Standard cosmology governed by General Relativity $G_{\mu\nu}(g) = 8\pi G T_{\mu\nu}$

Acceleration implies some form of unknown matter with negative pressure: **dark energy**

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Acceleration

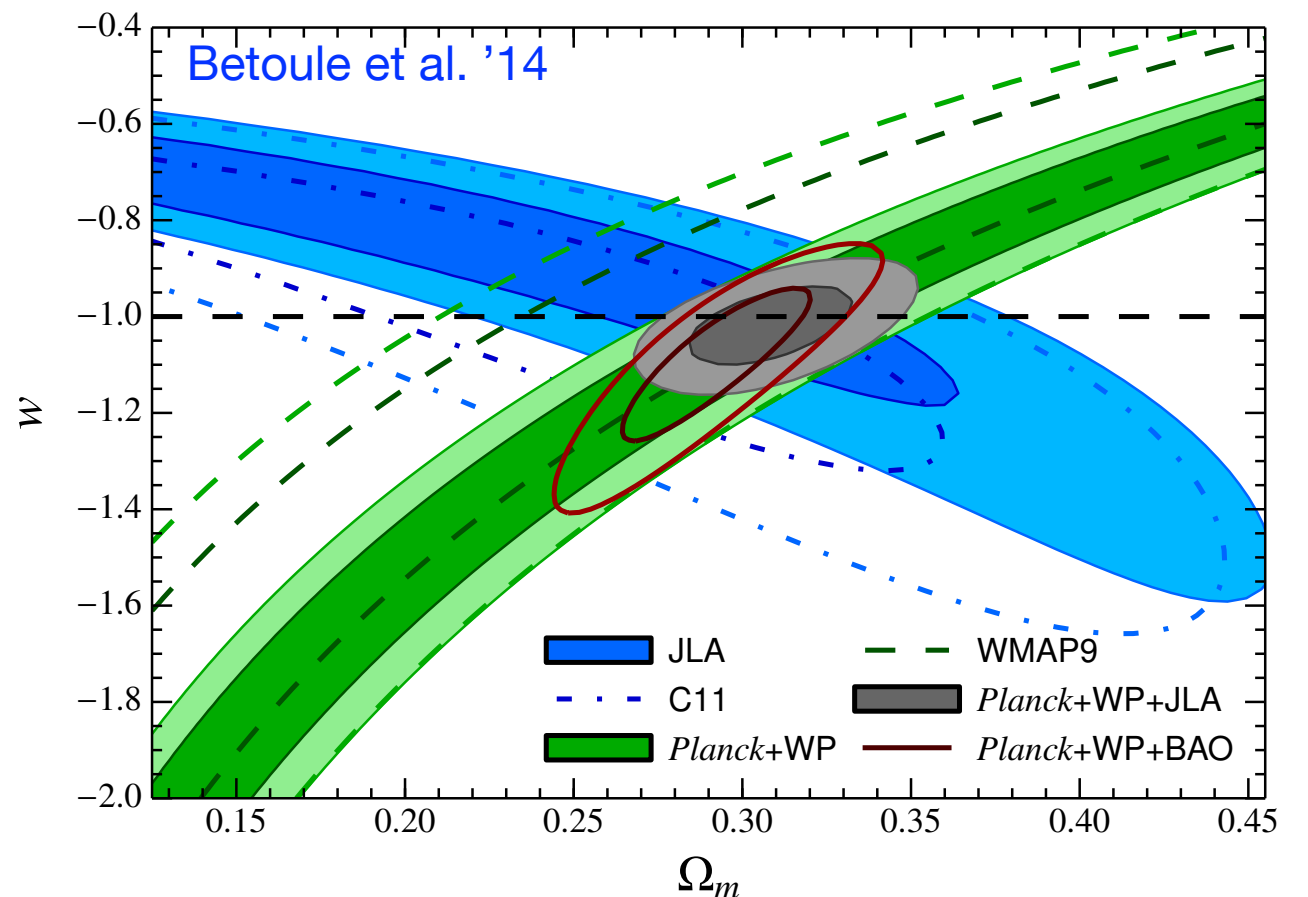
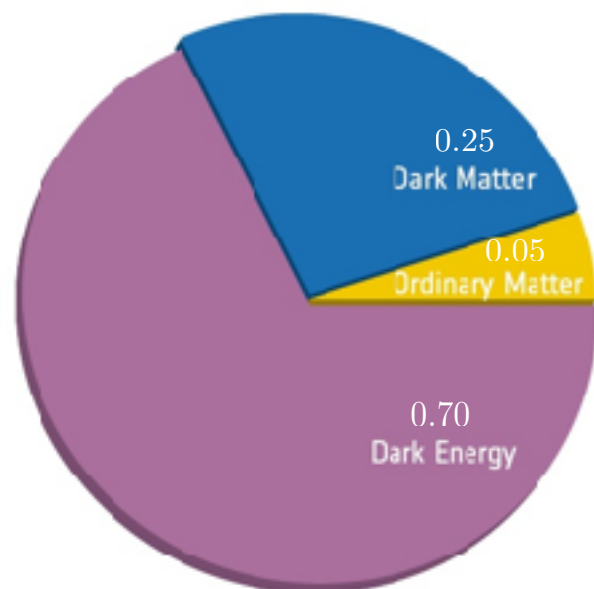
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$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

$$w = \frac{p_{\text{de}}}{\rho_{\text{de}}} \approx -1 \quad \Omega_{\text{de}} \approx 0.7$$



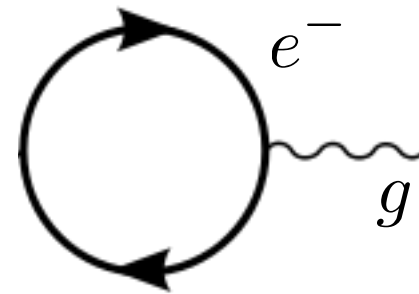
Cosmological Constant

$T_{\mu\nu}^{(\text{de})} = -\Lambda g_{\mu\nu}$: CC, **simplest** explanation, **consistent** with all data

But $\Lambda = \rho_{\text{de}} \simeq (10^{-3} \text{eV})^4$ unnaturally small

Extremely **sensitive** to **UV physics**. Cancellation with vacuum energy of each particle at any loop-order in perturbation theory

$$\Lambda_{\text{obs}} = \Lambda_{\text{bare}} + \sum_i c_i m_i^4$$



e.g. Burgess 13; Padilla 15

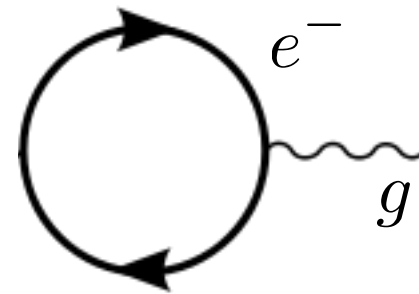
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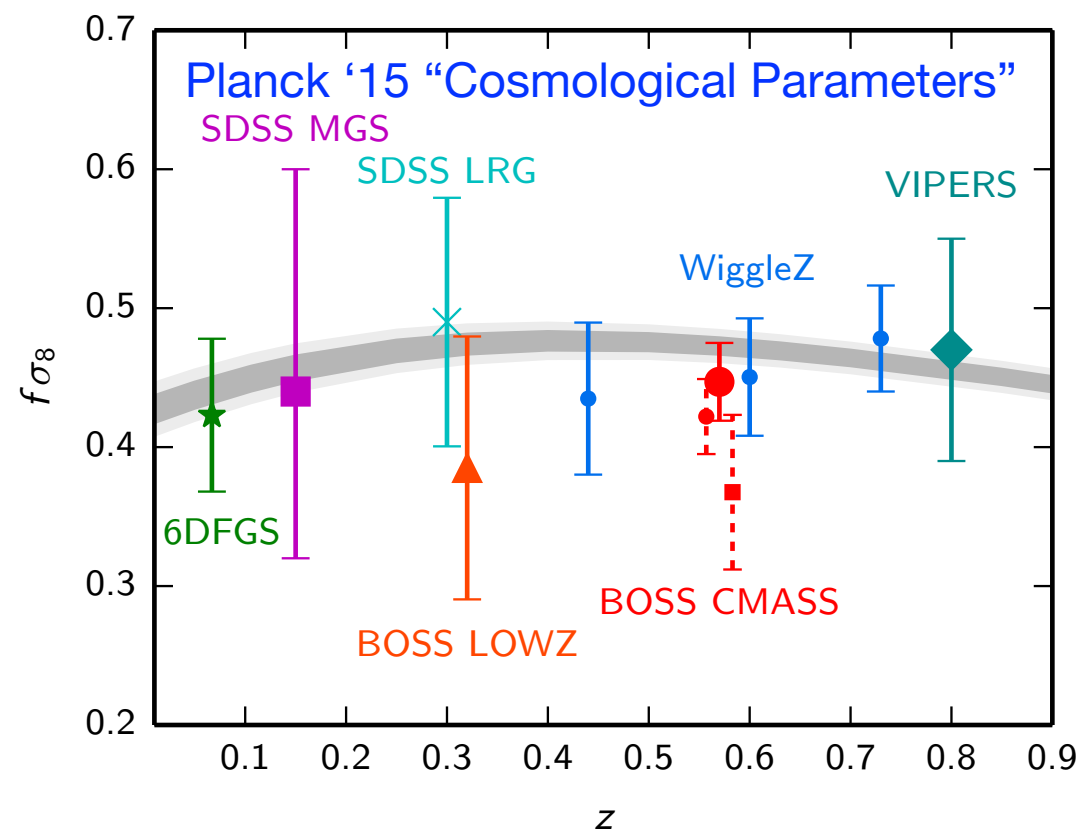


e.g. Burgess 13; Padilla 15

Several attempts to explain smallness. E.g., **Anthropic** (Weinberg 89), **Relaxation** (Abbott 85; Alberte et al 16), **Sequestering** (Kaloper & Padilla 13), **Nonlocal** (Carroll and Remmen 17), etc

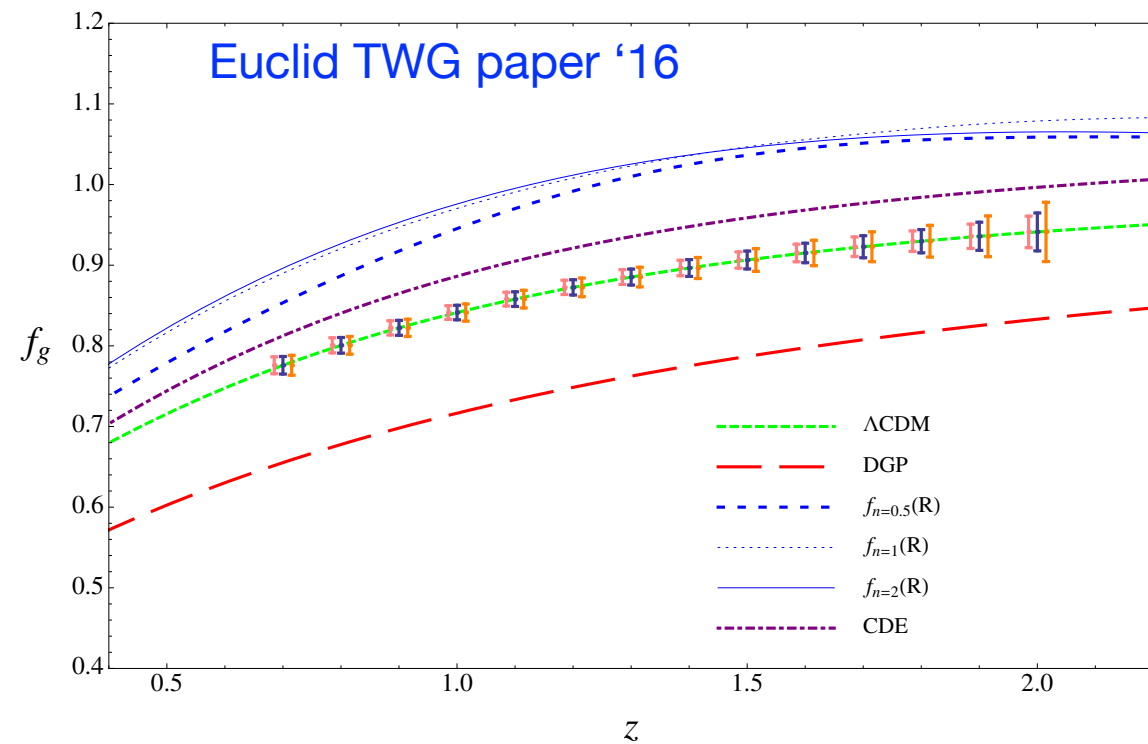
Not a CC

Explanation may be associated to some **dynamical mechanism**: new field or modification of gravity on large scales.



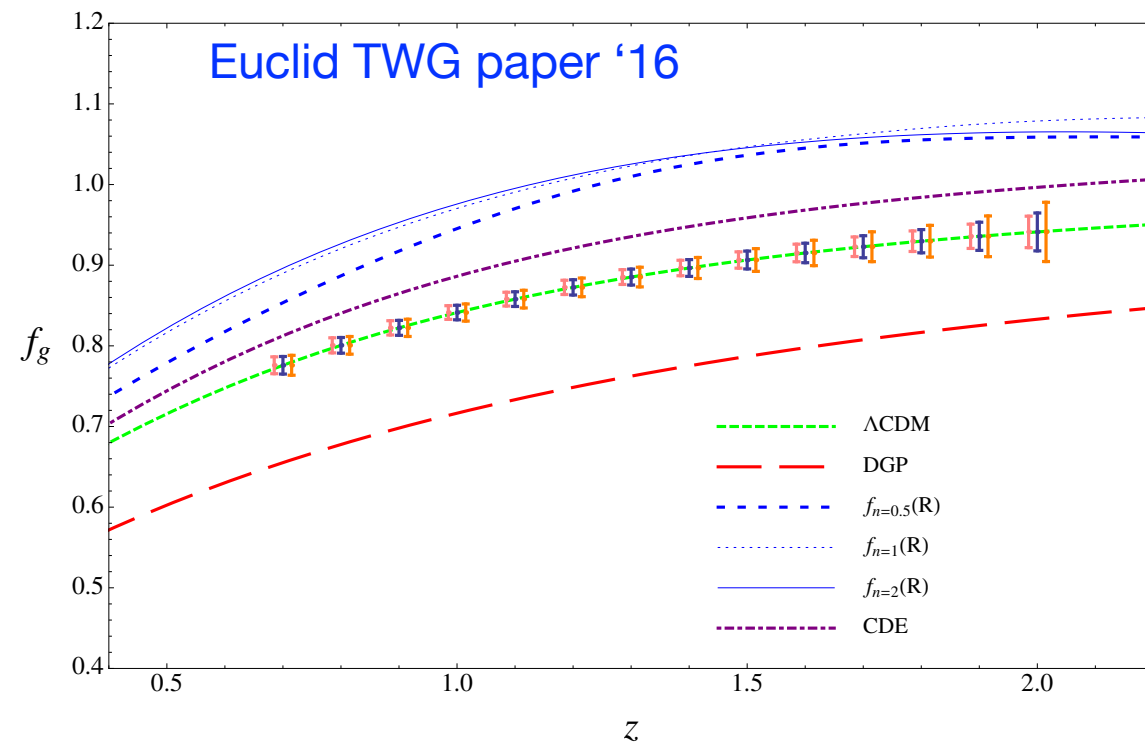
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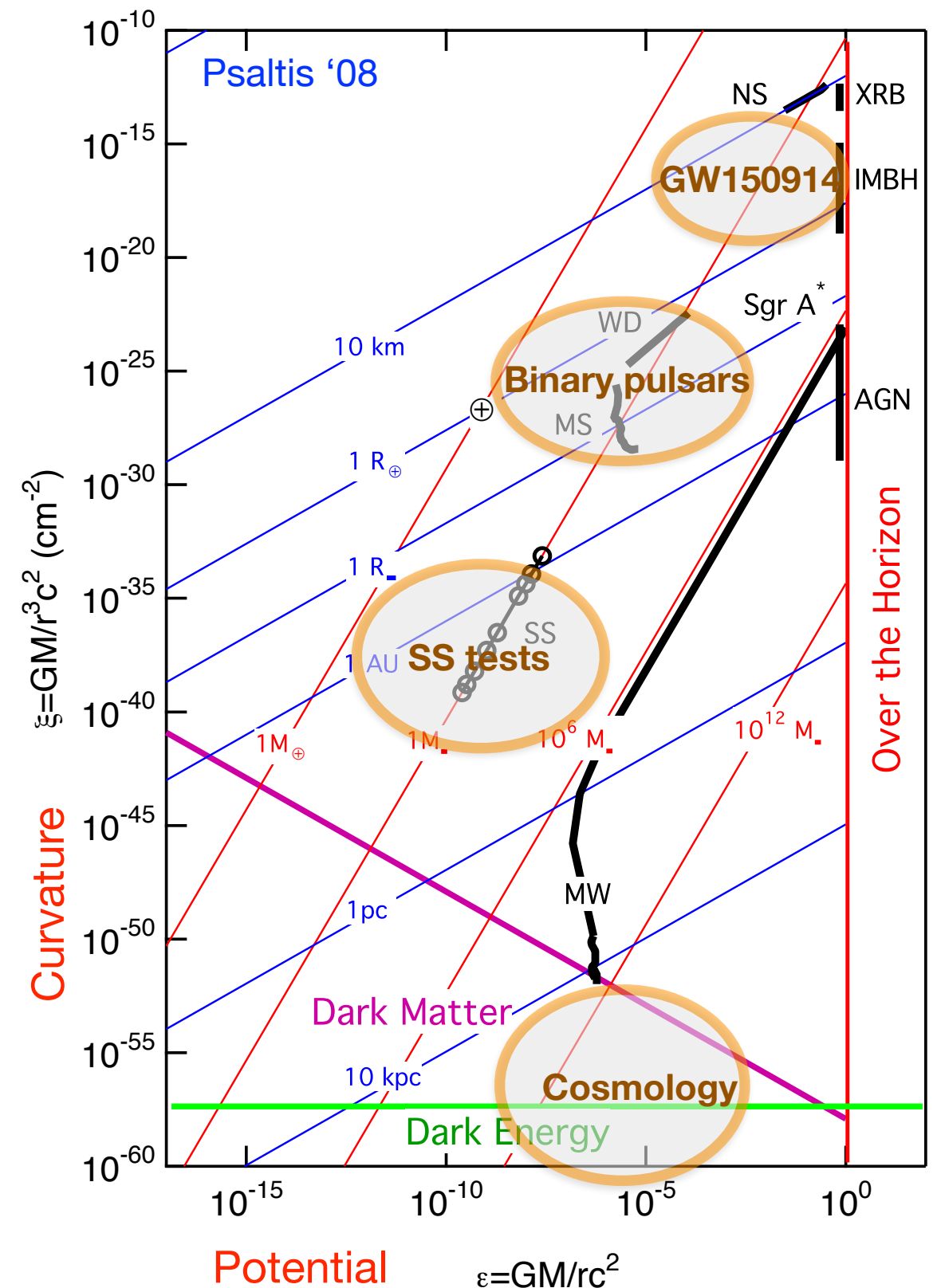


Not a CC

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Gravity **tested** over special ranges of scales and masses. Cosmology is a **window** for testing it on very large distances



Tessa Baker

Einstein-Dilaton-
Gauss-Bonnet

Cascading gravity

Strings & Branes

Randall-Sundrum I & II

DGP

2T gravity

Higher dimensions

Kaluza-Klein

Generalisations
of S_{EH}

Gauss-Bonnet

Lovelock gravity

Emergent
Approaches

CDT

Padmanabhan
thermo.

$$f\left(\frac{R}{\Box}\right) \quad R_{\mu\nu}\Box^{-1}R^{\mu\nu}$$

Non-local

Some
degravitation
scenarios

Lorentz violation

Hořava-Lifschitz

Conformal gravity

$$f(G)$$

Higher-order

$$f(R)$$

General $R_{\mu\nu}R^{\mu\nu}$,
 $\Box R$, etc.

Modified Gravity

TeV

Add new field content

Vector

Einstein-Aether

Lorentz violation

Massive gravity

Bigravity

Tensor

EBI

Bimetric MOND

Scalar-tensor & Brans-Dicke

Ghost condensates

Galileons

the Fab Four

KGB

Coupled Quintessence

Horndeski theories

Scalar

Chern-Simons

Cuscuton

Chaplygin gases

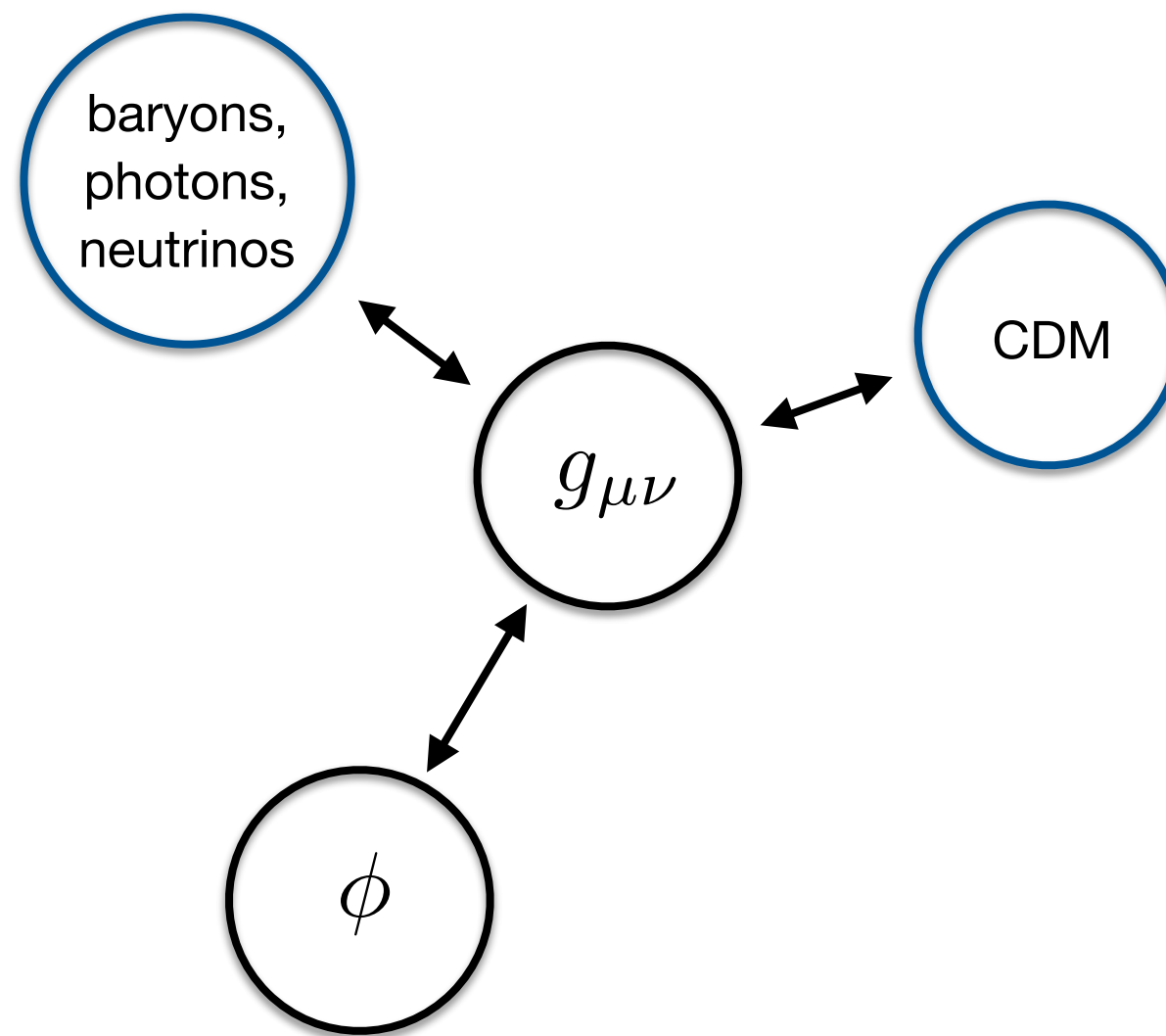
$$f(T)$$

Einstein-Cartan-Sciama-Kibble

Torsion theories

Focus on **single scalar** DOF: scalar-tensor theories.

(see Naruko's talk for vector-tensor)



Theoretical requirements

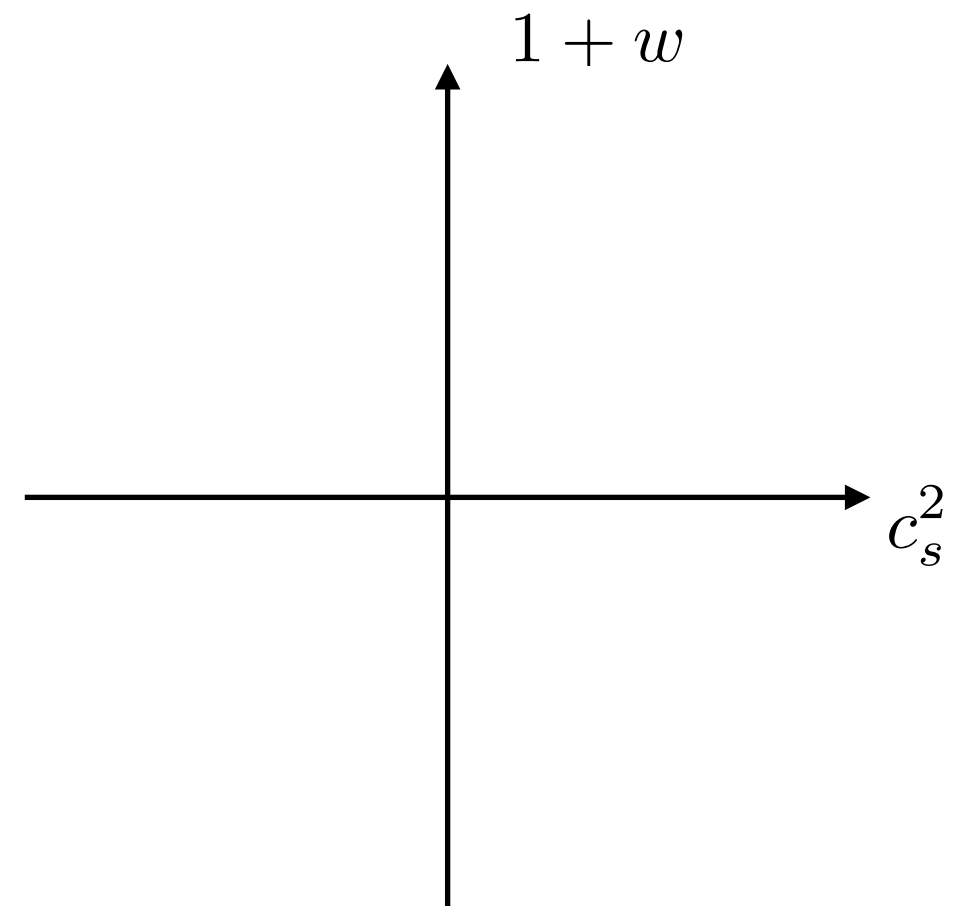
$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, X)\partial_\mu\phi\partial_\nu\phi - V(\phi) \quad X = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

Kinetic termPotential

Expand kinetic term around an FRW background: $\phi = \bar{\phi}(t) + \varphi(t, \vec{x})$

$$\mathcal{L} = \frac{1}{2}Z(\bar{\phi}) [\dot{\varphi}^2 - c_s^2(\bar{\phi})(\nabla\varphi)^2] \quad c_s^2 = \frac{\rho_{\text{de}} + p_{\text{de}}}{Z}$$

Time-dependent kinetic energy and sound speed



Theoretical requirements

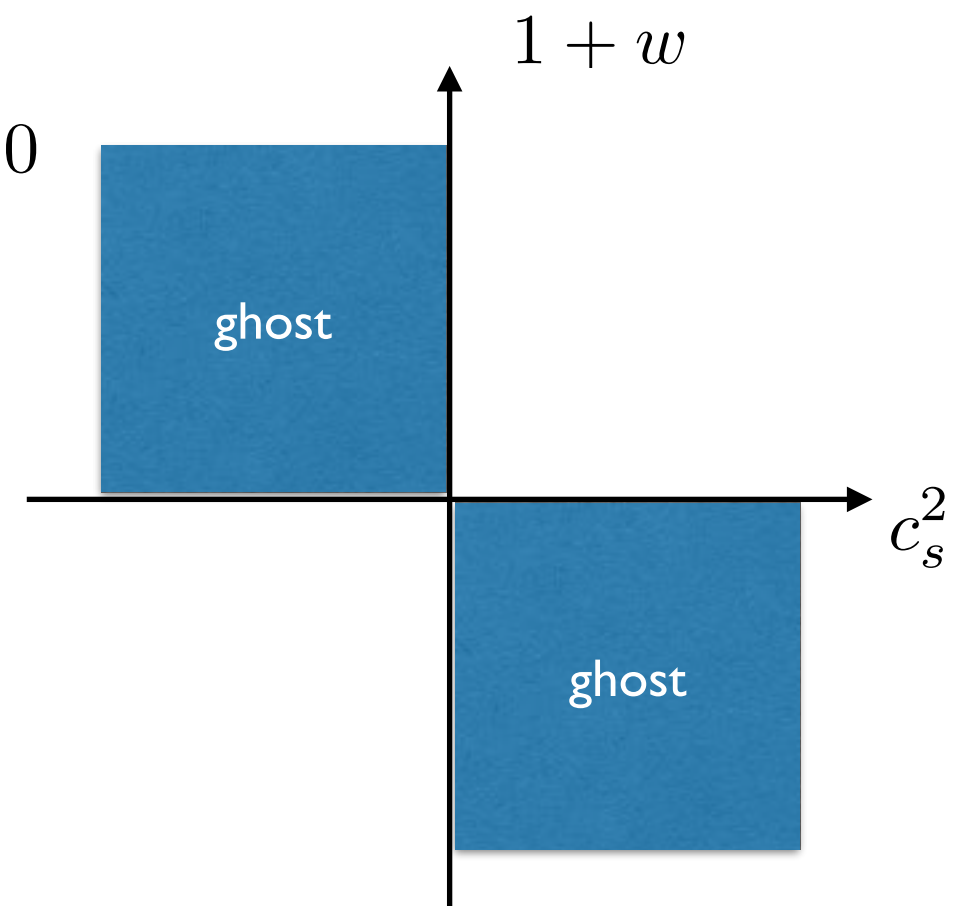
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Time-dependent kinetic energy and sound speed

- **Avoid negative energy states (ghosts):** $Z(\bar{\phi}) > 0$



Theoretical requirements

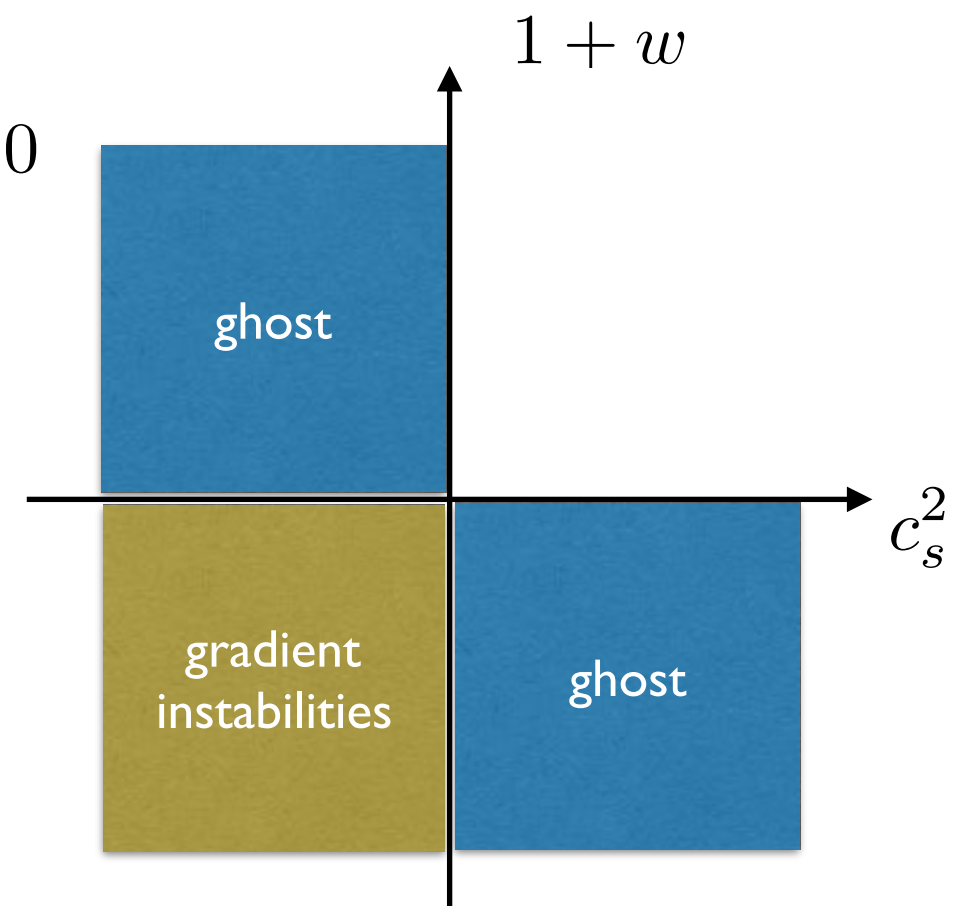
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Time-dependent kinetic energy and sound speed

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- **Avoid gradient instabilities:** $c_s^2(\bar{\phi}) > 0$



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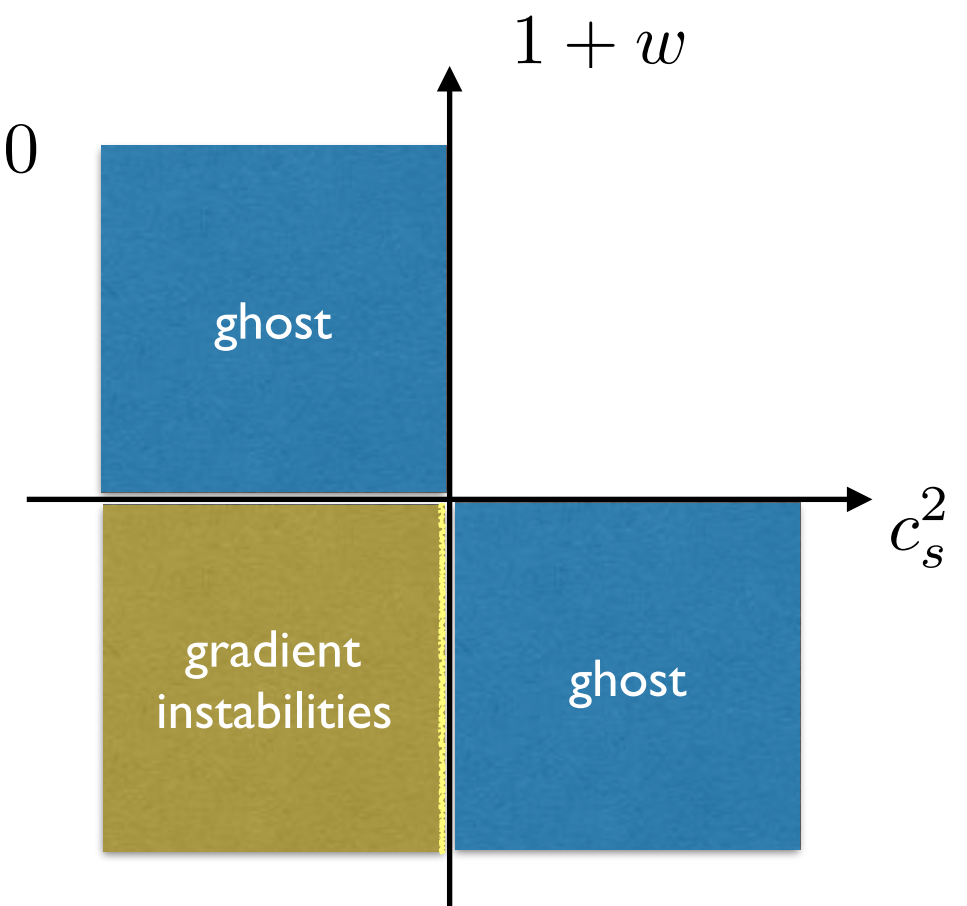
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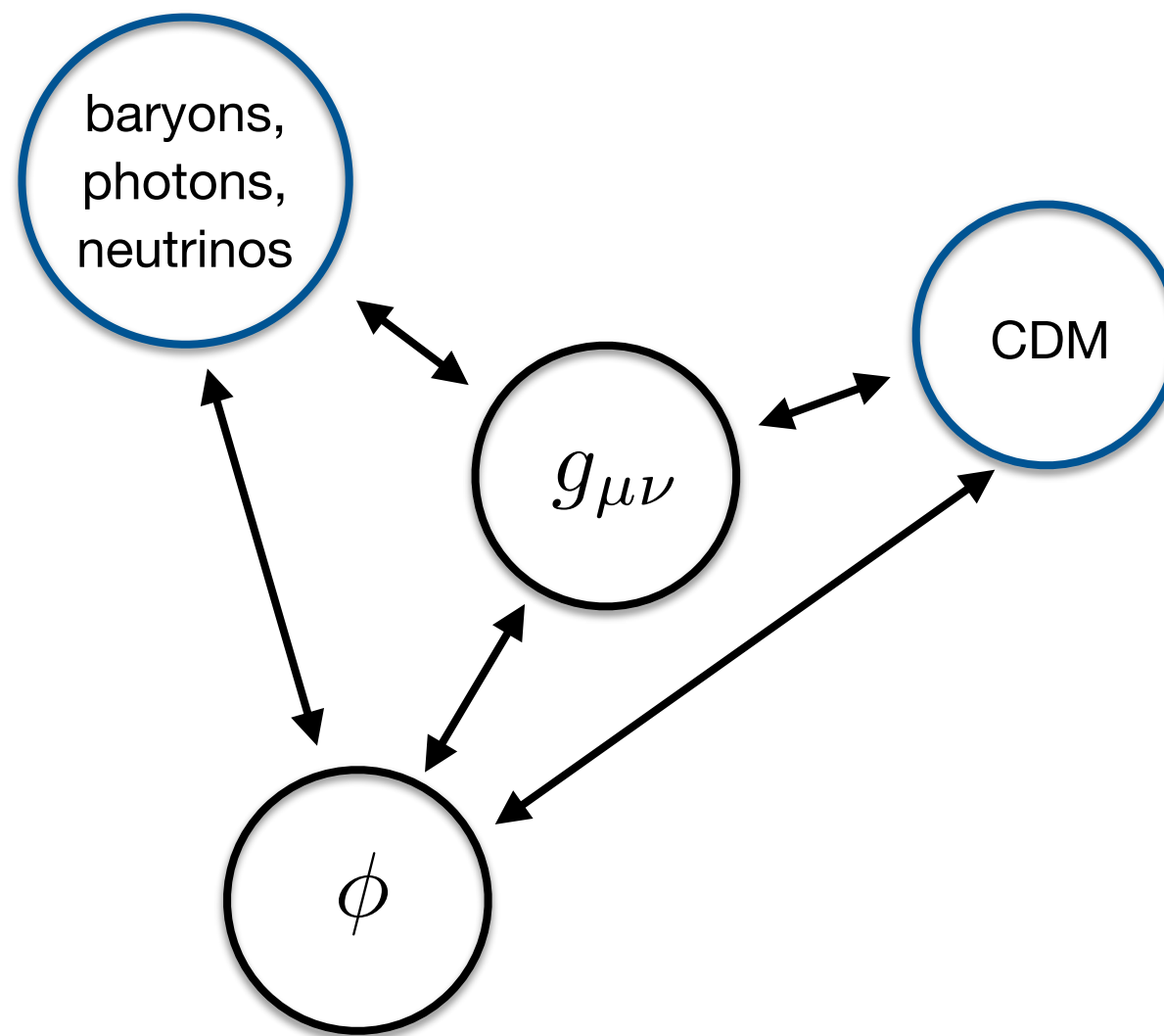
**Gradient instabilities can be cured by
higher-order operators for $c_s^2 \approx 0$**

Arkani-Hamed et al. 03; Creminelli et al 06

(see also Melville's talk)

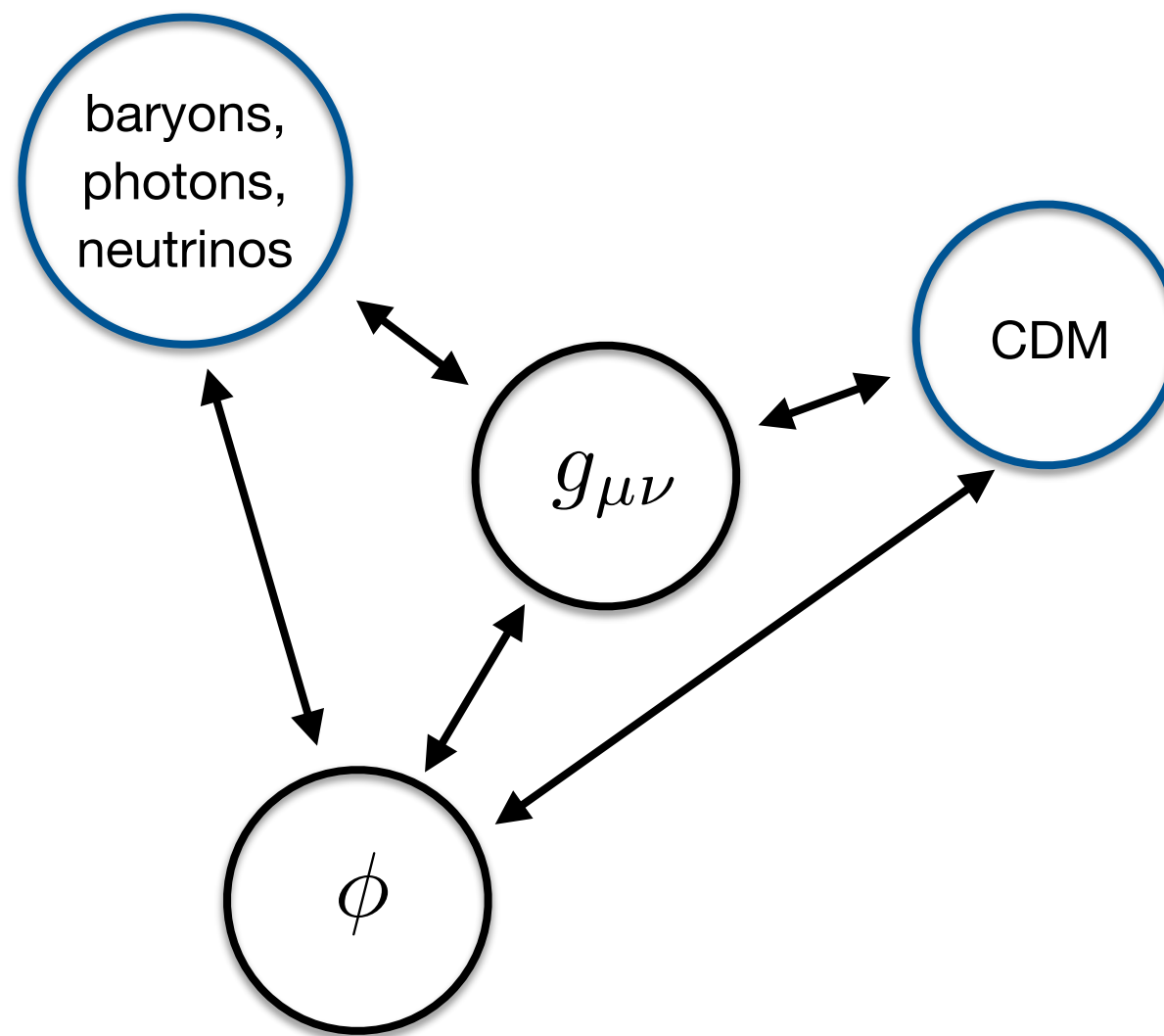


$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, X)\partial_\mu\phi\partial_\nu\phi - V(\phi) + \beta(\phi)T^\mu_\mu \quad \text{Coupling to matter}$$



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Acceleration can be explained by non-minimal coupling: **self-acceleration**



Fifth force

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, X)\partial_\mu\phi\partial_\nu\phi - V(\phi) + \beta(\phi)T^\mu_\mu$$

Expand solution and specialise to point source and quasi-static approximation

$$Z(\bar{\phi}) (\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi) + m^2(\bar{\phi})\varphi = \beta'(\bar{\phi})M\delta^{(3)}(\vec{x})$$

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$$U_5(r) = -\frac{\beta'^2(\bar{\phi})}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})c_s(\bar{\phi})}}r}}{4\pi r} M$$



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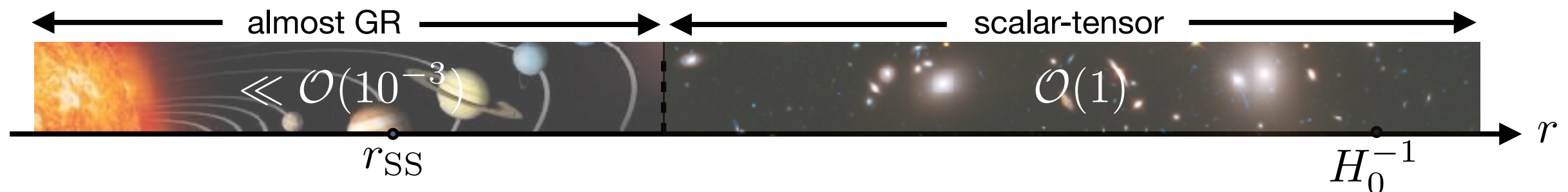
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Chameleon: scalar acquires a large mass in high density region, due to coupling to matter

[Khoury and Weltman 03](#)



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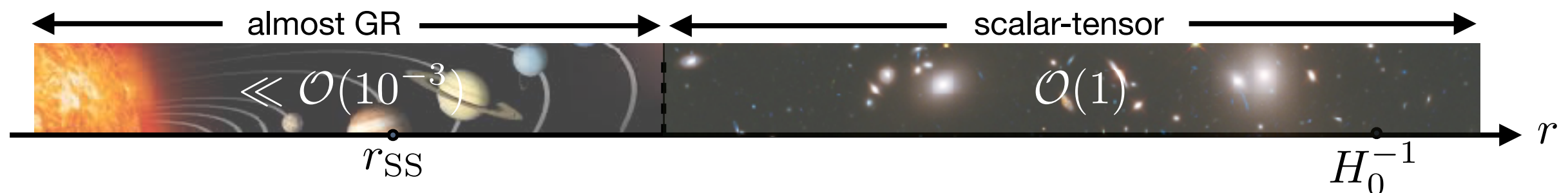
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Chameleon: scalar acquires a large mass in high density region, due to coupling to matter

Symmetron: coupling vanishes in high-density region, where symmetry is restored

Hinterbichler and Khoury 10



Fifth force

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}(\phi, X, \square\phi)\partial_\mu\phi\partial_\nu\phi - V(\phi) + \beta(\phi)T^\mu_\mu$$

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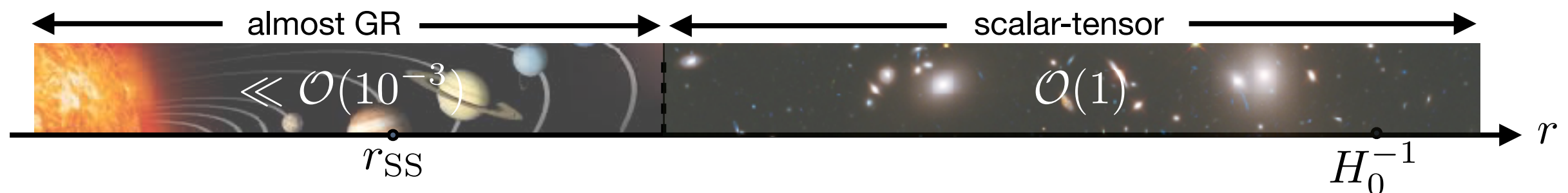
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Chameleon: scalar acquires a large mass in high density region, due to coupling to matter

Symmetron: coupling vanishes in high-density region, where symmetry is restored

Vainshtein: higher-derivative self-interactions suppress the scalar at short scales

Vainshtein 72



Vainshtein screening

Review by Babichev and Deffayet 13

- Originally introduced in Massive Gravity, rediscovered in DGP

Ex:
$$\mathcal{L} = -(\partial\phi)^2 + \frac{(\partial\phi)^2 \square\phi}{\Lambda^3} + \beta(\phi)T^\mu_\mu$$

Vainshtein screening

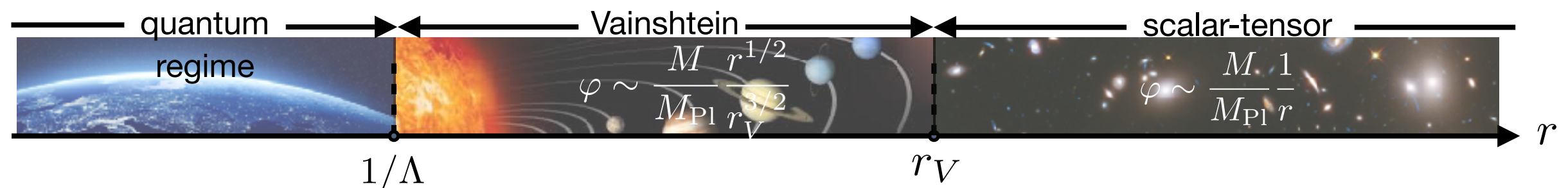
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Classical nonlinearities $\frac{\square\phi}{\Lambda^3} \sim 1 \Rightarrow r_V \sim \left(\frac{M}{M_{\text{Pl}}\Lambda^3} \right)^{\frac{1}{3}} \quad \varphi \sim \frac{M}{M_{\text{Pl}}} \frac{1}{r}$

Quantum corrections $\frac{\partial}{\Lambda} \sim 1 \Rightarrow \frac{1}{r\Lambda} \sim 1 \quad \Lambda \sim (M_{\text{Pl}}H_0^2)^{\frac{1}{3}} \sim \frac{1}{10^7 \text{ cm}}$



Horndeski theories

Most general Lorentz-invariant scalar-tensor theories with 2nd-order EOM. **No extra modes:**
1 scalar + 2 tensor polarisations

Horndeski 73, Deffayet et al. 11
(see Nishi and Ramirez's talk)

$$\mathcal{L}_H^{(2)} = G_2(\phi, X) \quad X = \nabla_\mu \phi \nabla^\mu \phi$$

$$\mathcal{L}_H^{(3)} = G_3(\phi, X) \square \phi$$

$$\mathcal{L}_H^{(4)} = G_4(\phi, X) R - 2G_{4,X}(\phi, X) [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2]$$

$$\mathcal{L}_H^{(5)} = G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{1}{3} G_{5,X}(\phi, X) [(\square \phi)^3 - 3\square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2(\nabla_\mu \nabla_\nu \phi)^3]$$

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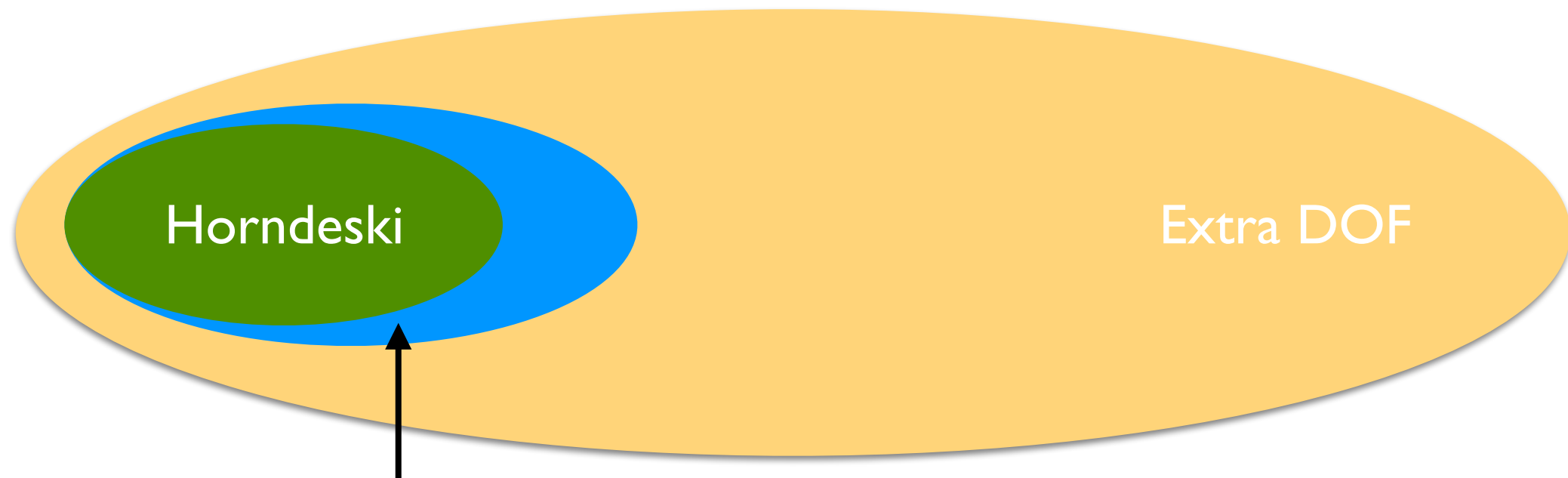
second-order
equations of motion

Beyond Horndeski

Add a new combination $\mathcal{L}_{\text{BH}} = \sum_i \mathcal{L}_{\text{H}}^{(i)}(\phi, X) + \mathcal{L}_{\text{GLPV}}(\phi, X)$

$$\begin{aligned} \mathcal{L}_{\text{GLPV}} = & F_4(\phi, X) \epsilon^{\mu\nu\rho}_{\sigma} \epsilon^{\mu'\nu'\rho'\sigma} \nabla_{\mu} \phi \nabla_{\mu'} \phi (\nabla_{\nu} \nabla_{\nu'} \phi) (\nabla_{\rho} \nabla_{\rho'} \phi) \\ & + F_5(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \nabla_{\mu} \phi \nabla_{\mu'} \phi (\nabla_{\nu} \nabla_{\nu'} \phi) (\nabla_{\rho} \nabla_{\rho'} \phi) (\nabla_{\sigma} \nabla_{\sigma'} \phi) \end{aligned}$$

with $X G_{5,X} F_4 = 3 F_5 (G_4 - 2 X G_{4,X})$



beyond Horndeski

Zumalacarregui, Garcia-Bellido 13

Gleyzes, Langlois, Piazza, FV 14;

Degeneracy

Higher derivatives \Rightarrow extra ghost DOF, only for **non degenerate** theories

Ex 1: **1 variable** mechanical system

$$\mathcal{L} = \frac{1}{2}\ddot{\phi}^2 + \frac{m}{2}\dot{\phi}^2$$

$$Q = \dot{\phi}$$

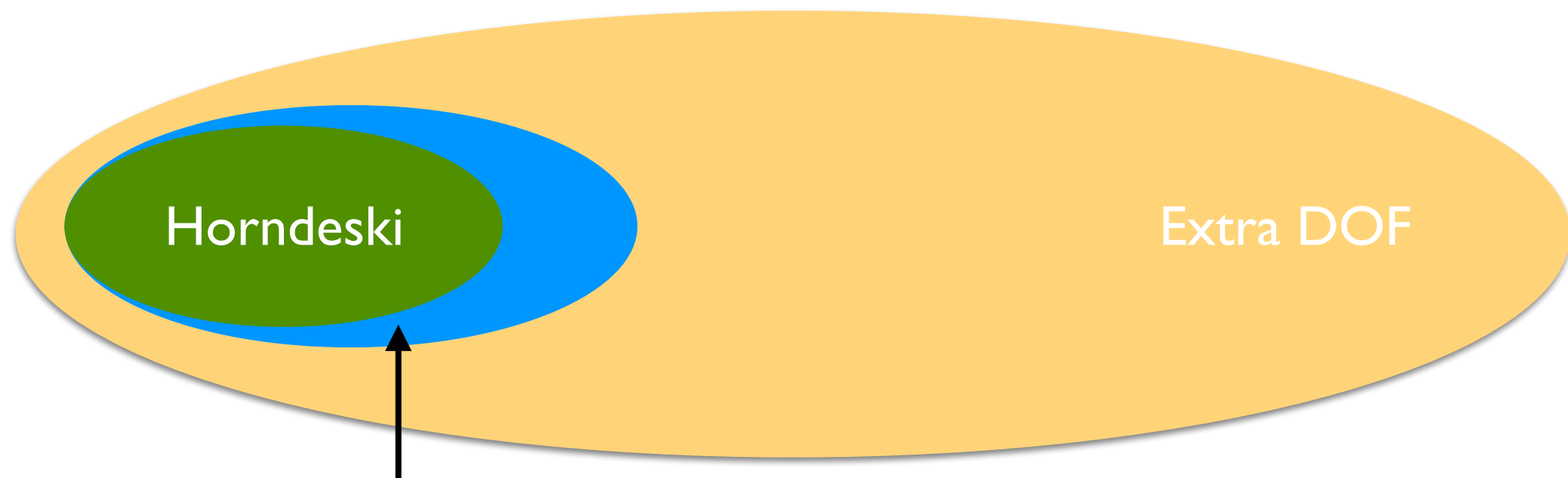
\Rightarrow

$$\ddot{Q} = mQ + \lambda$$

$$\dot{\lambda} = 0$$

$$\dot{\phi} = Q$$

2 DOF!



beyond Horndeski

Zumalacarregui, Garcia-Bellido 13

Gleyzes, Langlois, Piazza, FV 14;

Degeneracy

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Ex 2: **2 variables** mechanical system

$$\mathcal{L} = \frac{1}{2}\ddot{\phi}^2 + \frac{m}{2}\dot{\phi}^2 + \frac{k}{2}\dot{\chi}^2 + b\ddot{\phi}\dot{\chi}$$

$$Q = \dot{\phi}$$

$$\Rightarrow \begin{pmatrix} \ddot{Q} \\ \ddot{\chi} \end{pmatrix} \begin{pmatrix} 1 & b \\ b & k \end{pmatrix} = \begin{pmatrix} mQ + \lambda \\ 0 \end{pmatrix}$$

$$\dot{\lambda} = 0$$

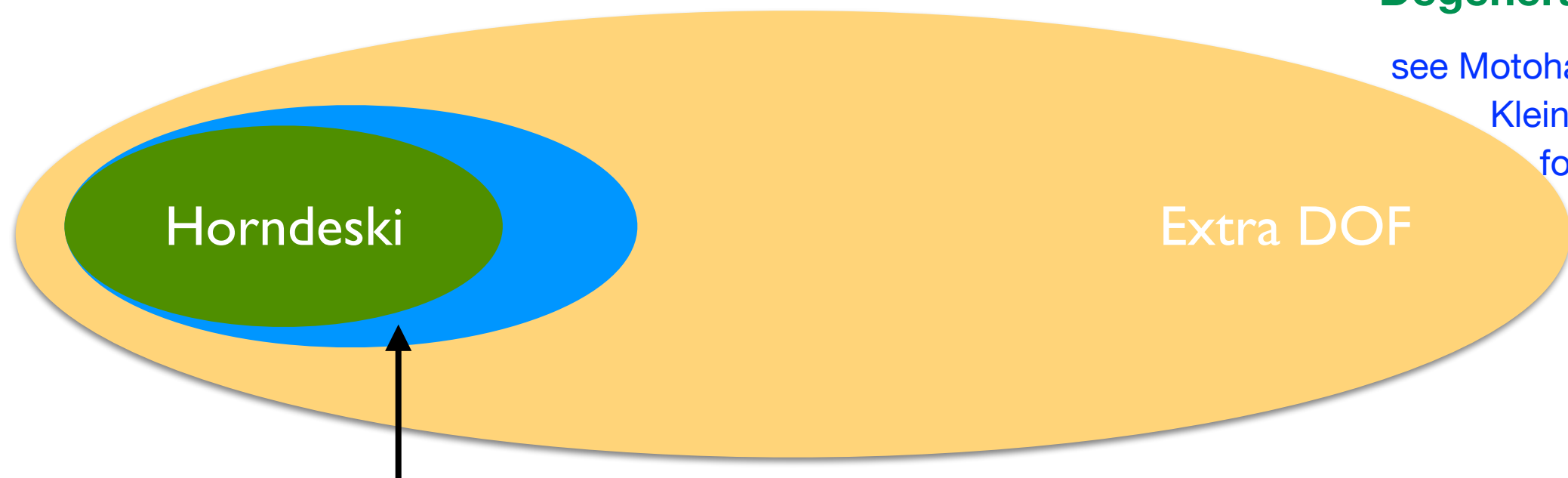
$$\dot{\phi} = Q$$

$$k \neq b^2 \quad \text{3 DOF}$$

$$k = b^2 \quad \text{2 DOF!}$$

Degenerate!

see Motohashi et al 16
Klein & Roest 16
for multifields



beyond Horndeski

Zumalacarregui, Garcia-Bellido 13
Gleyzes, Langlois, Piazza, FV 14;

$$\phi(t) \leftrightarrow \phi(x^\rho)$$

$$\chi(t) \leftrightarrow g_{\mu\nu}(x^\rho)$$

Degeneracy

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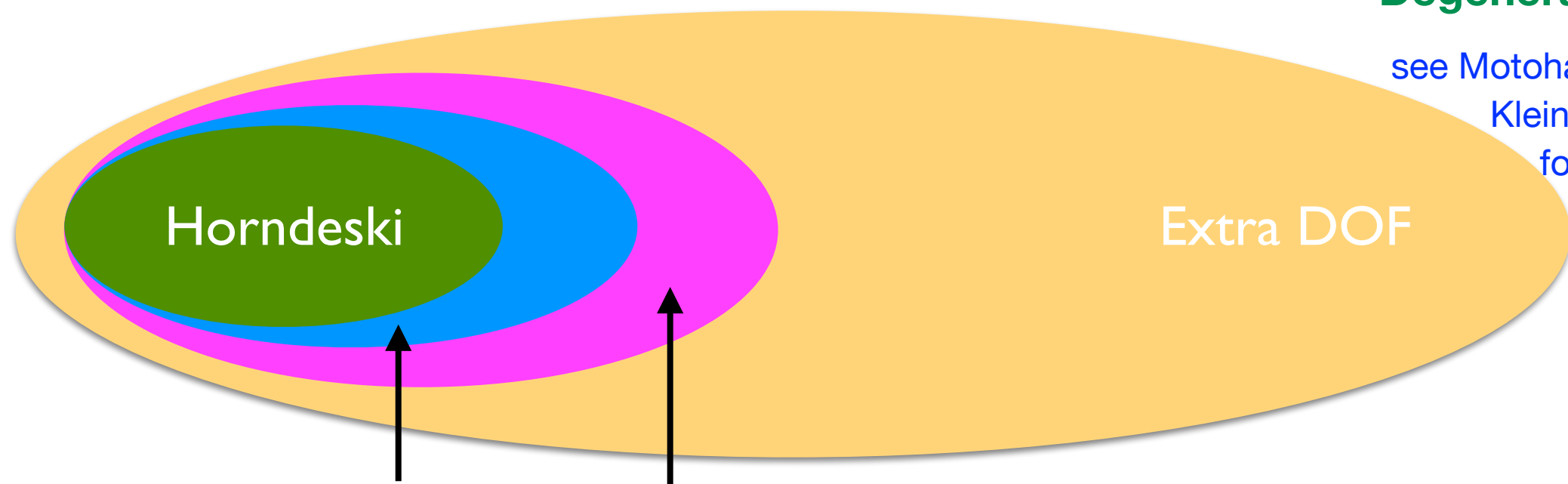
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beyond Horndeski

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Gleyzes, Langlois, Piazza, FV 14;

More degenerate theories:

DHOST/EST

Langlois, Noui 15, 16;
Crisostomi, Koyama, Tasinato 16;

DHOST/EST theories

Degenerate **H**igher-**O**rder **S**calar-**T**ensor theories or **E**xtended **S**calar **T**ensor theories

$$\mathcal{L}_{\text{DHOST}}^{(2)} = f_2(\phi, X)R + \sum_i^5 C_i^{(2)\mu\nu\rho\sigma}(\phi, X) \nabla_\mu \nabla_\nu \phi \nabla_\rho \nabla_\sigma \phi$$

Langlois, Noui 15;
Crisostomi et al. 16;
de Rham, Matas 16

(see Crisostomi's talk; see also Saito's talk)

DHOST/EST theories

Degenerate **H**igher-**O**rder **S**calar-**T**ensor theories or **E**xtended **S**calar **T**ensor theories

$$\mathcal{L}_{\text{DHOST}}^{(2)} = f_2(\phi, X)R + \sum_i^5 C_i^{(2)\mu\nu\rho\sigma}(\phi, X) \nabla_\mu \nabla_\nu \phi \nabla_\rho \nabla_\sigma \phi$$

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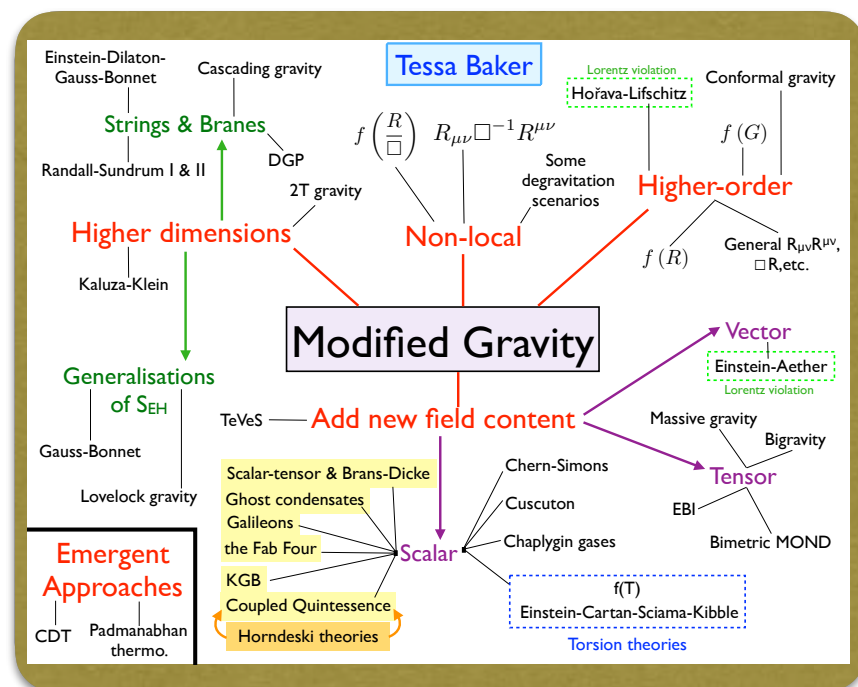
- In general, **3 degeneracy** conditions (**7 classes**) associated to second-class constraints
- Structure preserved by general **disformal** transformations of the metric:

$$g_{\mu\nu} \rightarrow C(\phi, X)g_{\mu\nu} + D(\phi, X)\partial_\mu\phi\partial_\nu\phi$$

(see also Takahashi's talk)

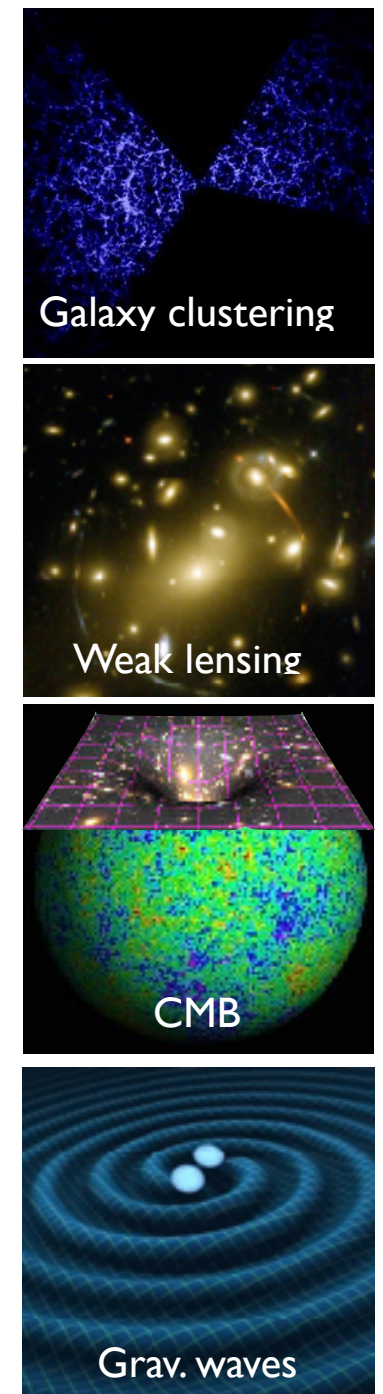
- **Quadratic + cubic** theories: **9 subclasses**, **25 combinations** of quadratic and cubic theories

Models

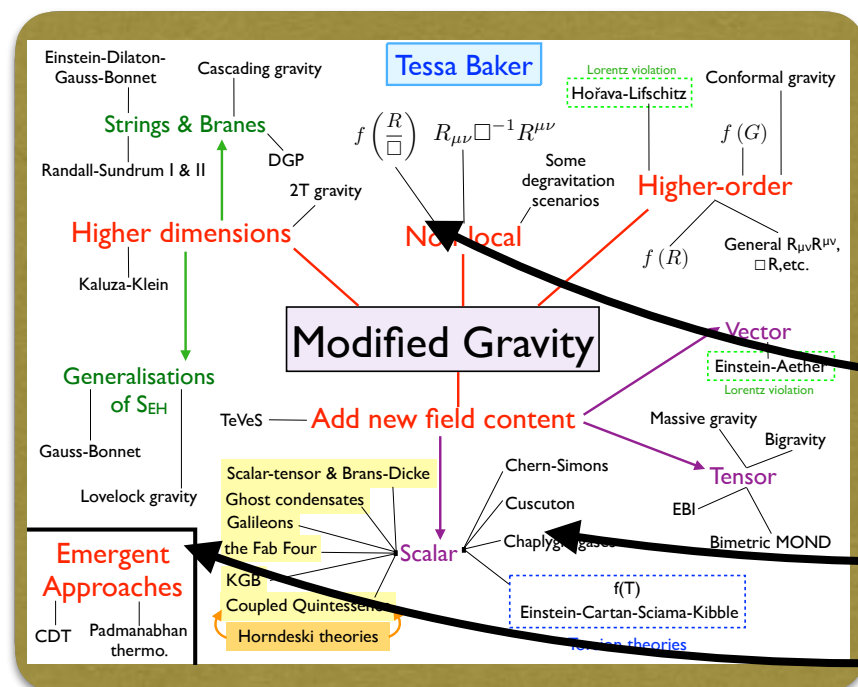


Many models of modified gravity, each with its own theoretical motivation and phenomenology

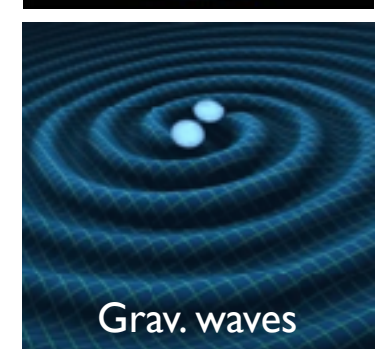
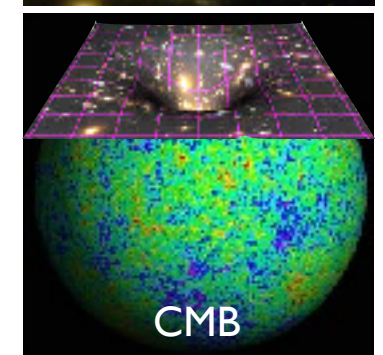
Observations



Models

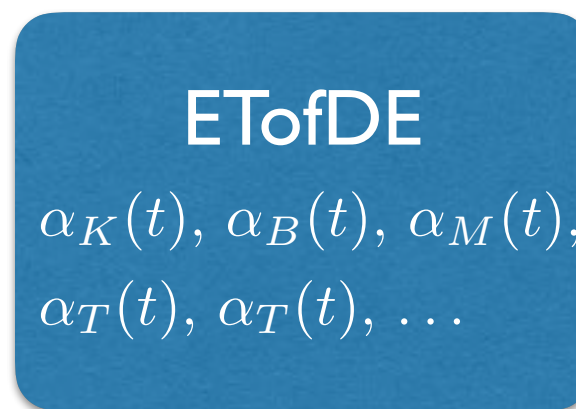
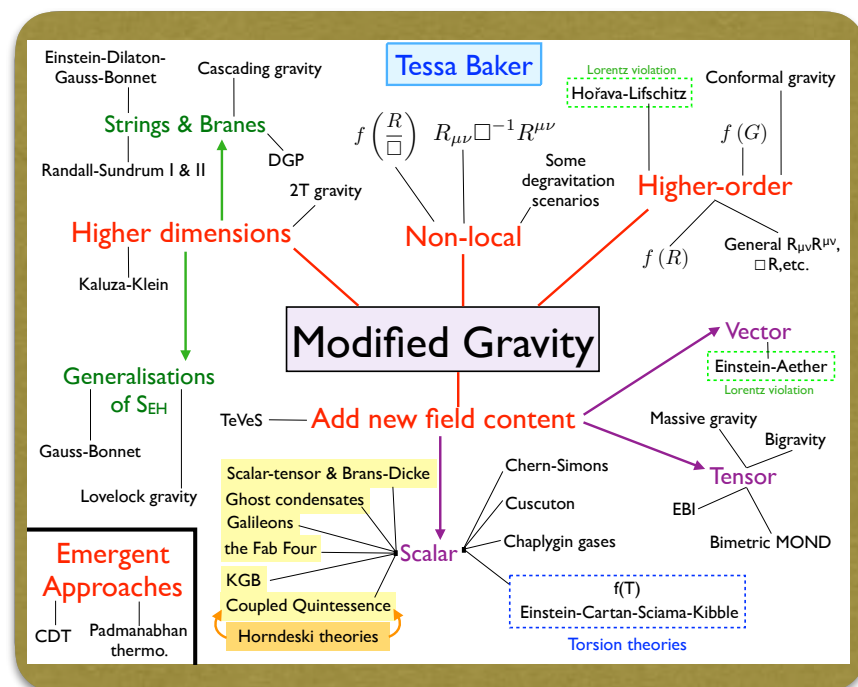


Observations



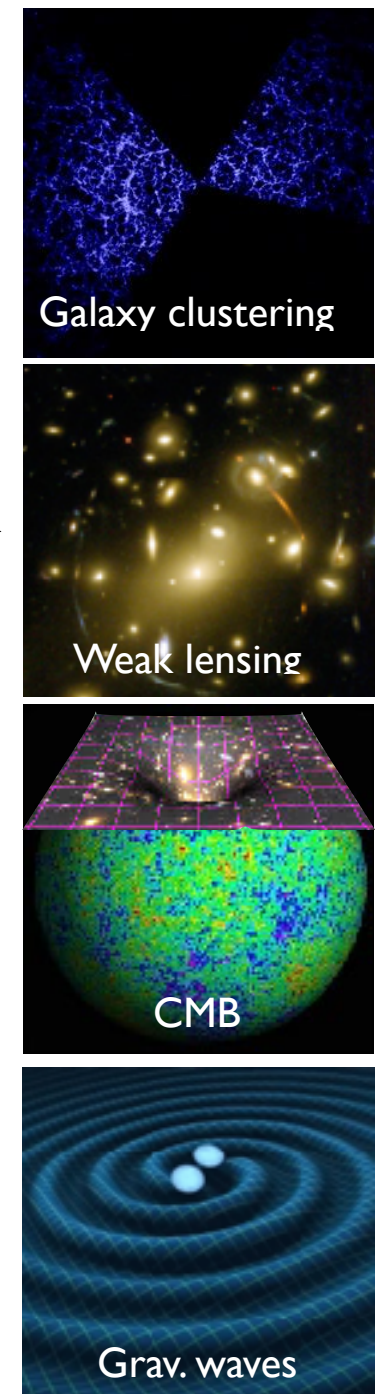
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Models

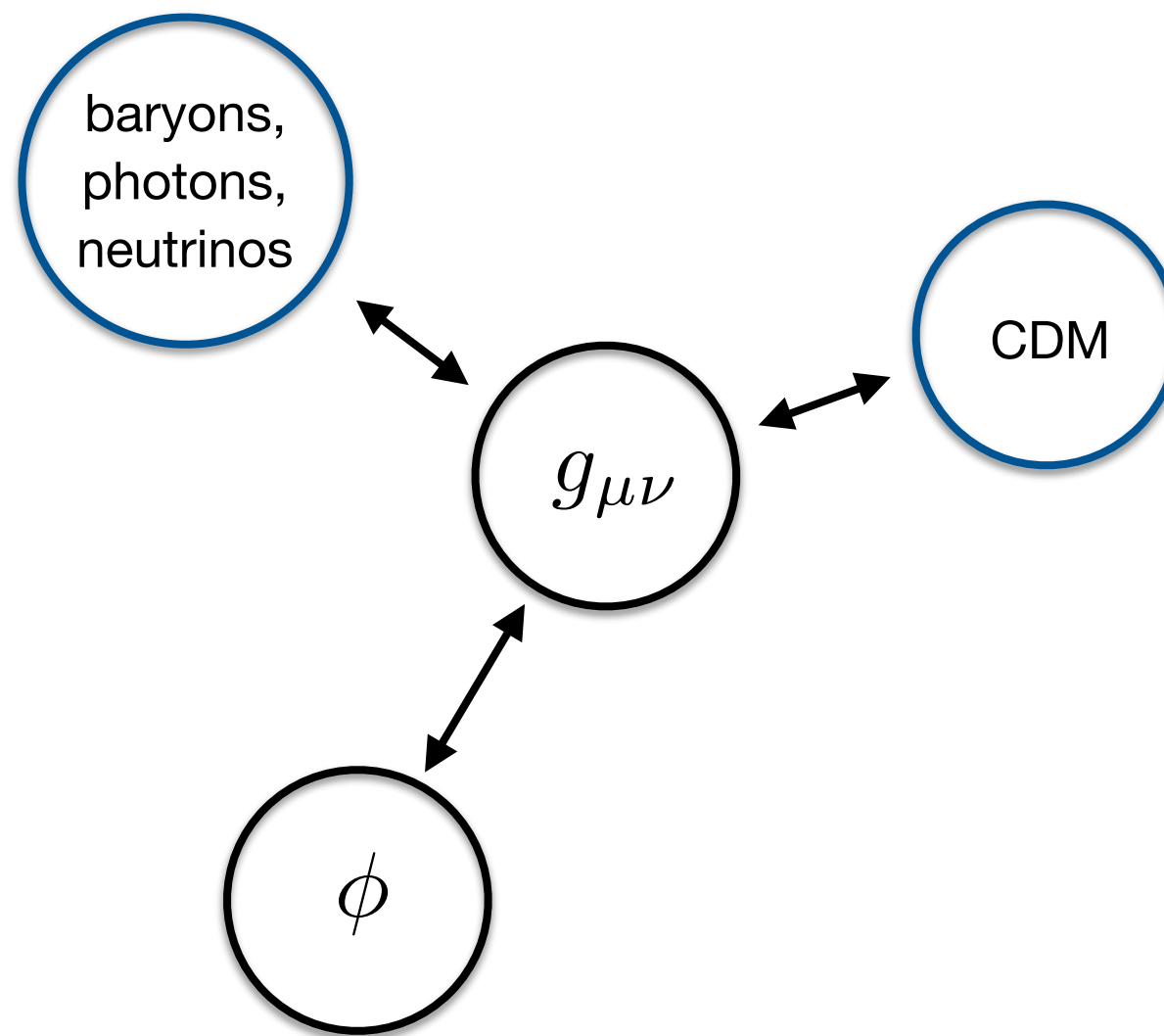


Bridge models and observations in a minimal and systematic way

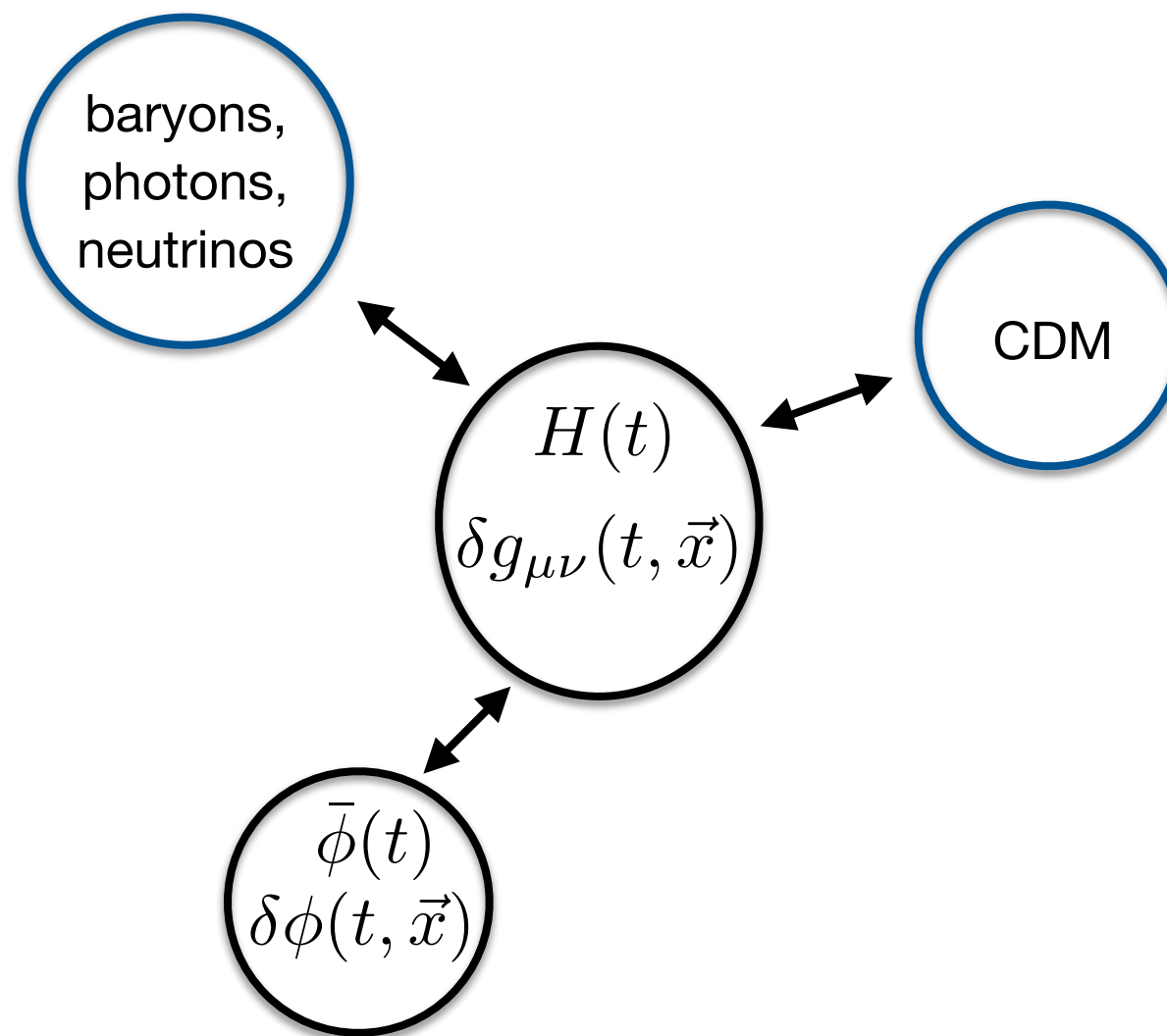
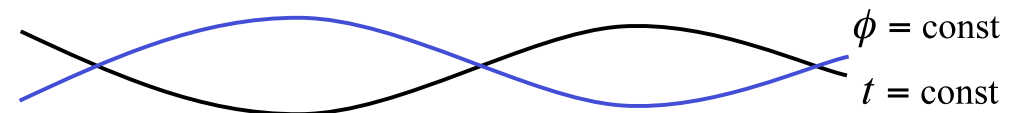
Observations



Jordan frame

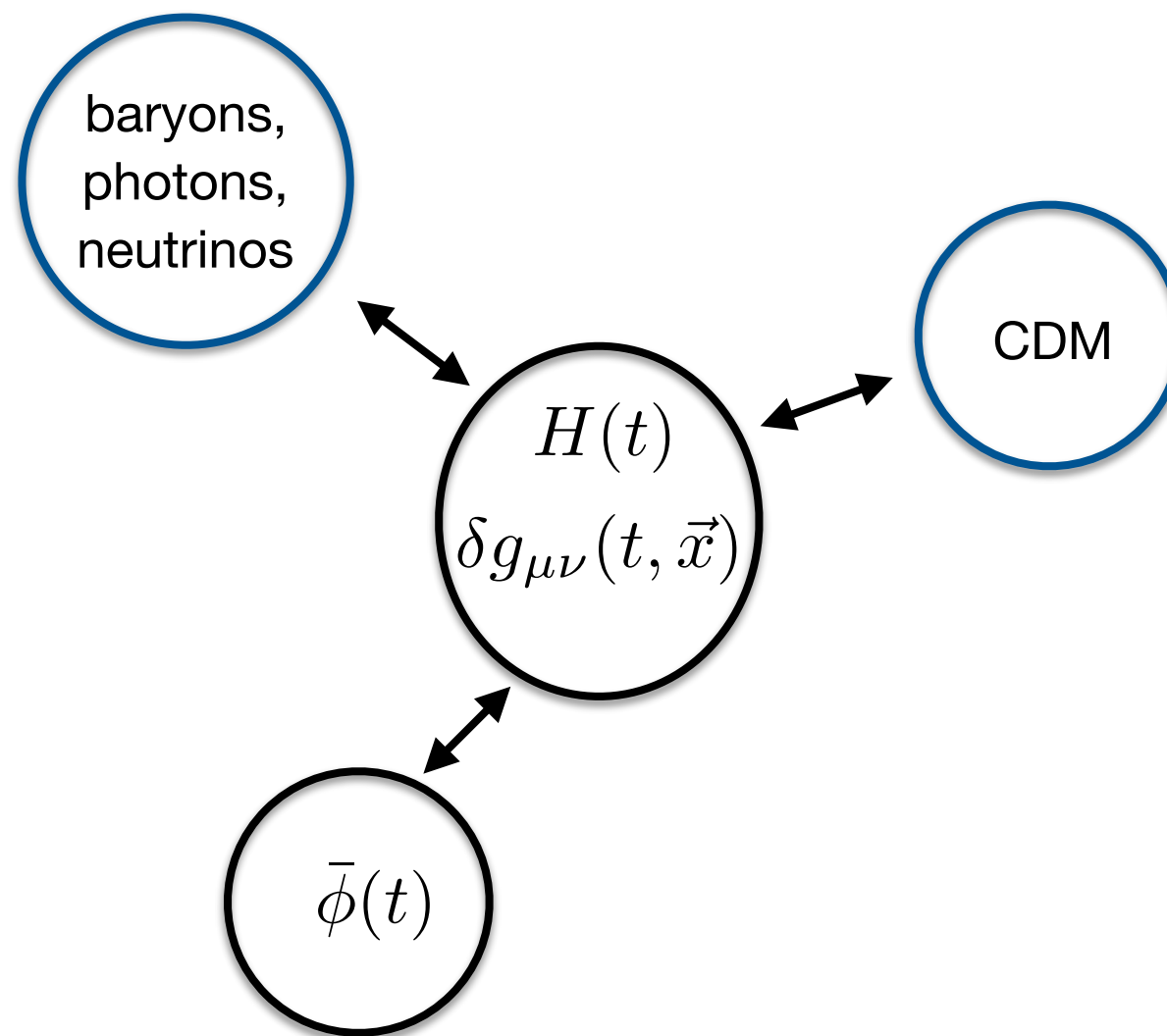
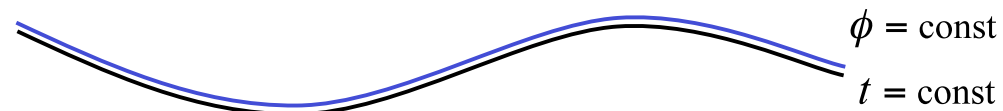


FLRW background



Time reparametrisation invariance broken, $\dot{\phi}(t) \neq 0$

Uniform field slicing $\delta\phi(t, \vec{x}) = 0$



Spatial reparametrisation invariance preserved on these hypersurfaces

Action: most general function of the metric perts, **preserving spatial-diff** invariance

EFT Lagrangian

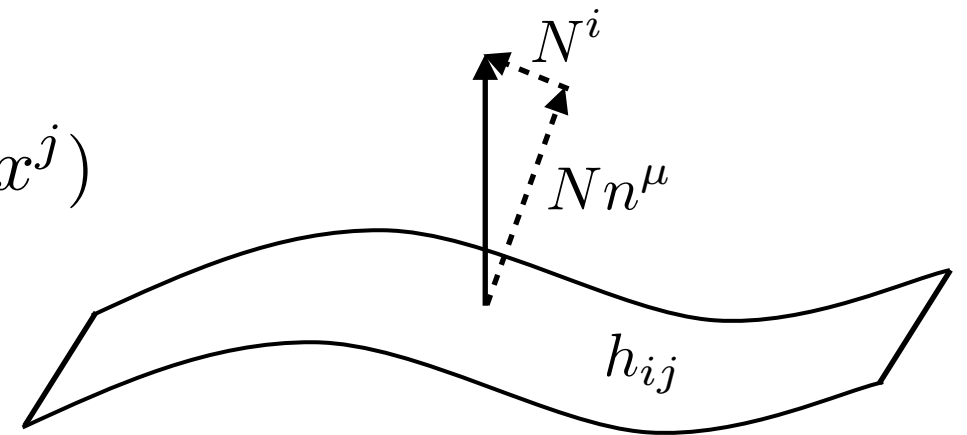
Gubitosi, Piazza, FV 12
Gleyzes, Langlois, Piazza, FV 13
Bloomfield et al. 12, 13

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

ADM decomposition

$$ds^2 = -N^2 dt^2 + h_{ij} (N^i dt + dx^i) (N^j dt + dx^j)$$

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$



EFT Lagrangian

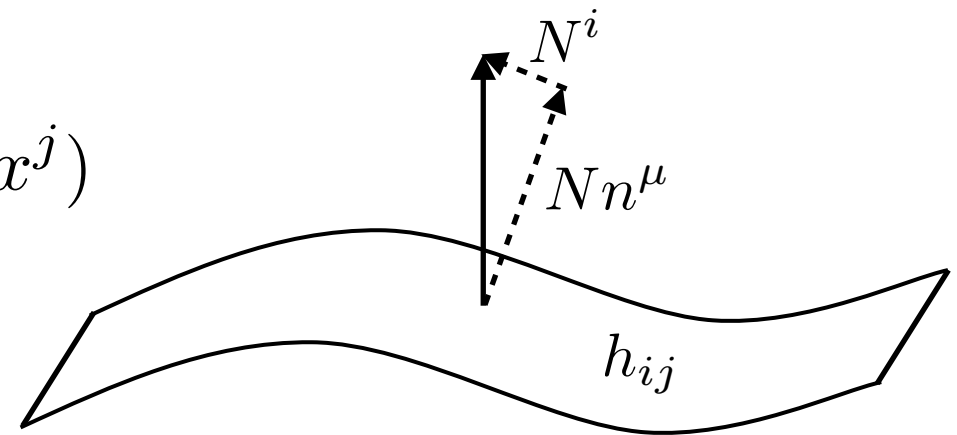
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- New operators describe **deviations from GR** (Λ CDM). Ordered in **number of perturbations** and **derivatives**
- **Time-dependent couplings** (functions $\alpha_i(t)$), due to expansion around FLRW background
- Functions $\alpha_i(t)$ **independent** of background evolution $H(t) = \dot{a}/a$

We fit to data $H(t)$ and $\alpha_i(t)$ (agnostic of their time dependence and parametrization)

EFT Lagrangian

Gubitosi, Piazza, FV 12
Gleyzes, Langlois, Piazza, FV 13
Bloomfield et al. 12, 13

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Notation of Bellini, Sawicki '14 for the alphas

α_i	α_K	α_B	α_M	α_T	α_H
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	${}^{(3)}R$	$\delta N {}^{(3)}R$
quintessence, k-essence	✓				
Cubic Galileon	✓	✓			
Brans-Dicke, f(R)	✓	✓	✓		
Horndeski	✓	✓	✓	✓	
Beyond Horndeski	✓	✓	✓	✓	✓

5 functions of time instead of 5
functions of $\phi, (\partial\phi)^2$; minimal
number of parameters

$$N \sim \dot{\phi}, \quad K_{ij} \sim \dot{h}_{ij}, \quad {}^{(3)}R \sim \partial^2 h$$

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Beyond Horndeski	✓	✓	✓	✓	✓

5 functions of time instead of 5 functions of $\phi, (\partial\phi)^2$; minimal number of parameters

Stability conditions easy to implement

	Scalar	Tensor
No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient inst.	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

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 Bloomfield et al. 12, 13

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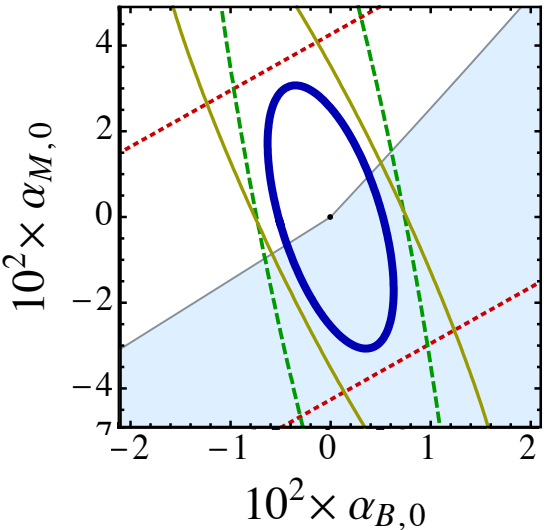
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✓

Gleyzes, Langlois, Mancarella FV '15



- Galaxy Clustering
- Weak Lensing
- ISW–Galaxy
- GC+ISW–Gal+WL

Euclid-like specifications

EFT Lagrangian

Gubitosi, Piazza, FV 12
 Gleyzes, Langlois, Piazza, FV 13
 Bloomfield et al. 12, 13

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No ghosts	$\alpha_K + 6\alpha_B^2 > 0$	$M^2 > 0$
No gradient inst.	$c_s^2(\alpha_i) \geq 0$	$\alpha_T \geq -1$

Can be applied to **non-singular cosmologies**
 (see Akama and Kobayashi's talk)

Higher-Order theories

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

All quadratic operators **up to two** derivatives

Langlois, Mancarella, Noui, FV 17

α_i	α_K	α_B	α_M	α_T	α_H	α_L	β_1	β_2	β_3
$\mathcal{O}_i^{(2)}$	δN^2	$\delta N \delta K$	$\frac{dM^2}{d \ln a}$	${}^{(3)}R$	$\delta N {}^{(3)}R$	δK^2	$\delta \dot{N}^2$	$\delta \dot{N} \delta K$	$(\partial_i \delta N)^2$

• Generic scalar dispersion relation: $\mathcal{E}_1 \omega^4 + \mathcal{E}_2 \omega^2 k^2 + \mathcal{E}_3 \omega^2 + \mathcal{E}_4 k^4 + \mathcal{E}_5 k^2 = 0$

• **Two types** of degeneracy conditions lead to $\omega^2 - c_s^2 k^2 = 0$

$$\mathcal{C}_I : \quad \alpha_L = 0, \quad \beta_2 = f_2(\beta_1), \quad \beta_3 = f_3(\beta_1)$$

$$\mathcal{C}_{II} : \quad \beta_1 = f_1(\alpha_T, \alpha_H, \alpha_L), \quad \beta_2 = f_2(\alpha_T, \alpha_H, \alpha_L), \quad \beta_3 = f_3(\alpha_T, \alpha_H, \alpha_L)$$

$$c_s^2 \propto -c_T^2 \quad \text{ruled out!}$$

Higher-Order theories

$$S^{(2)} = \int d^4x \sqrt{h} \frac{M^2}{2} \left[\delta K_i^j \delta K_j^i - \delta K^2 + {}^{(3)}R + \delta N {}^{(3)}R + \sum_i \alpha_i(t) \mathcal{O}_i^{(2)}(\delta N, \delta K, \dots) \right]$$

All quadratic operators **up to two** derivatives

Langlois, Mancarella, Noui, FV 17

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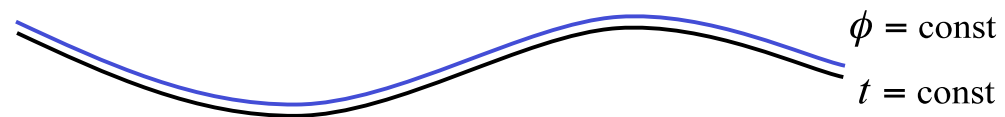
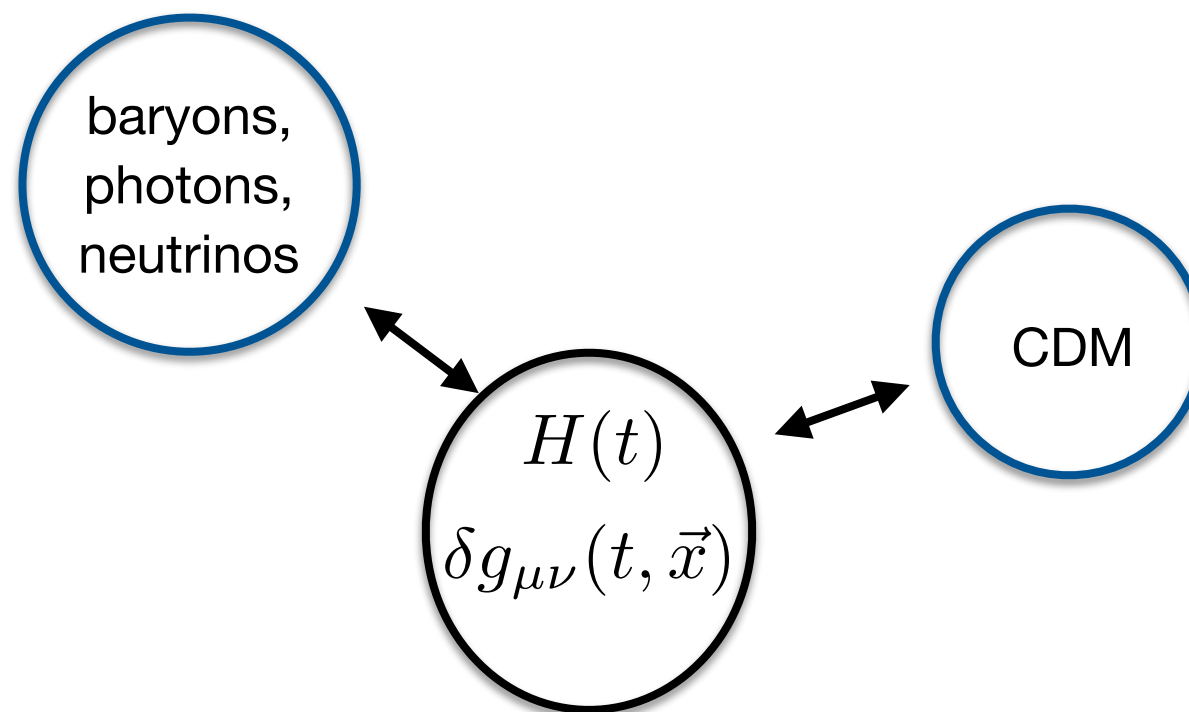
$$\mathcal{C}_I : \quad \alpha_L = 0, \quad \beta_2 = f_2(\beta_1), \quad \beta_3 = f_3(\beta_1)$$

- Class \mathcal{C}_I can be brought to **Horndeski frame**: $\alpha_H = 0, \beta_J = 0$



- Changing frame changes matter couplings (Horndeski vs Jordan): **Matter matters!**

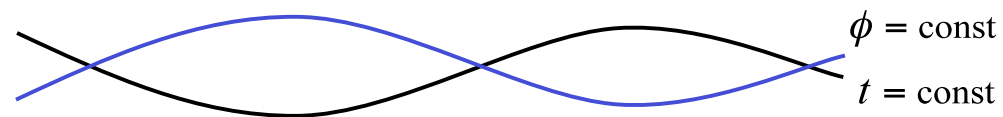
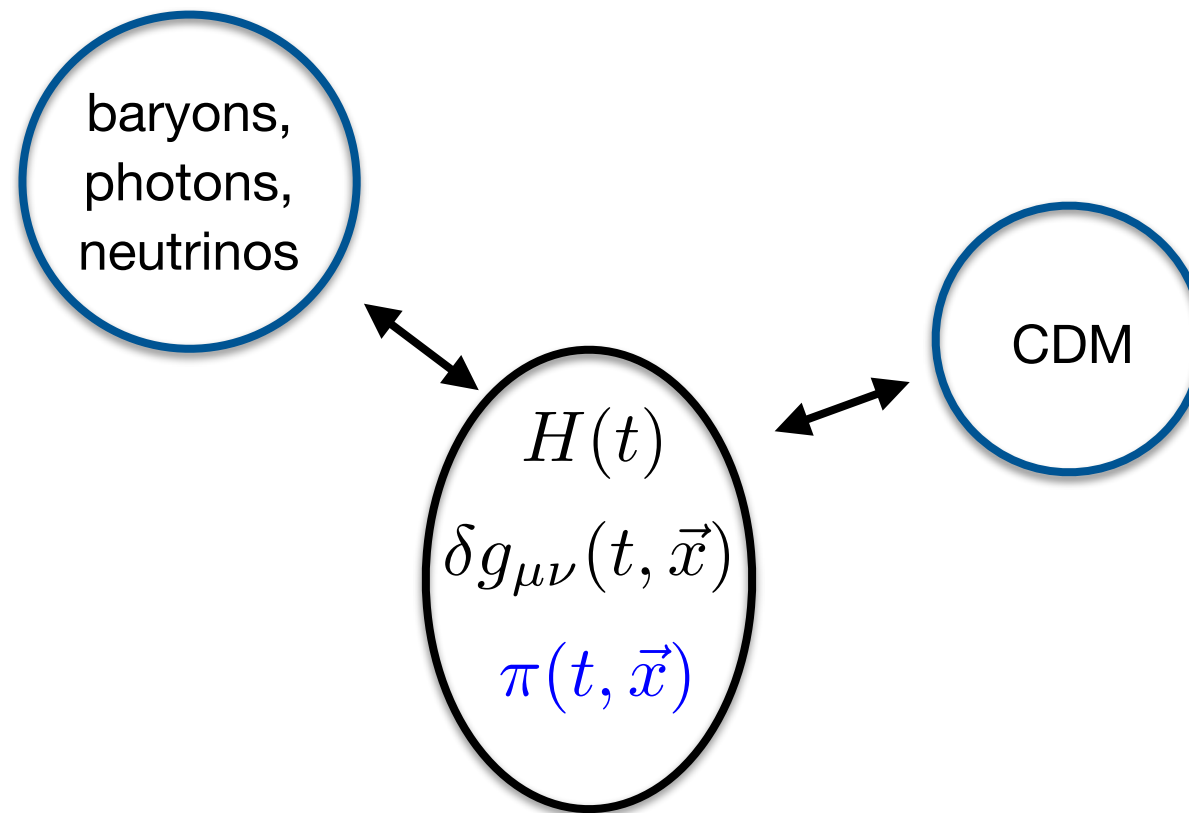
Uniform field slicing $\delta\phi(t, \vec{x}) = 0$



Uniform field slicing $\delta\phi(t, \vec{x}) = 0$

Newtonian gauge $ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$

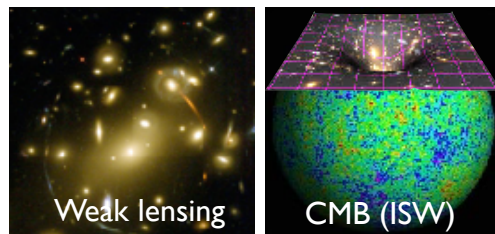
$$t \rightarrow t + \pi(t, \vec{x})$$



Phenomenology

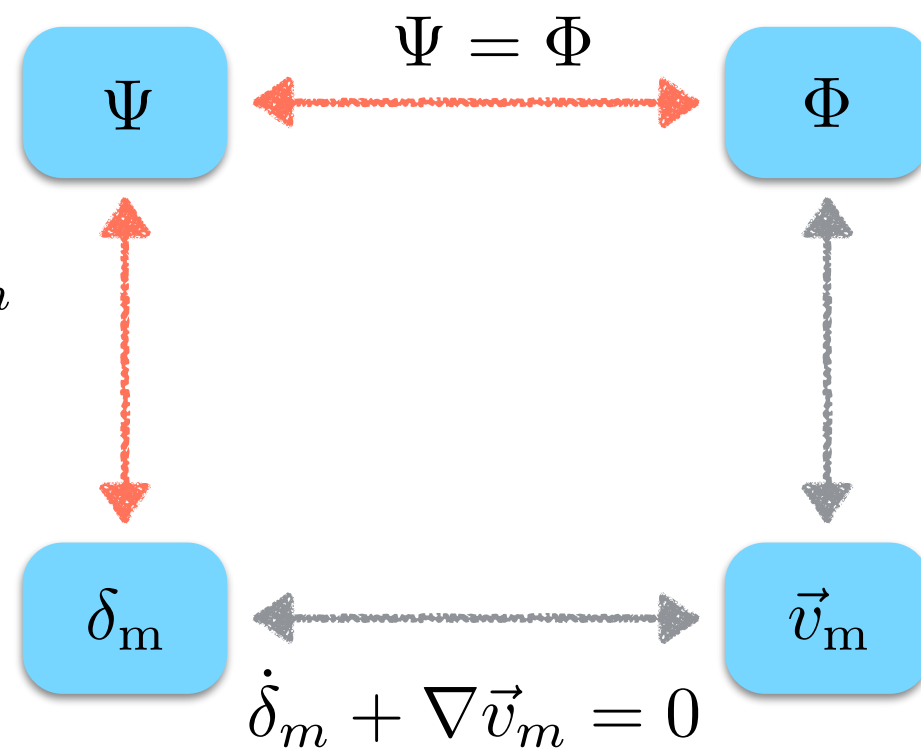
$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

Quasi-static approximations



$$\nabla^2(\Psi + \Phi) = 8\pi G a^2 \rho_m \delta_m$$

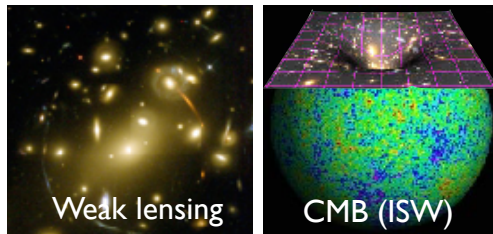
$$\nabla^2 \Psi = 4\pi G a^2 \rho_m \delta_m$$



Phenomenology

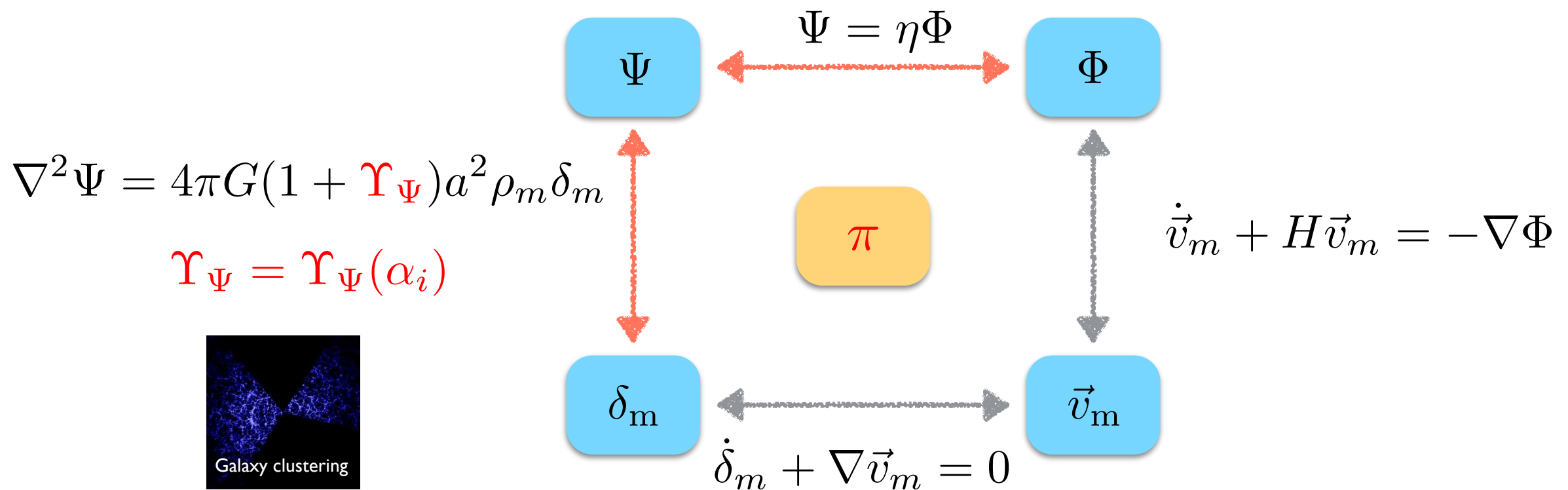
$$dt^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Psi)d\vec{x}^2$$

Quasi-static approximations



$$\nabla^2(\Psi + \Phi) = 8\pi G(1 + \Upsilon_{\text{lens}})a^2\rho_m\delta_m$$

$$\Upsilon_{\text{lens}} = \Upsilon_{\text{lens}}(\alpha_i)$$



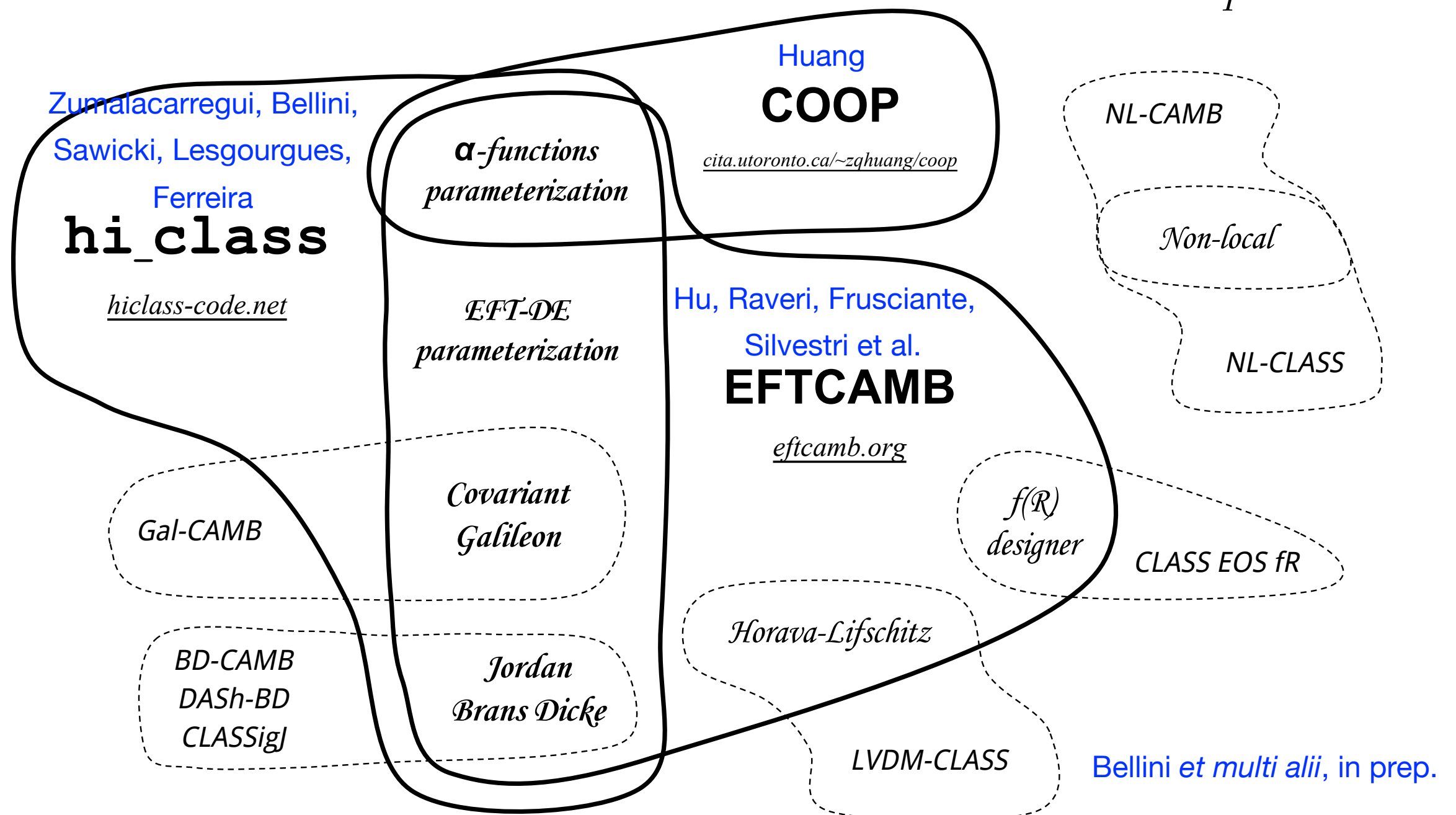
Einstein-Boltzmann solvers

- Full Einstein-Boltzmann solver: $\frac{df_I}{d\eta} = C_I[f_I]$, $I = \gamma, \nu, b, \text{CDM}$
 $\frac{\delta S^{(2)}}{\delta \pi} = 0$ & $G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$

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Einstein-Boltzmann solvers

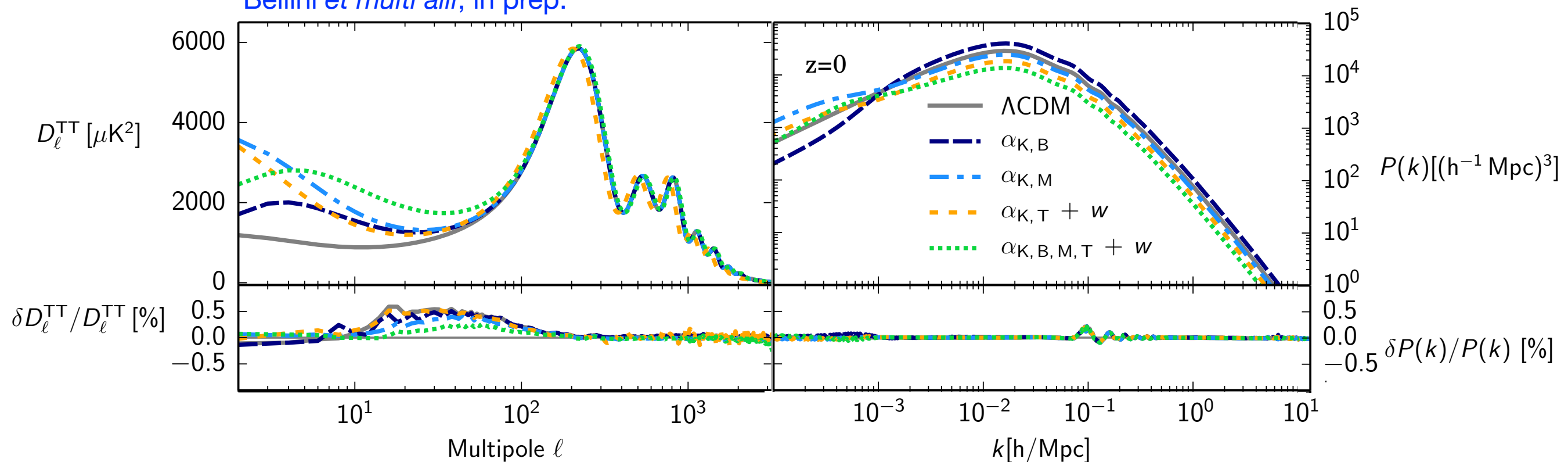
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$$\frac{\delta S^{(2)}}{\delta \pi} = 0 \quad \& \quad G_{ij}^{\text{modified}} = 8\pi G \sum_I T_{ij}^{(I)}$$

Codes **agree** at **sub-percent** level, in most cases

- EFTCAMB (from CMBFAST) (Hu, Raveri, Frusciante, Silvestri et al.)
- hi_class (from CLASS) (Zumalacarregui, Bellini, Sawicki, Lesgourgues, Ferreira et al.)

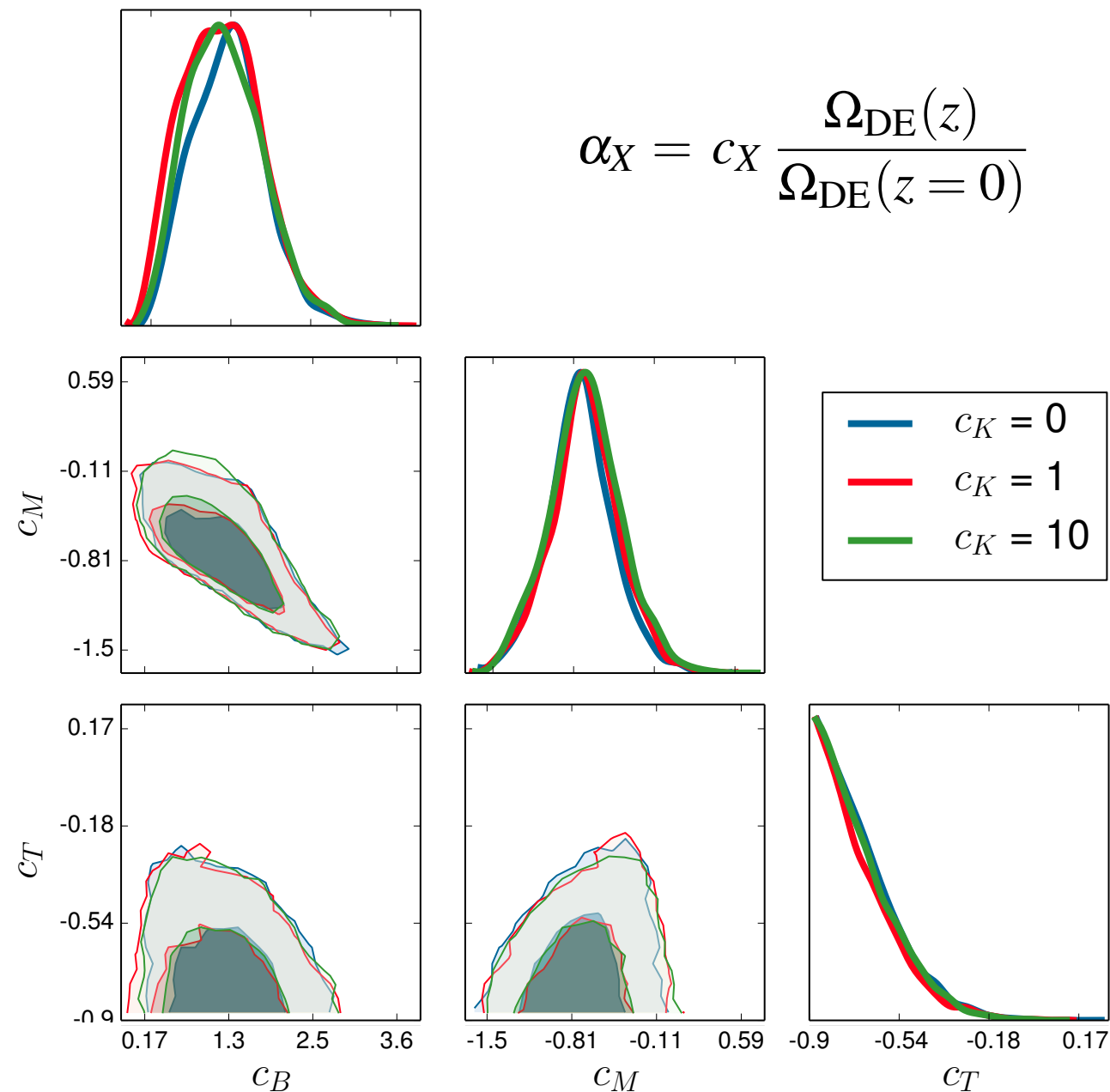
Bellini et multi alii, in prep.



Current Constraints

MCMC analysis using hi_class, from a combination of **CMB** (Planck2015WP), **P(k)** (WiggleZ), **BAO** (6dFGS, SDSS-MGS, BOSS) and **RSD** (6dFGS, MGS, LRG, Vipers, BOSS, WiggleZ)

Bellini, Cuesta, Jimenez, Verde 15



“The improvement in the fit at the expense of adding extra parameters, quantified in terms of difference of log likelihood is not significant”

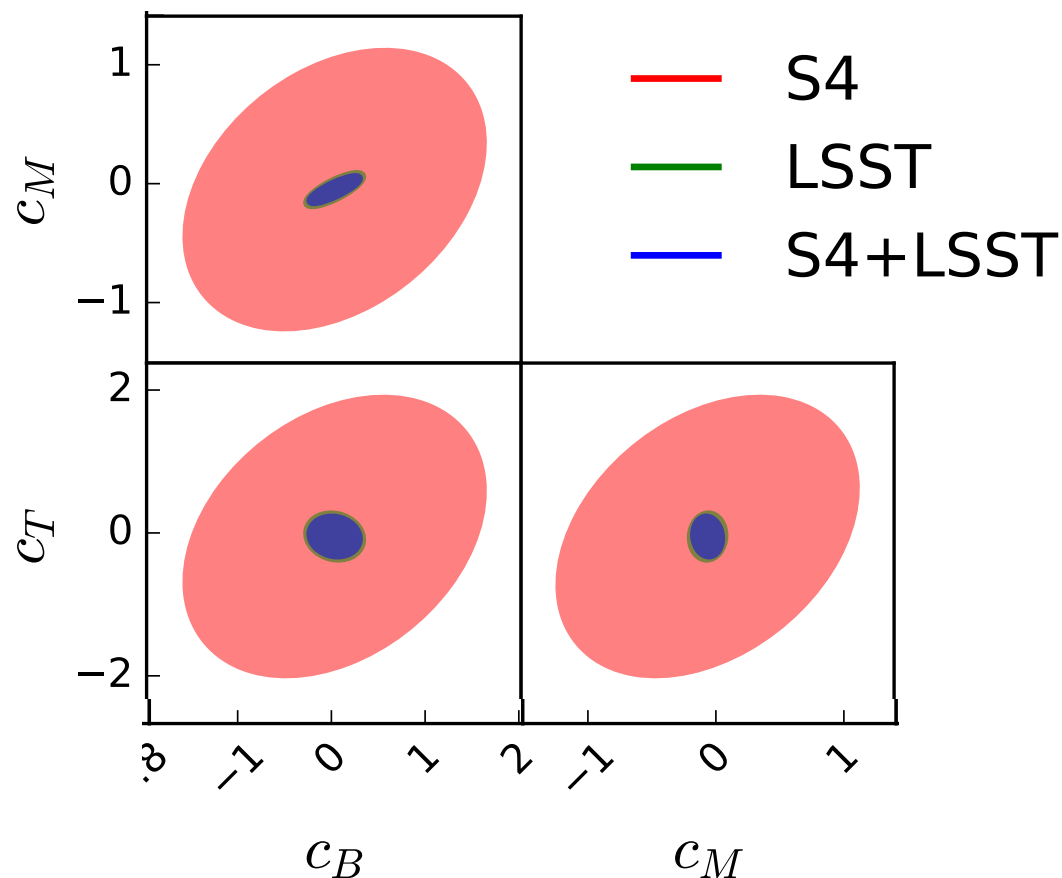
Log ($E_{\text{Hordenski}}/E_{\Lambda\text{CDM}}$) = 0.09
No evidence against ΛCDM

Future constraints

Fisher matrix analysis using hi_class, from a combination of **stage 4 CMB** experiment and **LSST** telescope

Alonso et al. 16

Case	$> \omega_{\text{BD}}, 95\% \text{ C.L.}$	$\sigma(c_B)$	$\sigma(c_M)$	$\sigma(c_T)$	$\sigma(c_K)$	$\sigma(w)$	$\sigma(\sum m_\nu) [\text{meV}]$
S4	2.9×10^3	0.796	0.746	1.26	4.9	0.112	71
LSST	1.2×10^4	0.193	0.089	0.205	8.8	0.016	45
S4+LSST	1.3×10^4	0.169	0.072	0.179	3.5	0.011	22



$$\alpha_X = c_X \frac{\Omega_{\text{DE}}(z)}{\Omega_{\text{DE}}(z=0)}$$

$$\sigma(\alpha_X) \sim \mathcal{O}(0.1)$$

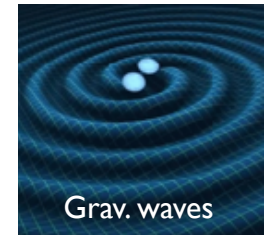
Cassini (Bertotti et al. 03): $\omega_{\text{BD}} > 40\,000$

This work: $\omega_{\text{BD}} > 20\,000$

GW complementarity

Friction term and **speed** of gravitational waves is affected

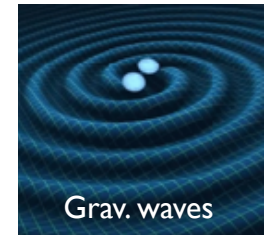
$$\ddot{\gamma}_{ij} + H(3 + \alpha_M)\dot{\gamma}_{ij} - (1 + \alpha_T)\frac{\nabla^2}{a^2}\gamma_{ij} = 0 \quad c_T^2 = 1 + \alpha_T$$



GW complementarity

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Effect simultaneously in **slip** parameter Saltas et al. 14, 16

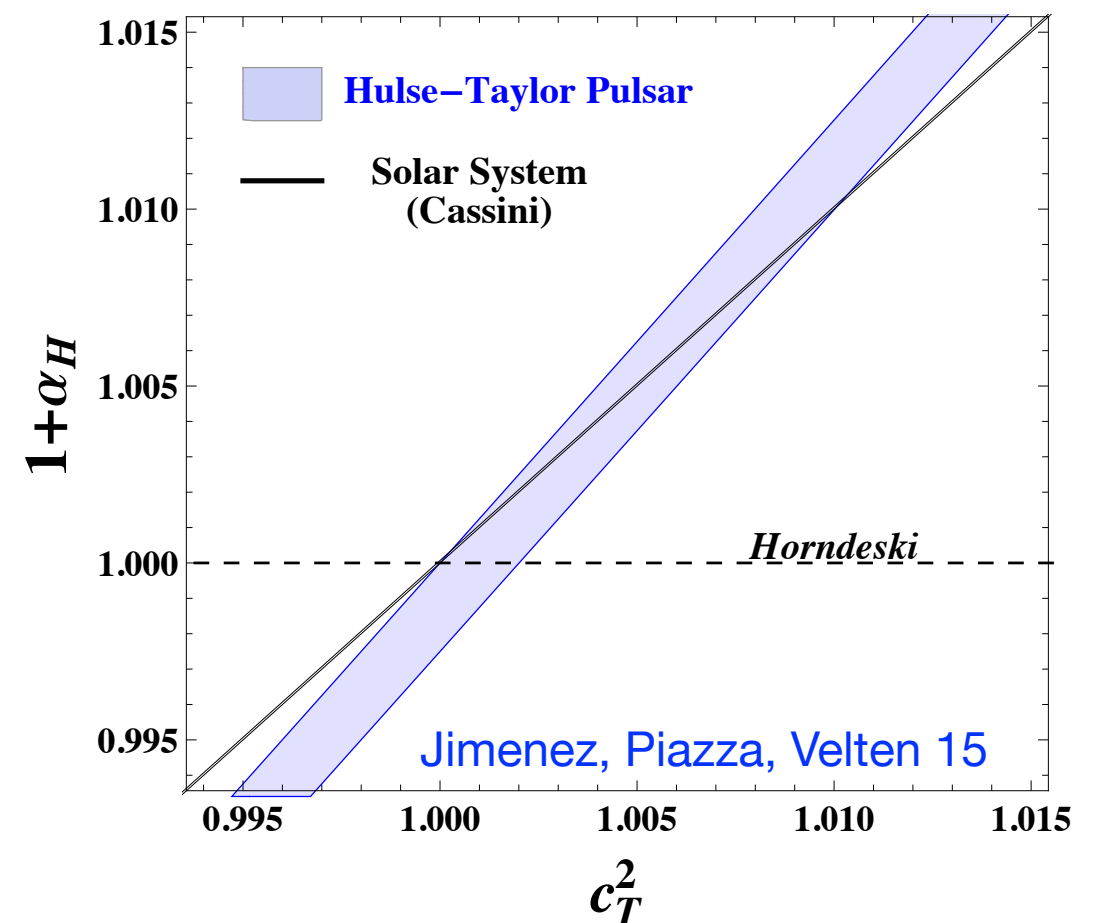
$$\Psi = \eta(\alpha_T, \alpha_M, \alpha_H)\Phi$$

Vainshtein screening ineffective for time-dependent cosmological VEV Babichev, Deffayet, Esposito-Farese 11

Jimenez, Piazza, Velten 15

$$\dot{P}_{\text{MG}} = \frac{G_{\text{gw}}}{G_N} \frac{c}{c_T} \dot{P}_{\text{standard}} = \frac{(1 + \alpha_H)^2}{c_T^3} \dot{P}_{\text{standard}}$$

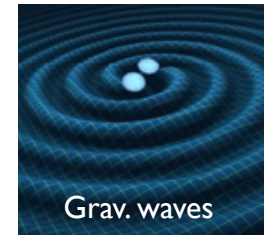
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GW complementarity

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Effect simultaneously in **slip** parameter [Saltas et al. 14, 16](#)

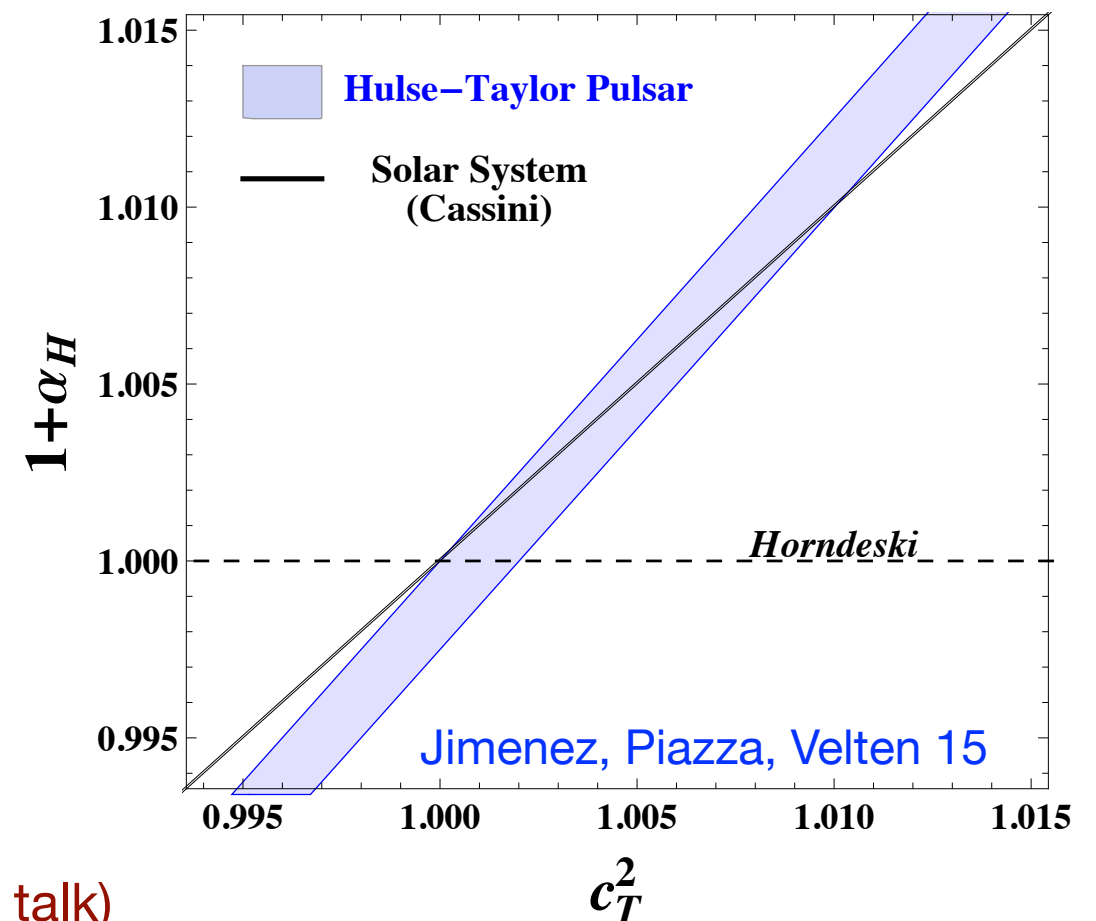
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$$\eta = \frac{1 + \alpha_H}{c_T^2}$$



Constraints from black holes and stars (see Sakstein's talk)

Conclusions

- ❖ Is Λ CDM the ultimate model or simplest approximation given the current precision of data?
- ❖ Scalar-tensor theories are testable candidate. Have extended beyond Horndeski with higher-order degenerate theories
- ❖ Unifying description, including higher-order degenerate scalar-tensor theories (and more). Preserves physical principles (locality, causality, unitarity, stability).
- ❖ Connection with linear observables (Einstein-Boltzmann codes) and GW (partially) worked out